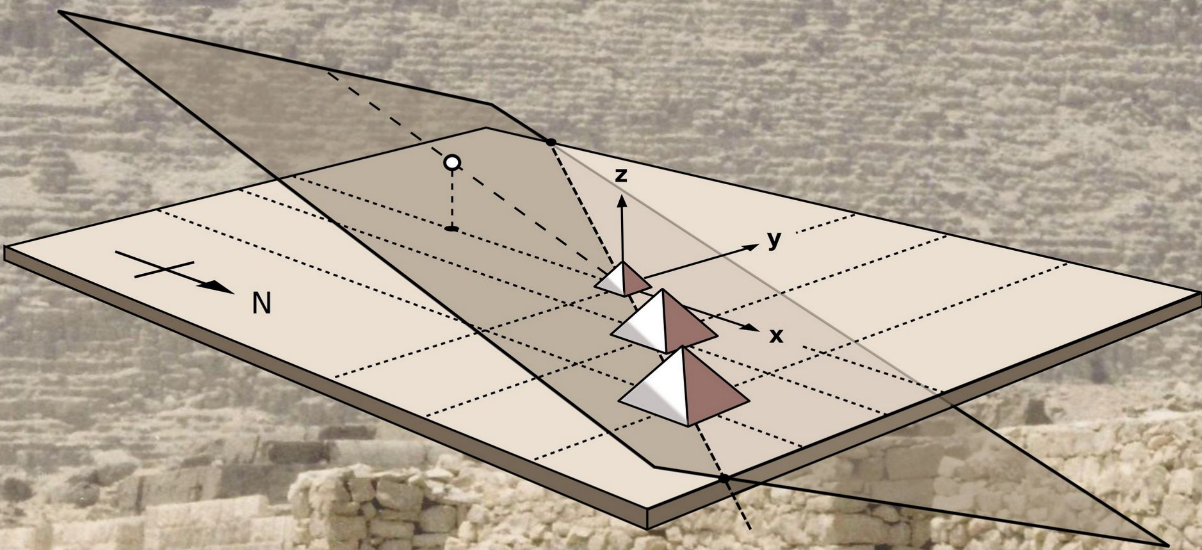
Hans Jelitto

Planetary Correlation of the Pyramids at Giza and Teotihuacán

P5 Program Description



4th extended edition

Hamburg, August 2025

Planetary Correlation of the Pyramids at Giza and Teotihuacán

P5 Program Description

Hans Jelitto

The main editions

Supplementary text P3 program, October 1999: This program was used for the calculations in the book *Pyramiden und Planeten* (1999). The description is only a brief text file (README-V3.TXT). The P3 program package from Sept. 2007, available on the Internet, is slightly updated.

1st Edition P4 program, September 2014: This is the first comprehensive description of both the program and background. Following on from P3, it includes the planet Mars and the chamber system of the Cheops Pyramid.

2nd Edition P4 program, June 2015: Besides minor revisions, the book title was reworded, section 4.7.4 covering transit series was added, the P4 source code was slightly revised, and a parallelized version for the effective use of a multi-core processor was made available.

3rd Edition P5 program, August 2022 (first P5 version: February 2022): The interpretation and priority in the celestial positions within the Cheops Pyramid has been changed; several quick start options concerning the chambers are accordingly adapted. Furthermore, the pyramid area in Teotihuacán is included as a new topic, and section 4.2.5 about the new theory version VSOP2013 has been added. See also the preface to the third edition.

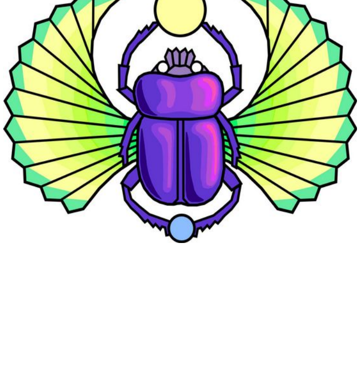
4th Edition P5 program, August 2025 (DOI: 10.13140/RG.2.2.22904.46081/8, first version: April 2024): Section 3.4 of exemplary program outputs is extended. The accuracy of equation (1) is examined in detail (section 4.10.6). With respect to the solar radius, a brief description of the different measurement techniques is given in section 4.10.7. Section 5.2.1 is extended and a possible meaning of the Citadel in Teotihuacán is added (sections 5.2.3 and 5.2.4). Concerning section 4.10.6, the source code of the TOPO program is provided in appendix A2. See also the preface to the current fourth edition.

Hans Jelitto, Ewaldsweg 12, D-20537 Hamburg, Germany

Hamburg, August 30, 2025

This work is dedicated
to my parents
Karl and Käthe Jelitto.

In ancient Egypt, the Scarabaeus was considered to have symbolic and magical properties. The ball of dung that the beetle rolls is likened to the daily motion of the Sun from east to west. Thus, it is also a symbol of reincarnation. As a seal or amulet, it serves as a lucky charm.



Preface to the 2nd edition (P4)

(slightly adapted)

A correlation between the pyramids of Giza and the inner planets of our solar system has been found. This manual is not only a user guide for the P5 computer program regarding this correlation, it also provides some basic information about the technical and theoretical background, including archaeological, mathematical, and astronomical aspects. Further details and several other related results, which are not included here, are presented in the book Pyramiden und Planeten *(in German). A subsequent book (in preparation) will provide more details about the results given here. We have, however, included, as far as possible, all relevant information so that the reader can work properly with the manual and the program. This manual is intended for scientists and for anyone that is interested in the secret of the pyramids.*

For a basic overview of the planetary correlation concerning Giza, it is sufficient to read chapter 1 (introduction), sections 3.1.1–3.1.3, 4.1, 4.6.3, 4.10, and the summary. Related lecture videos by the author on the Internet (with English subtitles) can be found via the website pyramiden-jelitto.de or using the search items "pyramiden planeten jelitto." For the essential concepts of the calculations, chapter 4 provides information on the underlying basics.

Additionally, the appendix A1 contains the entire source code of the program, which is provided mainly for programmers and scientists. When printing the manual, the printout should be in color and double-sided, if the printer supports this feature. In this way, an adequate ring-bound copy can be made.

Preface to the 3rd edition (P5)

Concerning Giza, the extension relates mainly to the comparison between the planetary constellations and the chamber positions in the Great Pyramid. This implies an alternative "Sun position" and a second "Mars position" within the pyramid. The quick start options have been accordingly adapted and the results and text have been revised where necessary. Nevertheless, the astronomical basis of the calculations remains unchanged.

Another planetary correlation has been found with respect to the pyramid area at Teotihuacán in Mexico. This correlation is of a different kind compared to the situation at Giza, but could be easily included in the program because the astronomical calculations can be performed on the basis of the VSOP theory. Besides the new program name P5 and other changes, chapter 5 about Teotihuacán has been added. Information relating to Teotihuacán can be found in the last two paragraphs of chapter 1, sections 3.1.4, 3.3.19–3.3.23, 3.4.9, 3.4.10, chapter 5, and the last paragraphs of the summary.

The program package of the third edition, p5-program-08-2022.zip, no longer comprises the executable 32-bit file. If the user has a 32-bit computer, they can compile the new source code p5.f95 with any Fortran compiler and thus create a 32-bit version. Another possibility is to use the previous P4 program package, since it contains a 32-bit version of the executable file (p4-32). In contrast to the P4 program, P5 has only one source code, p5.f95, which can be used both for creating an executable single-thread and a multi-thread program file. (Since February 2022, the P5 program and this manual have only slightly been revised.)

With regard to the Giza pyramids, any results from the new options in the third edition can also be obtained with the second edition (P4). If a certain quick start option does not exist in the second edition, the input parameters must be defined individually at program start, beginning with the option "0." Because of the change in format, the user should not mix the files of the second and third editions.

Remark to the Internet links

Internet addresses can sometimes change years after the date of publication, and, in principle, links could become a safety concern. Therefore, most of the active hyperlinks are replaced by a given URL + number, e.g., (URL 20), and the corresponding Internet address (URL) is listed as plain text at the end of this document – see section "Internet addresses."

Preface to the 4th edition (P5)

The description of the planetary correlations is generally extended. Section 3.4 of the program outputs is broadened, whereas the P5 program itself (appendix A1) is only slightly adapted. The previous program package p5-program-08-2022.zip is replaced by the new one: p5-program-08-2025.zip. The main equation (1) defining the size of the Cheops Pyramid is analyzed in detail. In order to obtain the required volume of Earth including all ice and land masses, the volume is calculated on the basis of the topographic data of Worldbath (section 4.10.6). The source code of the corresponding program, TOPO, is provided in appendix A2 and the program package, topo-program-08-2025.zip, can be downloaded from the website pyramiden-jelitto.de. In this context, the different measurements of the solar radius are also briefly discussed (section 4.10.7). For Teotihuacán, a time scan of the astronomical correlation is added in section 5.2.1. Additionally, a tentative and hypothetical interpretation of the Citadel is provided in section 5.2.3, and the section 5.2.4 concerning the TNO Sedna is added. From September 2024 to August 2025, this edition got some slight modifications and extensions again; the title remained unchanged.

Contents

Appendix A1 – P5 Source Code 126


Giza, Egypt


Teotihuacán, Mexico

1. Introduction

The original purpose of the P5 program was to perform astronomical calculations with respect to the planets of our solar system and the three pyramids of Giza (Fig. 1). Surprisingly, a similar correlation has been found for the pyramids in Teotihuacán (Mexico). This is described mainly in chapter 5. The P5 program (download: URL 1 – see the list of Internet addresses at the end of this document preceding the references) is based on the French planetary theory VSOP87 [1, 2]. The fundamental idea is that a correlation exists between the three inner planets and the three pyramids of Giza. The first papers on this hypothesis were published in the Austrian journal *Grenzgebiete der Wissenschaft* (in German) in 1995 [3, 4]. The development of the program P3 began about two years earlier, allowing for the mathematical comparison of pyramid positions with planetary positions. Because of three equations that define the size of each pyramid, it seems that the Cheops Pyramid, the Chefren Pyramid, and the Mykerinos Pyramid represent the planets Earth, Venus, and Mercury, respectively. Furthermore, the pyramid positions correlate with the planetary positions. Because the planets are moving all the time, their arrangement and distances between each other change continuously. This implies that the geometric arrangements of pyramids and planets match for only one or a few points of time. These dates were found, and depend on the mathematical approach and further boundary conditions. Thus, among other things, the program calculates the dates when Mercury, Venus, and Earth stand in a constellation according to the arrangement of the Giza pyramids (see Fig. 1 [5, p. 95]). The data in Fig. 1 were measured by Sir W. M. F. Petrie [6]. Excellent geographical maps reproducing the pyramids in Egypt are available, e.g., in Cairo [7, 8].

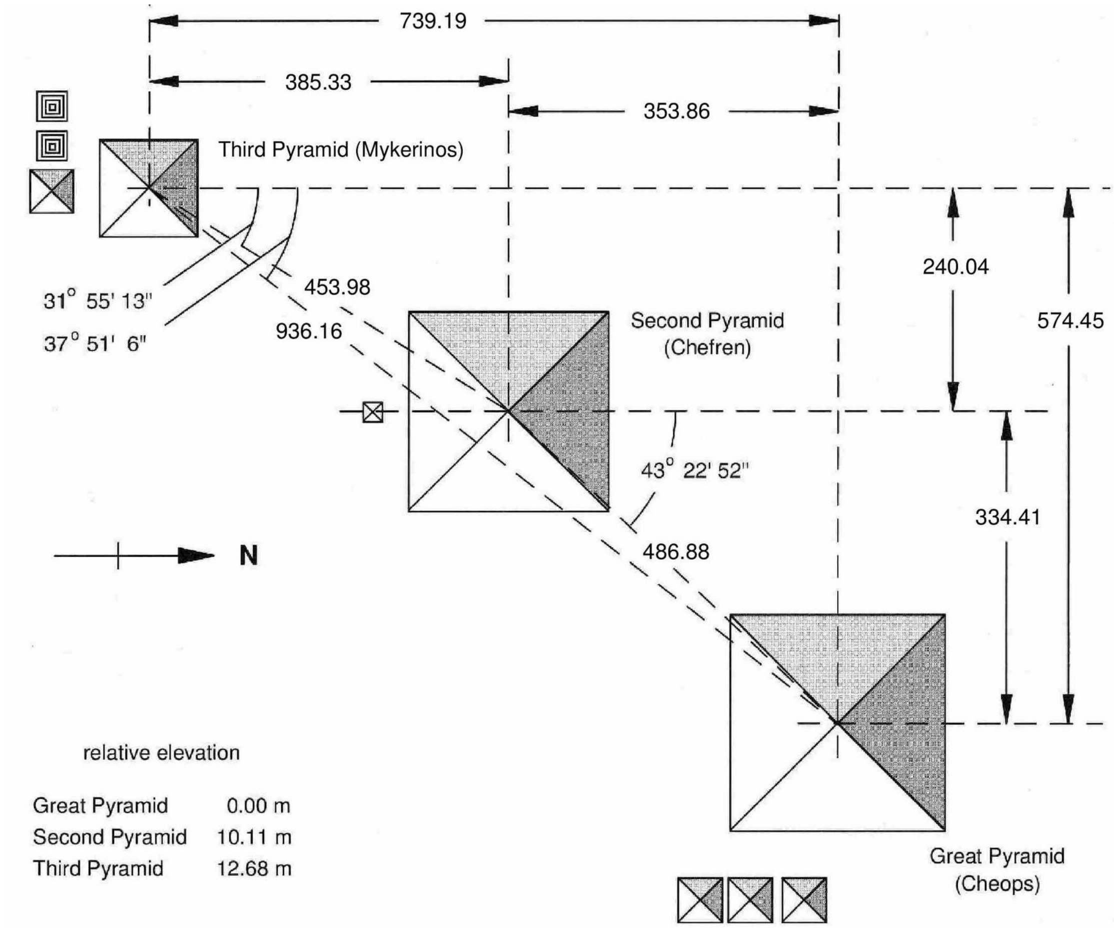


Figure 1: *Alignment of the Giza pyramids with data measured by Sir W. M. F. Petrie [6] (distances in m). Two numbers have been slightly corrected: The long diagonal is 936.16 m rather than 936.19 m, and one angle is 31° 55' 13" rather than 34° 10' 11" (explanation in [5, p. 96; 6, p. 125]). The relative elevations stem from J. S. Perring (see: [9, part IV, map 1]). Detailed information is provided in the drawings of Maragioglio and Rinaldi [9]. The angles were calculated from the original distances, given in inches (1 inch = 2.54 cm).*

The archaeological state of knowledge is that the three great pyramids of Giza were built by the Egyptian pharaohs Chufu, Chaefre, and Menkaure in the 4th dynasty. In addition to these Egyptian names, the Greek names for these are Cheops, Chefren, and Mykerinos. In archaeological chronology, the 4th dynasty is dated roughly between the years 2600 and 2480 BC [10, vol. I, p. 970]. (BC = before Christ.) On the other hand, in 1987 and 1994 it was reported that the age of several buildings of the Old Kingdom, including the pyramids of Giza, was determined independently using accelerator mass spectrometry (AMS) [11, 12]. This project was organized by the ETH Zürich in Switzerland. AMS is a modern variant of radiocarbon dating in which a particle accelerator is used to determine the amount of radioactive ^{14}C isotopes. The result is that, for example, the Cheops Pyramid must be dated between the years 3030 and 2905 BC with a probability of 95 %. This is a discrepancy of approximately 400 years! Because it is impossible to shift the chronology of the pharaohs by 400 years, the reader should keep this point in mind (see details in [5, pp. 361 ff.]).

The first program version was named P3 because of the 3 large pyramids at Giza and the 3 planets Mercury, Venus, and Earth. It was used for computing the astronomical tables in the book *Pyramiden und Planeten* [5]. After this book was published in 1999, another correlation was found, namely between the planetary positions and the chamber positions in the Great Pyramid – with an unexpected connection between both correlations. This led to an extension of the program P3 with several other options. The next program name was P4 [13], which was an upgrade of P3 and includes the fourth planet Mars. The processing speed was optimized and the application was made much easier. Now, P5 – considerably extended – covers all the features of P4. The results, which cannot all be provided in this manual, are described in detail in the subsequent book [14]. Because, until now, most of the publications are in German, this description is written in English.

The comparison of the arrangements is performed mathematically by a coordinate transformation. An interesting point of this correlation is that, by using the transformation of the planetary arrangement, the position of the Sun can be precisely transferred to the pyramid area (Fig. 2). This means that we have a "Sun position" at the Giza plateau. Furthermore, the positions of the chambers define another "Sun position" inside the Cheops Pyramid. In this work, "Sun position" is written in quotation marks because here we do not refer to the real Sun but to the corresponding position in the pyramid area. We will later also find a "Mars position" in the Cheops Pyramid.

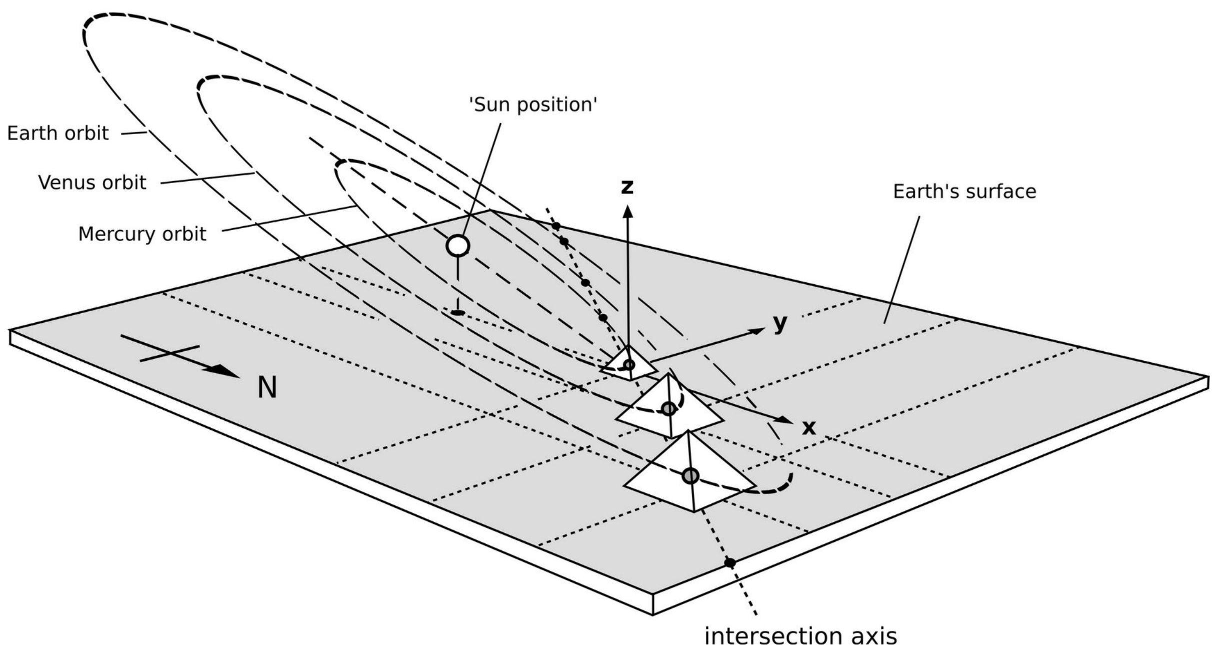


Figure 2: *Schematic representation of the Earth's surface around the Giza pyramids and the orbits of the three inner planets, Mercury, Venus, and Earth, after adapting the pyramid and planetary positions. The geometric arrangement of constellation number 12 (section 3.4.3) looks very similar. Due to different inclinations, the orbits are slightly tilted against each other. This fact is neglected in the drawing but is taken into account in the calculations.*

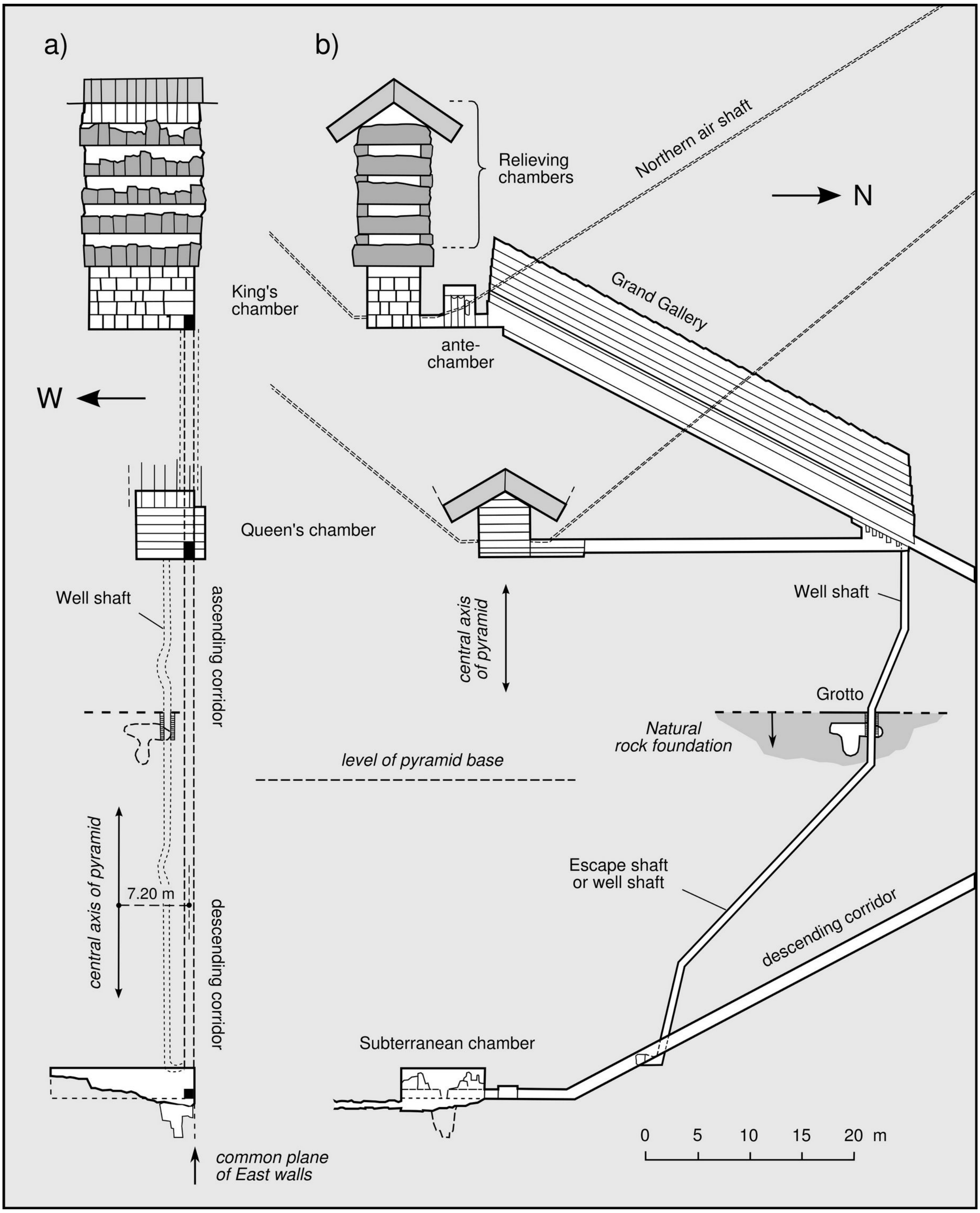


***Figure 3:** Chamber and corridor system inside the Great Pyramid, as seen from the south **(a)** and from the east **(b)**. The figure is based on drawings of Maragioglio and Rinaldi [9, part IV, maps 3–7] (arrangement of (a) and (b) as in R. Stadelmann [15]).*

In Fig. 3, the remarkable system of chambers and corridors in the Cheops Pyramid is given, which also plays a major role in the correlation between pyramids and planets. The names of the chambers, such as "King's chamber" and "Queen's chamber," originate from classical archaeology and are based on the explanation that the pyramids were tombs of the pharaohs. At first glance, this explanation seems reasonable because it is written in countless books. However, this interpretation is not necessarily correct because a mummy has never been found in an Egyptian pyramid! Numerous mummies of kings and queens have been discovered, but all of them were found in hidden tombs in the desert, as in the Valley of the Kings. More about this is provided in [5].

Some of the boundary conditions for the comparison of pyramid and planetary positions are as follows: The “Sun position” can be fixed by placing the “Sun” on the Earth’s surface exactly on the center line 726 m south of the Mykerinos Pyramid (see Fig. 5). The “Sun position” can also be free in the two horizontal coordinates and would be fixed only to the Earth’s surface by adapting the planetary positions, or it can be free in all three dimensions. In order to get a better idea, an example of the two systems, “pyramids” and “planets,” using a 3-dimensional fit, is shown in Fig. 2. The two planes, Earth’s surface and ecliptic plane (plane of the Earth’s orbit), are not coplanar but tilted against each other. More about this is given in section 4.10.2 and by Eq. (45) in section 4.7.3.

Furthermore, the P5 program computes the dates of the linear constellations of the celestial bodies Mercury, Venus, Earth, Mars, and the Sun, which means that the planets have nearly the same ecliptic longitude. These linear arrangements of the celestial bodies (conjunction and opposition) are called also syzygies (singular: syzygy). In addition, the exact geocentric transit phases, when Mercury or Venus passes the Sun's disk, can be determined. All options and parameters of the P5 program are provided in chapter 3.

The calculations were performed as accurately as possible. Strong emphasis was put on the use of the most recent and precise scientific data. This refers to the astronomical data and computations as well as to the archaeological data. Concerning the exact dimensions of the Cheops Pyramid, the latest results were not always the most accurate. The reason is that, due to weathering effects, the measurement conditions at the pyramid about 100 years ago were partly better than today (this is discussed in detail in [5, pp. 249–255]). Although the corresponding small differences are important – not for the size of the Cheops Pyramid, but rather for its exact shape – they do not have any effect on the results in this manual.

An additional topic, which was not originally intended, is Teotihuacán in Mexico. On the occasion of a conference in Mexico, the pyramid site of Teotihuacán (Fig. 4) was visited and a very interesting planetary correlation was found, being completely different to the correlation at Giza.

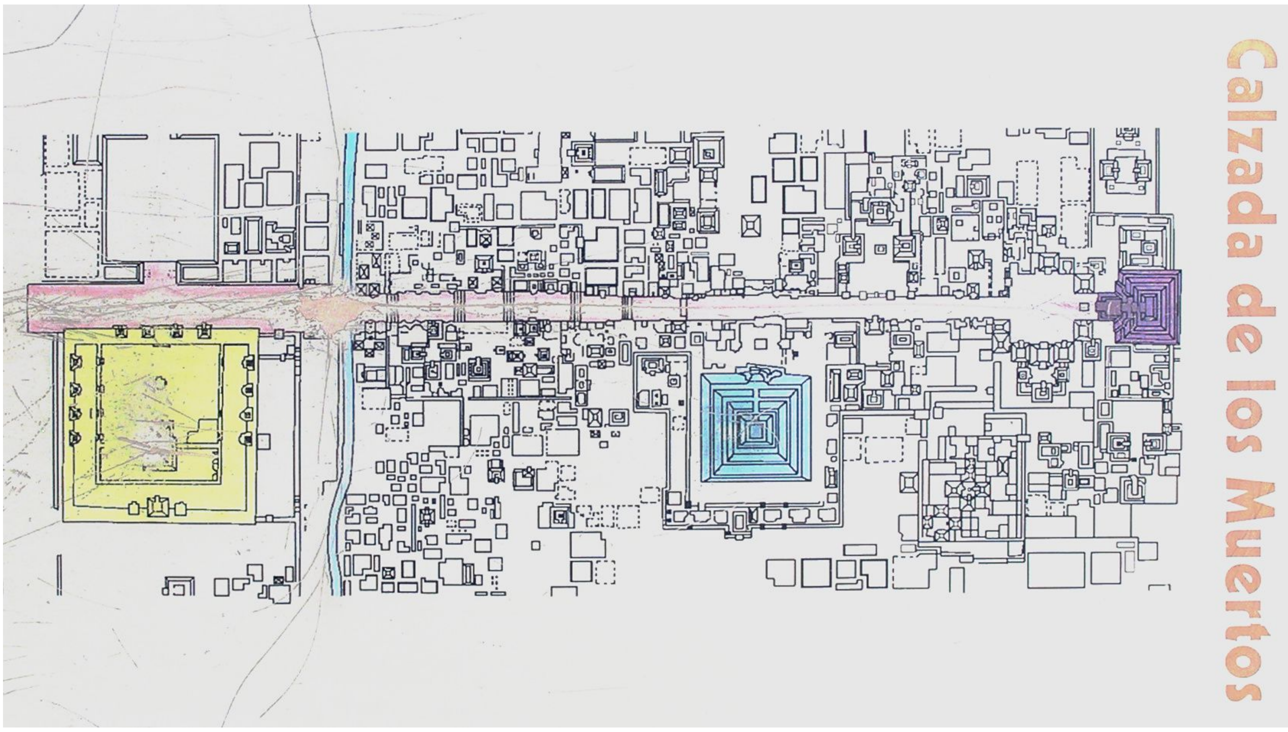


Figure 4: *Pyramid site in Teotihuacán. This photo, taken at the pyramid site of Teotihuacán in 2005, is a poster providing information for tourists that visit the archaeological area. Here, the drawing has been rotated by 90°. (SECRETARIA DE CULTURA.-INAH.-MEX. Reproduction Authorized by the Instituto Nacional de Antropología e Historia – see page 187.)*

The calculations concerning Teotihuacán are not detailed in a separate program but are included in the P5 program. The reason for this is that the astronomical computations can also be performed using the VSOP theory and the corresponding subroutines were already implemented in P5. However, in chapter 5, it will become clear that this correlation can also be checked simply by a spreadsheet analysis (MS Excel or LibreOffice Calc). Almost all required numerical data provided in section 5.1.3 are also available on the Internet.

2. General technical information

The astronomical calculations are based on the planetary theory VSOP87 [1, 2], developed by P. Bretagnon and G. Francou at the Bureau des Longitudes, Paris, today the IMCCE, Institut de mécanique céleste et de calcul des éphémérides (URL 2). VSOP stands for Variations Séculaires des Orbites Planétaires and 87 for the year of publication (1987). The files for the VSOP87 theory can be downloaded from the FTP server of the IMCCE website (URL 3). Meanwhile, different new and similar versions have been published, e.g., VSOP2013, TOP13, INPOP19a, and INPOP21a, which are more precise for the near future and past, i.e., 4000 to 6000 years forwards and backwards from the present time. Nevertheless, for our purposes, VSOP87 is completely sufficient, and, especially for longer time spans, VSOP87 seems to be even more appropriate compared to the new versions. This is discussed in detail in section 4.2.5.

A multi-parameter fit program FITEX (fit experiment) [16, 17], included in P5, was developed by G. W. Schweimer, Zyklotron-Laboratorium, KfK (Kernforschungszentrum Karlsruhe, today the KIT, Karlsruhe Institute of Technology, URL 4). For calculating calendar dates, an algorithm from the book *Astronomical Algorithms* by J. Meeus [18] is converted into a subroutine of P5. The conversion of Terrestrial Time (TT) to Universal Time (UT) is performed using $\Delta T = TT - UT$, calculated by F. Espenak and J. Meeus (NASA Eclipse Web Site, URL 5, Polynomial Expressions for Delta-T, URL 6). The P5 program, all subroutines, and other programs from the author are written in Fortran. The whole package of programs, and all associated files, can be downloaded from the author's website (URL 1). Note: Through the links and URLs provided, many of the given references can also be downloaded elsewhere from the Internet. For details of the theoretical basis, see chapters 4 and 5.

The previous program P3 was originally developed with the IBM Professional Fortran 77 compiler (Version 1.0, Ryan-McFarland) using the SPF/PC editor and the Windows operating system. We later switched to the GNU Fortran compiler g77 with Ubuntu Linux, and then to (GNU) GFortran. It is of course possible to use other Fortran compilers and other operating systems.

It would also be of interest to port the program to languages such as C, C++, Java, or Python. However, because the architectures of these programming languages are quite different to Fortran, it is probably easier to write a new program code. In addition, it would be a good test of the results if the calculations were performed independently and based, for example, on a theory other than VSOP87. (In this case, several options in P5 used for test purposes can be omitted.)

2.1 Data files and other related programs

Table 1 contains a compilation of all files belonging to the astronomical program P5. A few comments about other available programs and more information about the files in Table 1 are provided in the following text, where program, catalogue, and file names are highlighted in blue. Reading README-P5 in the program package is also recommended.

Table 1: *All 36 files of the P5 program: program, text, and data files (4th Ed., download: author's website (URL 1)).*

File	Brief description
p5.f95	Fortran source code
p5-64	Executable singe-thread program file for a 64-bit system
p5-64-m	Executable multi-thread program file for a 64-bit system
p5-64.sh	Shell-script, clears monitor screen and starts p5-64. (If required, a 32-bit version can be created by compiling p5.f95 on a 32-bit computer.)
p5-64-m.sh	Shell-script, clears monitor screen and starts p5-64-m.
p5-manual-08-2025.pdf	User manual with details of the P5 program and main aspects of the planetary correlation concerning the pyramids of Giza and Teotihuacán (this book)
README-P5	Some general information concerning this program package
README-vsop87	Notice to the theory planetary solutions VSOP87 from Bretagnon and Francou
vsop87.doc	Technical information about the VSOP87 theory from Bretagnon and Francou
out.txt	Output file (If it does not exist, it will be created by the program.)
inedit.t	Ancillary input file (can be used to create a new set of parameters for inparm.t)
inparm.t	This file contains all input parameters for the quick start options 1 to 22, for most of the tables 39–51 in [5], and for most of the tables 17–38.B in [14].
ingiza.t	Parameters for FITEX and coordinates of pyramid and chamber positions at Giza
inserie.t	Dates of transit series for Mercury and Venus (used only at program start)
invsop1.t	Shortened VSOP87D data for the planets Mercury to Mars, typewritten manually from: J. Meeus, *Astronomical Algorithms* [18, pp. 381 ff.]
invsop3.t	Polynomial representation of orbital elements, derived from VSOP82 and taken from: J. Meeus, *Astronomical Algorithms* [18, pp. 200 ff.]
inteoti.t	Different data (e.g., GPS) for the positions in Teotihuacán
DATUM-2.f95	Additional calendar program, input: decimal year or JDE, source code
DATUM-2	Executable file of the calendar program (not needed for P5)
VSOP87A.mer	Mercury: VSOP87A, heliocentric rectangular coordinates, ecliptic J2000.0
VSOP87A.ven	Venus: . . . " . . .
VSOP87A.ear	Earth: . . . " . . .
VSOP87A.mar	Mars: . . . " . . .
VSOP87A.jup	Jupiter: . . . " . . .
VSOP87A.sat	Saturn: . . . " . . .
VSOP87A.ura	Uranus: . . . " . . .
VSOP87A.nep	Neptune: . . . " . . .
VSOP87A.emb	Earth-Moon barycenter: . . . " . . .
VSOP87C.mer	Mercury: VSOP87C, heliocentric rectangular coordinates, dynamical equinox
VSOP87C.ven	Venus: . . . " . . .
VSOP87C.ear	Earth: . . . " . . .
VSOP87C.mar	Mars: . . . " . . .
VSOP87C.jup	Jupiter: . . . " . . .
VSOP87C.sat	Saturn: . . . " . . .
VSOP87C.ura	Uranus: . . . " . . .
VSOP87C.nep	Neptune: . . . " . . .

The files README-vsop87 and vsop87.doc in Table 1 provide details about the theory versions of VSOP87 and are given directly by the authors Bretagnon and Francou (as a download from the IMCCE website). The file out.txt contains the results after running P5 if the output parameter is not set otherwise. The next seven files in Table 1, beginning with “in...” are the input files necessary to run P5. All parameter sets for the quick start options are compiled in the file inparm.t. File inedit.t is a combined input-output file. During each run, all input parameters are stored at the end of this file. This ancillary file helps to create new parameter sets, which can be added as new quick start options to the file inparm.t. In this case, the subroutine inputdata in p5.f95 has to be properly adapted. The input parameters in the file inedit.t can also be edited manually and are adopted by the program with the quick start option 999. This allows for testing new parameter sets. The file ingiza.t contains parameters for the subprogram FITEX as well as the exact coordinates of the pyramid chambers and pyramid positions. In inserie.t, several dates (JDE) are listed to determine the serial numbers of the first Mercury or Venus transits found after program start. The shortened parameter series of the VSOP87D version are in invsop1.t, taken from [18, pp. 381 ff.]. The file invsop3.t contains coefficients for polynomials of the third degree for the elements of planetary orbits deduced from VSOP82 [18, pp. 200 ff.]. For Teotihuacán, the necessary geographical data are provided in inteoti.t. All remaining files from VSOP87A.mer to VSOP87C.nep represent full versions of the planetary theory [1, 2] with a very high accuracy. They are also available from the FTP server of the IMCCE website.

This paragraph provides some information about the other programs used in the current pyramid research; the P5 and TOPO programs are new and are used here and in [14]. TOPO calculates the exact volume of the Earth, including the volume of all ice and land masses. All other programs, including P3, are used and described in [5]. This includes the programs FORM-2, SEKAN-2, PYT-2, and 7916-2, which enable geometric calculations concerning the shapes of the three pyramids of Giza, especially their casing angles. The program DATUM-2 converts the time system Julian Ephemeris Day (JDE)[1] into a calendar date and is based on an algorithm from the book by Jean Meeus [18, p. 63]. The program SKYGLOBE [19] is a “planetarium” simulation of the sky and shows the celestial bodies, like stars, planets, Sun, and Moon, as well as the Milky Way and constellations for every date and location on Earth. It was written by Mark A. Haney as shareware (URL 7). In this project, it has been applied only to check the ORION correlation propounded by R. Bauval and A. Gilbert [20]. When the positions and proper motions of the corresponding stars were taken into account for a quantitative analysis, large errors and deviations were found. On the one hand, Bauval and Gilbert were the first to correlate the pyramids with celestial bodies. On the other hand, their ORION hypothesis did not pass the test [5, pp. 157 ff., 349 ff.]. The analysis in [5] is based on the PPM Star Catalogue (Positions and Proper Motions) [21, 22]. Meanwhile, a new powerful and free-software planetarium simulation, named Stellarium [23] (URL 8), has become available.

For those who are interested, the current text, formulas, and most figures, including the book cover, were created using Ubuntu with OpenOffice (now LibreOffice), Inkscape, and GIMP.

2.2 How to start the program

The P5 program does not require any installation. After downloading and unpacking the files (p5-program-08-2025.zip), the easiest way is to store all of them in the same folder (directory), which could be named “P5,” for example. It is assumed that the operating system is Linux because the P5 program was developed on the Linux distribution Ubuntu. If another operating system is installed, it is normally necessary to compile the source code p5.f95 again. In the case of a Windows system, p5.f95 can be compiled with a Windows compliant Fortran 95 compiler (e.g., ifort or GFortran with MinGW or Cygwin). Other possibilities are to create a Linux partition beside Windows, to use a Linux life CD like Knoppix, or to apply VirtualBox. Special characters are not used in the program output, so for character encoding, the Unicode UTF-8 or ISO 8859-15 can be applied.

1 In order to be consistent with [5] and with the notation of Meeus [18], JDE (Julian Ephemeris Day or Julian Day) was used, based on Terrestrial Time (TT). Today, JD and JD(TT) have the same meaning.

After creating the folder, we open a terminal in Ubuntu, e.g., with the right mouse button and *Open in Terminal*. (In most cases, a terminal window width of 80 characters is sufficient; only a few options need a line length of 148 characters – see section 3.3.5.) In the following, all texts on the monitor screen, e.g., commands, menus, input data, and program results, are printed in blue using a monospaced font (not program names or file names). If, for instance, the folder has the name "P5" and is located in the path `~/Desktop/P5$`, we type the following command at the command line in the terminal: `cd Desktop/P5` ↵. Now we are in the correct folder. The sign ↵ denotes the return key. To start the program on a 64-bit computer system, we type `./P5-64.sh` ↵ or `./P5-64-m.sh` ↵, which clears the screen, and the start menu appears. The command with the letter "m" should be used for multi-thread processors. Another possibility is to start P5 directly with `./P5-64` ↵ or with `./P5-64-m` ↵ without clearing the screen. If the program does not start, type `chmod +x P5-64*` ↵. In the case of a new compilation of the source code with GNU Fortran, use the command `gfortran -static -O3 -Wall p5.f95` ↵. For a multiple core application, use `gfortran -fopenmp -static-libgfortran -O3 -Wall p5.f95` ↵. For a 32-bit system and without compilation, also the corresponding executable files of the previous version P4 can be used. And that's it! In the next chapter we will see how to proceed.

3. Program features

After typing the start command, the main menu appears on the monitor:

```
                       -----------------------
                        PLANETARY CORRELATION
                        P5 Program, Aug. 2025
                       -----------------------

   Giza pyramids       Great P. chambers      transits syzygy     Teotihuacan
 -------------------------------------------------------------------------------
  3D Mer at aph (1)    3D Mer at per (6)     Mercury tr  (11)    GPS m km  (16)
  2D Mer at aph (2)    Keplers equ    (7)    Venus tr    (12)    Map mm km (17)
  constell 3088 (3)    constell 3088 (8)     syzygy 3 pl (13)    GPS log3  (18)
  1.5 days 3088 (4)    1.5 days 3088 (9)     syzygy 4 pl (14)    Map log3  (19)
  near aphelion (5)    F minimized   (10)    TYMT test   (15)    24000 y.  (20)
 -------------------------------------------------------------------------------
        info (111)   detailed options (0)   (1..20 or book options) : _
```

The date in the title indicates the most recent update of the program. In the main table there are four different categories. The first options, 1 to 5, belong to the pyramids of Giza, the options 6 to 10 concern the chambers in the Great Pyramid, the options 11 to 15 represent planetary conjunctions, and the options 16 to 20 refer to Teotihuacán. The third category includes different astronomical events: The three or four inner planets of our solar system stand in conjunction (syzygy). Additionally, if Mercury or Venus are in conjunction with the Sun, it sometimes happens that they pass in front of the solar disc, which is called a transit. To avoid confusion, the astronomical relationships are explained in more detail in the following sections.

3.1 Quick start options 1–22

Normally, about 10 to 15 different parameters have to be specified before the astronomical calculation starts. These parameters determine, for example, the kind of astronomical event, the VSOP version used, the coordinate system, the mode of calculation for the "Sun position" in Giza, the time period to be examined, the complexity of the output, and so on. In order to avoid this, the general quick start options 1 to 22 start the program with predefined parameters immediately after typing a short number. (The options 21 and 22 are not displayed in the main menu.) For example, typing 12 ↵ makes the program calculate all Venus transits for the years from AD 1500 to AD 4000. (AD = *anno Domini* or after Christ.) The program output is:

```
                              TRANSITS OF VENUS
               (geocentric transit phases, terrestrial time TT)
                              < P5-option  12 >

 VSOP87C, comb. search,     ecliptic of date,               all Venus transits
      Period   (years) from   1500.00  to   4000.00,  Jul./Greg. calendar

co/p    date/  time:   I        II      nearest     III        IV    sep["]a  S
===============================================================================
    26. May  1518  22:32: 2  22:49:14   1:59:45   5:10:16   5:27:28 -505.3   3
    23. May  1526  16:17:38  16:38: 9  19:14:43  21:51:18  22:11:49  666.7   5
---- (Greg. cal.) -------------------------------------------------------------
  v  7. Dec. 1631   3:53:17   5: 2:16   5:20:49   5:39:22   6:48:20  939.3/  6
     4. Dec. 1639  14:58: 4  15:16:26  18:26:47  21:37: 8  21:55:29 -523.6/  4
     6. June 1761   2: 2:20   2:20:35   5:19:30   8:18:25   8:36:40 -570.4   3
     3. June 1769  19:15:49  19:34:52  22:25:36   1:16:20   1:35:23  609.3   5
     9. Dec. 1874   1:49:12   2:18:56   4: 7:22   5:55:49   6:25:33  829.9/  6
     6. Dec. 1882  13:56:41  14:17:10  17: 5:54  19:54:38  20:15: 7 -637.3/  4
     8. June 2004   5:14:47   5:34:13   8:20:49  11: 7:24  11:26:51 -626.9   3
     6. June 2012  22:10:56  22:28:53   1:30:43   4:32:33   4:50:30  554.4   5
    11. Dec. 2117   0: 2:31   0:25:39   2:52: 8   5:18:38   5:41:46  723.6/  6
     8. Dec. 2125  13:19:29  13:43:10  16: 5:49  18:28:28  18:52: 8 -736.4/  4
    11. June 2247   8:51:10   9:12:31  11:42:27  14:12:24  14:33:45 -691.3   3
     9. June 2255   1:17:39   1:34:41   4:47:36   8: 0:31   8:17:33  491.9   5
    13. Dec. 2360  22:47:17  23: 7:30   1:58:44   4:49:57   5:10:10  625.7/  6
    10. Dec. 2368  12:44:56  13:15:24  15: 0:28  16:45:33  17:16: 0 -836.4/  4
    12. June 2490  12: 1:48  12:25:17  14:39:42  16:54: 7  17:17:36 -741.1   3
    10. June 2498   4:12: 4   4:28:32   7:48:35  11: 8:38  11:25: 6  442.7   5
    16. Dec. 2603  21:14:54  21:33: 8   0:44:29   3:55:49   4:14: 3  517.1/  6
  v 13. Dec. 2611  12:36:50  13:40:18  14: 6: 9  14:31:59  15:35:27 -934.8/  4
    15. June 2733  15:45: 8  16:13:18  18: 0:59  19:48:39  20:16:49 -808.3   3
    13. June 2741   7:17: 8   7:33: 5  11: 0:24  14:27:43  14:43:40  385.6   5
    17. Dec. 2846  20:24:29  20:41:44   0: 5:13   3:28:41   3:45:55  432.1/  6
  v 14. Dec. 2854     --        --     13:14:26     --        --   -1026.7/  4
    16. June 2976  18:54: 5  19:27:43  20:53: 7  22:18:32  22:52: 9 -850.5   3
    14. June 2984  10:10:33  10:26: 9  13:58:46  17:31:23  17:46:59  336.3   5
->  18. Dec. 3089  19: 1:49  19:18:10  22:53:36   2:29: 2   2:45:23  320.6/  6
  v 20. June 3219  22:31:18  23:28: 6   0: 0: 6   0:32: 6   1:28:55 -908.1   3
    17. June 3227  13: 3:37  13:18:56  16:55:19  20:31:43  20:47: 2  293.4   5
    20. Dec. 3332  18:14:30  18:30:23  22:12: 4   1:53:44   2: 9:38  235.5/  6
  v 22. June 3462   1:48:43     --      2:46:31     --      3:44:19 -948.1   3
    19. June 3470  15:51:28  16: 6:35  19:46:41  23:26:48  23:41:55  247.9   5
    23. Dec. 3575  17: 7:58  17:23:32  21:10:32   0:57:31   1:13: 5  131.5/  6
  v 24. June 3705     --        --      5:35:19     --        --    -989.3   3
    21. June 3713  18:30:27  18:45:25  22:27:21   2: 9:18   2:24:17  215.2   5
  c 25. Dec. 3818  16:23: 6  16:38:31  20:27:15   0:15:58   0:31:22   41.1/  6
    24. June 3956  21:17:37  21:32:30   1:16:53   5: 1:17   5:16:10  175.2   5
===============================================================================
 Computed constellations:      11183               ("/" means ascending node)
 Tested planet. passages:       1564
 Detected transits      :         37
 Centr./grazing transits:     1 /  6      CPU time  0: 0: 0.855
                                          run time  0: 0: 0.240 -- end of run.
```

The general appearance of the printed output is always similar. The first two lines show the title of the program run and possibly some additional information. In the third line we find the number of the selected option, which is often a quick start option. In most cases, two, and up to five, lines follow, providing the remaining information in a brief form so that it is later possible to understand what has been calculated. These two to five lines include the following data: the theory version of VSOP, the astronomical coordinate system, some data about the planets, pyramids, or chambers, the time period, the allowed angular range (e.g., the range of the ecliptic longitudes), and other information. The output is shown in a different way only for the Teotihuacán options.

In most cases, there are two kinds of output of different extent. At first, each astronomical event, like a transit, is written down in a single line as provided in the previous table. This kind of output is useful for an overview when large time periods are investigated and when many planetary constellations are found. Another possibility is to characterize every astronomical event with much more information in several lines. At the end of the output, one or more lines give a summary of the program run. This includes, for example, the number of calculated and detected astronomical events as well as the CPU time and the run time in hh:mm:ss.sss. More information about these different kinds of output is provided in section 3.4.

Next, an example of an extended output for each found constellation is given. In order to illustrate this and avoid an output that is too long, the time limits (AD 1000 to 3500) are chosen in such a way that only five planetary constellations are found. In this program run, the main condition is that the Sun and the four planets Mercury, Venus, Earth, and Mars stand in a straight line. This means that the planets have almost the same ecliptic longitude (*L*), which is called a conjunction or syzygy. The calculation is performed by iteratively solving Kepler's equation. The first line of numbers in the table contains the information that is also shown in a short program output. The additional lines provide the corresponding orbital elements of the eight planets. For more details, see sections 3.3 and 3.4. (The fourth conjunction – constellation 12 – seems to be an important event with respect to the Giza pyramids.)

```
                      PLANETS IN A LINE (SYZYGY)
        (angular range of eclipt. longitudes dL minimized, JDE)
                          < P5-option   0 >

 "Keplers equation",        ecliptic of date,        linear c. Mercury to Mars
 Period   (years)   1000.00  to   3500.00 (c2)      angular range:  4.0000 deg

co       k         JDE          year   dt[days]  Lm-Lv    Lm-Le  Lm-Lma    dLmin
================================================================================
       -1926  2282079.83512   1535.994 -37.470   0.452    2.431   0.0      2.431
--------------------------------------------------------------------------------
pla.   mean long.   a [AU]   eccentr.  asc.node    incl.   per.[°]   per.[AU]
--------------------------------------------------------------------------------
Mer     96.90363   0.38710   0.20554   42.83109  6.99614   70.24089   0.30754
Ven    110.19773   0.72333   0.00700   72.50776  3.38999  125.03525   0.71827
Ear    107.58062   1.00000   0.01690     ---     0.00000   94.96910   0.98310
Mar    103.64670   1.52368   0.09298   45.97593  1.85279  327.52126   1.38201
Jup    346.81208   5.20260   0.04773   95.73609  1.32887    6.87167   4.95430
Sat    133.36050   9.55492   0.05710  109.59369  2.50588   83.96302   9.00930
Ura    119.61985  19.21845   0.04642   71.61507  0.77042  166.11338  18.32624
Nep      4.16199  30.11039   0.00896  126.67581  1.81298   41.51439  29.84065
================================================================================
         476  2493450.15416   2114.732  30.471   1.304   -0.459    1.304    1.764
--------------------------------------------------------------------------------
pla.   mean long.   a [AU]   eccentr.  asc.node    incl.   per.[°]   per.[AU]
--------------------------------------------------------------------------------
Mer     23.94304   0.38710   0.20566   49.69204  7.00705   79.24226   0.30749
Ven      1.10802   0.72333   0.00672   77.71431  3.39581  133.17105   0.71847
Ear      4.17142   1.00000   0.01666     ---     0.00000  104.91077   0.98334
Mar    356.71340   1.52368   0.09350   50.44394  1.84905  338.17263   1.38121
```

```
Jup    277.90377   5.20260   0.04868  101.63631  1.29697   16.18287   4.94933
Sat     13.81307   9.55491   0.05511  114.67166  2.48457   95.31093   9.02834
Ura     87.23869  19.21845   0.04626   74.60563  0.77413  174.71076  18.32931
Nep    196.62132  30.11039   0.00900  133.04896  1.75926   49.76055  29.83953
===============================================================================
         795  2521489.31037   2191.501   7.405  -3.878   0.0    -1.506   3.878
-------------------------------------------------------------------------------
pla.   mean long.   a [AU]   eccentr.  asc.node    incl.   per.[°]   per.[AU]
-------------------------------------------------------------------------------
Mer    290.74352   0.38710   0.20567   50.60306  7.00841   80.43781   0.30748
Ven    284.56818   0.72333   0.00668   78.40703  3.39658  134.24494   0.71850
Ear    280.89243   1.00000   0.01663     ---     0.00000  106.23188   0.98337
Mar    291.22522   1.52368   0.09357   51.03670  1.84862  339.58626   1.38110
Jup     88.78364   5.20260   0.04881  102.42102  1.29276   17.42327   4.94869
Sat    233.06731   9.55491   0.05484  115.34468  2.48167   96.82045   9.03089
Ura     57.23326  19.21845   0.04624   75.00893  0.77482  175.85232  18.32971
Nep      5.41993  30.11039   0.00900  133.89570  1.75210   50.85634  29.83938
===============================================================================
12      4519  2849066.03400   3088.376 -13.729  -3.366  -2.601   0.0     3.366
-------------------------------------------------------------------------------
pla.   mean long.   a [AU]   eccentr.  asc.node    incl.   per.[°]   per.[AU]
-------------------------------------------------------------------------------
Mer    218.24880   0.38710   0.20585   61.26192  7.02274   94.43121   0.30741
Ven    237.78862   0.72333   0.00626   86.53534  3.40547  146.69081   0.71880
Ear    236.06015   1.00000   0.01624     ---     0.00000  121.70696   0.98376
Mar    244.75076   1.52368   0.09438   57.96608  1.84469  356.11360   1.37988
Jup    320.07889   5.20261   0.05021  111.62429  1.24399   31.99909   4.94137
Sat     46.26121   9.55489   0.05166  123.19409  2.44653  114.53506   9.06126
Ura    312.53062  19.21845   0.04601   79.86021  0.78595  189.20817  18.33424
Nep    177.49252  30.11039   0.00906  143.80993  1.66784   63.69143  29.83766
===============================================================================
        5548  2939566.31412   3336.157 -33.910   3.840   0.0     0.574   3.840
-------------------------------------------------------------------------------
pla.   mean long.   a [AU]   eccentr.  asc.node    incl.   per.[°]   per.[AU]
-------------------------------------------------------------------------------
Mer    139.53770   0.38710   0.20590   64.21181  7.02622   98.30563   0.30740
Ven    154.52658   0.72333   0.00615   88.79244  3.40790  150.09487   0.71888
Ear    157.44059   1.00000   0.01612     ---     0.00000  125.99570   0.98388
Mar    153.34969   1.52368   0.09459   59.88266  1.84396    0.68371   1.37955
Jup    283.32994   5.20261   0.05059  114.17870  1.23065   36.05183   4.93942
Sat    197.86228   9.55488   0.05077  125.35747  2.43646  119.45650   9.06980
Ura    297.65003  19.21845   0.04595   81.25222  0.79002  192.90443  18.33545
Nep      2.32976  30.11038   0.00907  146.55586  1.64439   67.24816  29.83718
===============================================================================
 Computed constellations:      11248
 Number of syzygies     :          5       CPU time  0: 0: 0.025
                                           run time  0: 0: 0.026 -- end of run.
```

The 22 quick start options, representing typical program runs, are specified in more detail in the next sections. The mode in which the parameters are defined individually one after the other can be entered with the option 0 (input of previous program run: 0, 3, 4, 3, 1, 2, 1000, 3500, 0, 4, 2, 2). A detailed description of all corresponding menus is provided in section 3.3. It is anticipated here that there are many more quick start options than 22. The additional quick start options with three digits are intended to reproduce the results in the tables of the two books [5, 14].

For a better understanding, we should mention that in [5], 14 different dates and associated planetary constellations in the period 13,000 BC to AD 17,000 were analyzed in detail, depending on the geometric approach when comparing pyramid positions (Giza) with the planetary positions. These constellations were numbered 1 to 14. Five of them were more significant than the others, but it became clear later on that the constellation number 12 is the most important [14, 24, 5].

3.1.1 Pyramid positions (Giza)

One of the main results of the first book [5] is that the three inner planets correlate with the three pyramids of Giza (Fig. 5). More precisely, the Cheops Pyramid represents the planet Earth, the Chefren Pyramid represents Venus, and the Mykerinos Pyramid represents Mercury. The book describes how the correlation between pyramids and planets was discovered. Three basic equations were found that define the sizes of the pyramids. The maximum relative error of these equations is 0.2 %. From recent systematic studies, it was found that the relative uncertainty of the first equation is approximately 0.001 % (see sections 4.10.6, 4.10.7, and Ref. [14]). With *S* being the base length of the pyramid, *V* the volume, *Q* the aphelion distance (largest planet's distance from the Sun), and *c* the speed of light, these equations are as follows:

$$\frac{S_{Cheops}}{c \cdot 1\,\mathrm{s}} = \frac{V_{Earth}}{V_{Sun}} \;, \qquad \frac{V_{Cheops}}{V_{Chefren}} = \frac{V_{Earth}}{V_{Venus}} \;, \qquad \frac{S_{Cheops}}{S_{Mykerinos}} = \frac{Q_{Earth}}{Q_{Mercury}} \qquad (1)-(3)$$

(Cheops Pyramid) (Chefren Pyramid) (Mykerinos Pyramid)

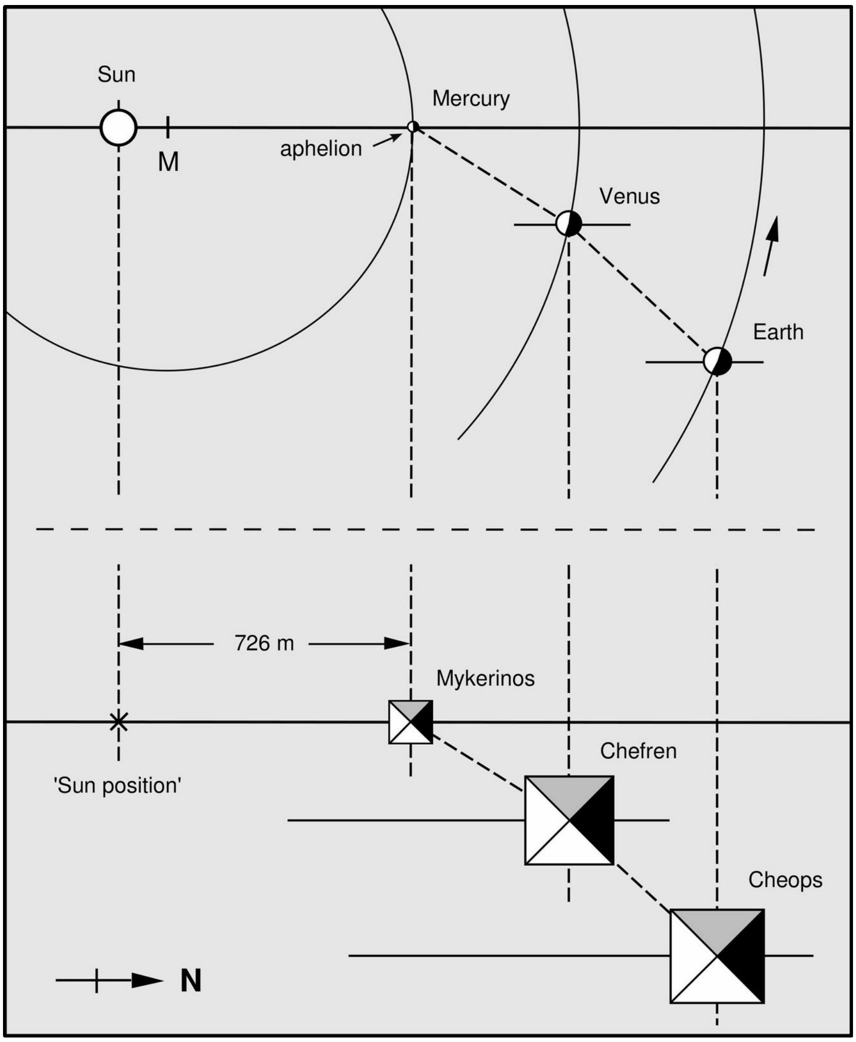


Figure 5: *Correlation between the inner three planets of our solar system and the three pyramids of Giza. The positions are each projected vertically into the main plane. Mercury is placed exactly at the aphelion. For improved visibility, the Sun is magnified by a factor of 6 in relation to the planetary orbits, and the planets by a factor of 500 [5] – see also Fig. 11. The distance of 726 m corresponds to a (one-dimensional) comparison of certain angles – see section 4.3.1.*

Option 1: `"3D Mer at aph (1)"` (Compare with main menu at the beginning of chapter 3.)

"3D" refers to a 3-dimensional calculation. The comparison of the positions of pyramids and planets is performed by considering all 3 dimensions. The vertical position of a pyramid is given here by its center of mass, which is located at a quarter of the pyramid's height. The date is restricted in the way that Mercury is always placed at the aphelion of the orbit, having the largest distance to the Sun. The investigated time period covers the years 13,000 BC to AD 17,000. The results of the VSOP87 theory become less precise if the date proceeds thousands of years into the past or into the future. Nevertheless, an estimate of the precision [2] (see also section 4.2.4) shows that within the given years the accuracy is entirely sufficient for our purposes. In the P5 program, the dates for the application of VSOP87 are mostly restricted to the described time period. This means that start and end dates can be chosen freely within this period, but cannot exceed these limits.

The detected dates are listed in a table, where every date is represented by one line. Special dates are marked at the beginning of the line with the number of the corresponding constellation. These numbers 1 to 14 indicate certain planetary constellations, which are defined and described primarily in [5]. The output table using this option is also given in [5, p. 346, upper part of Tab. 50]. For more details, see sections 3.3 and 3.4.2.

Option 2: `"2D Mer at aph (2)"`

This calculation is similar to that of option 1, with the distinction that the calculation is restricted to 2 dimensions. This means that the positions of the pyramids are projected into the horizontal plane of the Earth's surface. Accordingly, the positions of the planets are projected into the main plane, given by the plane of the Earth's orbit. Therefore, the vertical coordinates are not taken into account (see also [5, Tab. 45 on p. 327]).

Option 3: `"constell 3088 (3)"`

This option calculates (3D) all relevant quantities for the constellation 12. This planetary constellation at May 31, 3088, 6:19:09 a.m. (TT, Terrestrial Time) represents the most relevant event of the 14 constellations concerning the pyramid positions at Giza. Mercury is again placed at the aphelion. (Note that in Eq. (3) above Fig. 5, the aphelion distance $Q_{Mercury}$ appears.) Additionally, the heliocentric coordinates of all planets from Mercury to Neptune for this special date are transformed to coordinates at the Giza plateau (see also [14, Tab. 24 in app. A2]). The program output is given in section 3.4.3 (see also [14, chapter 4]).

Option 4: `"1.5 days 3088 (4)"`

In this case a time scan around the date of constellation 12 (pyramid positions) is created. The positions of the planets are given in time steps of one hour beginning 18 hours before and ending 18 hours after the date of constellation 12 (therefore "`1.5 days`"). Thus, the slow change of all the important parameters can be followed easily when time passes through the main moment. Compare with [14, Tab. 22.B in app. A2] and see also section 3.4.4.

Option 5: `"near aphelion (5)"`

This search for planetary constellations represents the pyramid positions at Giza without the restriction that Mercury is placed at the aphelion. It was found that the planets Mercury, Venus, and Earth are in line with the pyramid positions only when Mercury is placed not too far away from its aphelion position. Thus, in order to keep the computation time short, the constellations are firstly checked with Mercury in the aphelion. If the agreement of the positions is good enough, Mercury is placed outside (but near) the aphelion position (short version VSOP87). At the beginning of each line, F means relative error ≤ 0.5 %; M means error of scale factor ≤ 2 %; and >>> means both errors ≤ 0.1 %. The errors and especially the theoretical scale factor M are described in [5].

3.1.2 Chamber positions (Giza)

Interestingly, 44 days before the "pyramid date" of constellation 12, the planets Mercury, Venus, and Earth represent the arrangement of the chambers in the Great Pyramid. At this moment, Mercury is placed exactly at the perihelion of its orbit, the nearest point to the Sun. The correlation between planets and chambers can be seen in Fig. 6. Notice that the "chamber constellation" also defines a "Mars position" within the Great Pyramid above the King's chamber. Additionally, the "Sun position" could be the place of another (secret) chamber. For detailed information and exact coordinates of the new locations, see sections 3.4.11 and 3.4.12. Between the two dates of chamber and pyramid positions, the five celestial bodies (Sun, Mercury, Venus, Earth, and Mars) are placed nearly in a straight line. This linear constellation (syzygy) is examined in section 3.1.3.

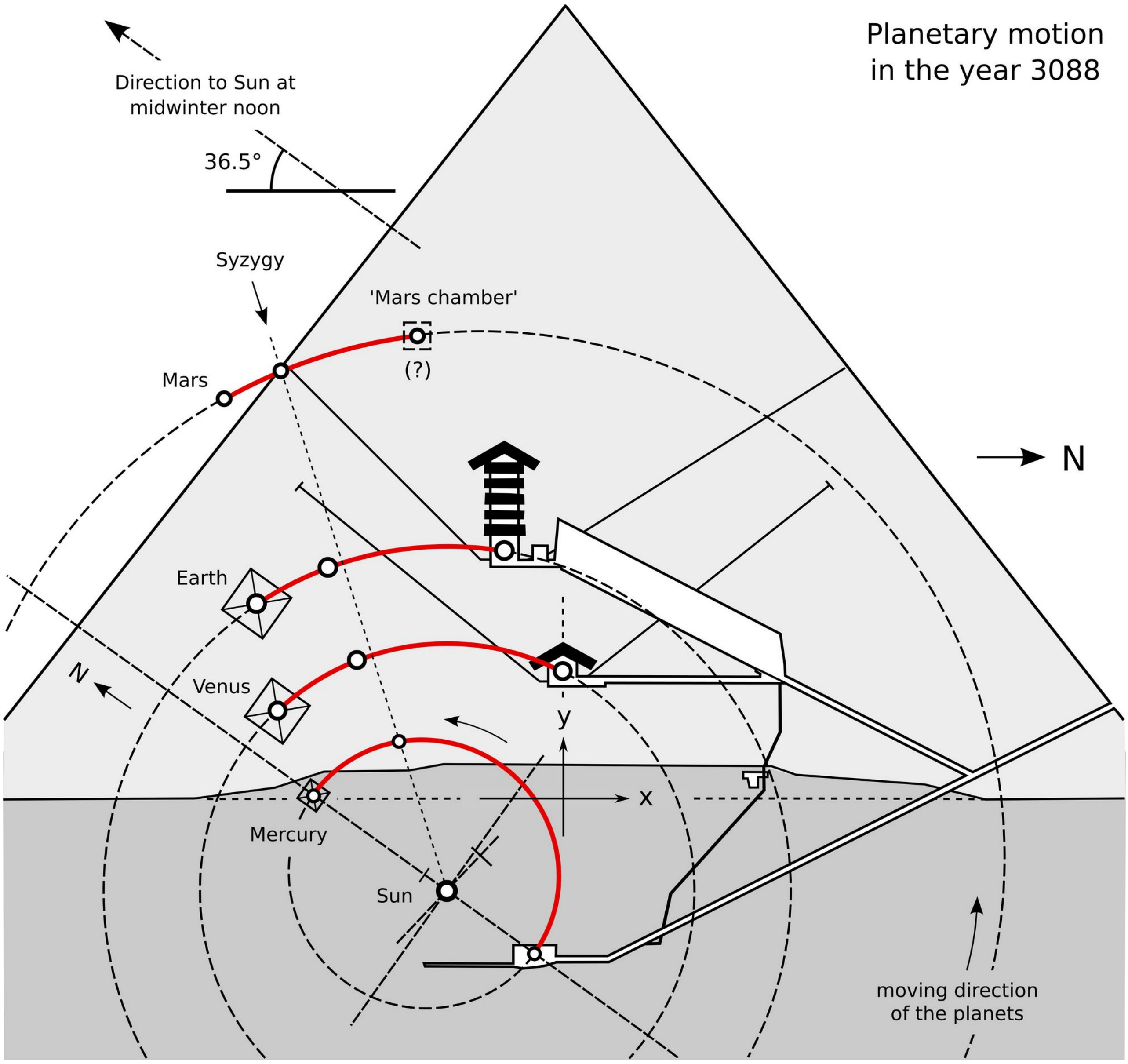


Figure 6: *Cross-sectional area of the Great Pyramid (Cheops Pyramid) as seen from the east [24]: common representation of the known original chamber system in the pyramid, the arrangement of the pyramids themselves, and the planetary orbits. The time span between the constellations of chambers and pyramids is 44 days (red paths), which means half of Mercury's orbit. On the right side of the "Sun," Mercury is placed at perihelion; on the left side at aphelion. The whole figure corresponds to constellation 12. All "planetary positions" can be calculated with P5 using the options 330–335. The configurations in the figure are roughly true to scale. For the exact positions, use the calculated coordinates.*

The origin of the coordinate system is placed at the vertical middle axis of the east wall of the Queen's chamber on the level of the pyramid base. The x-axis points to the north, the y-axis points upward (see Fig. 6), and the z-axis points to the east. The quick start options, which have been implemented as main examples, are as follows:

Option 6: `"3D Mer at per (6)"`

The calculation is an analog to option 1. The positions of the pyramids are replaced by the positions of the chambers in the Great Pyramid, and Mercury is always located at its perihelion. The investigated time period again covers 13,000 BC to AD 17,000.

Option 7: `"Keplers equ   (7)"`

Here, the planetary positions are not calculated with the short or the full version of VSOP87. Instead, the positions are determined with the orbital elements by solving Kepler's equation (sections 3.4.5 and 4.2.3). The orbital elements are derived from the VSOP82 theory [18, pp. 197 ff.], and the transcendental equation of Kepler is solved numerically. The other boundary conditions are similar to option 6. This method does not have the accuracy of the full version of VSOP87, but, nevertheless, since the calculation is different to that of option 6, it is a good test of the previous results. When this time period of 30,000 years is investigated, 124,558 constellations are calculated and checked, and the overall computation time is less than 1 second.

Option 8: `"constell 3088 (8)"`

This computation is an analog to option 3 – only the positions of the pyramids are replaced by the positions of the chambers in the Great Pyramid, and Mercury is placed at its perihelion (section 3.4.11). The exact date is April 17, 3088, 6:41:13 a.m. (TT, Terrestrial Time). Now, the planetary positions are all transformed to the coordinate system of the Great Pyramid. In the previous version P4, the 3D positions correspond to the spatial middle of each chamber. In P5, the positions mean the center of the east walls of the chambers, which is the major change in this quick start option – see also section 4.10.5. The origin of the coordinate system can be seen in Fig. 6 on the ground level of the pyramid, as described previously. From this calculation it was determined that the “Mars position” is also placed inside the Great Pyramid about 40 m above the King's chamber (Fig. 6 and [14, section 4.5, Tab. 23 in app. A2]).

Option 9: `"1.5 days 3088 (9)"`

Analogously to option 4, this is a 36-hour time scan around the “date of the chambers.” This constellation was also given the number 12 because it is closely related to the “date of the pyramids.” The time difference of 44 days is very short when compared to astronomical time scales.

Option 10: `"F minimized  (10)"`

The search here is similar to that of option 5, but the algorithm is more sophisticated. For the time period from the year AD 2500 to the year AD 3900, the planetary positions are compared with the chamber positions and the date is not restricted in any way. This means that Mercury can be placed anywhere on its orbit. For each date where the positions match with each other and the relative error is below a certain value (0.5 %), this error is minimized and the constellation is counted only if the minimized error is smaller than another limit (0.4 %). Within the investigated 1,400 years, the result is 40 dates in which these conditions are met (section 3.4.6). Of course, other boundary conditions imply that ultimately only one date is left (see option 8, and also [14, Tab. 20.A in app. A2]).

3.1.3 Planetary conjunctions and transits

Conjunction means either that two or more celestial bodies have almost the same position in the sky, or that, for example, two or more planets have the same ecliptic longitude. The latter case can be seen in Fig. 7. The figure shows roughly the correct dimensions of the orbits in 3088, when the four planets Mercury, Venus, Earth, and Mars together with the Sun are aligned nearly in a straight line. As mentioned before, such an arrangement is called syzygy, being a generic term for conjunction and opposition.

From time to time, Mercury and Venus pass in front of the Sun's disc, which is called a transit. In Fig. 8 the typical lapse of time is shown for the Venus transit in the year 2012. Here we take Venus instead of Mercury because of the recent Venus transit, which was a rare event.

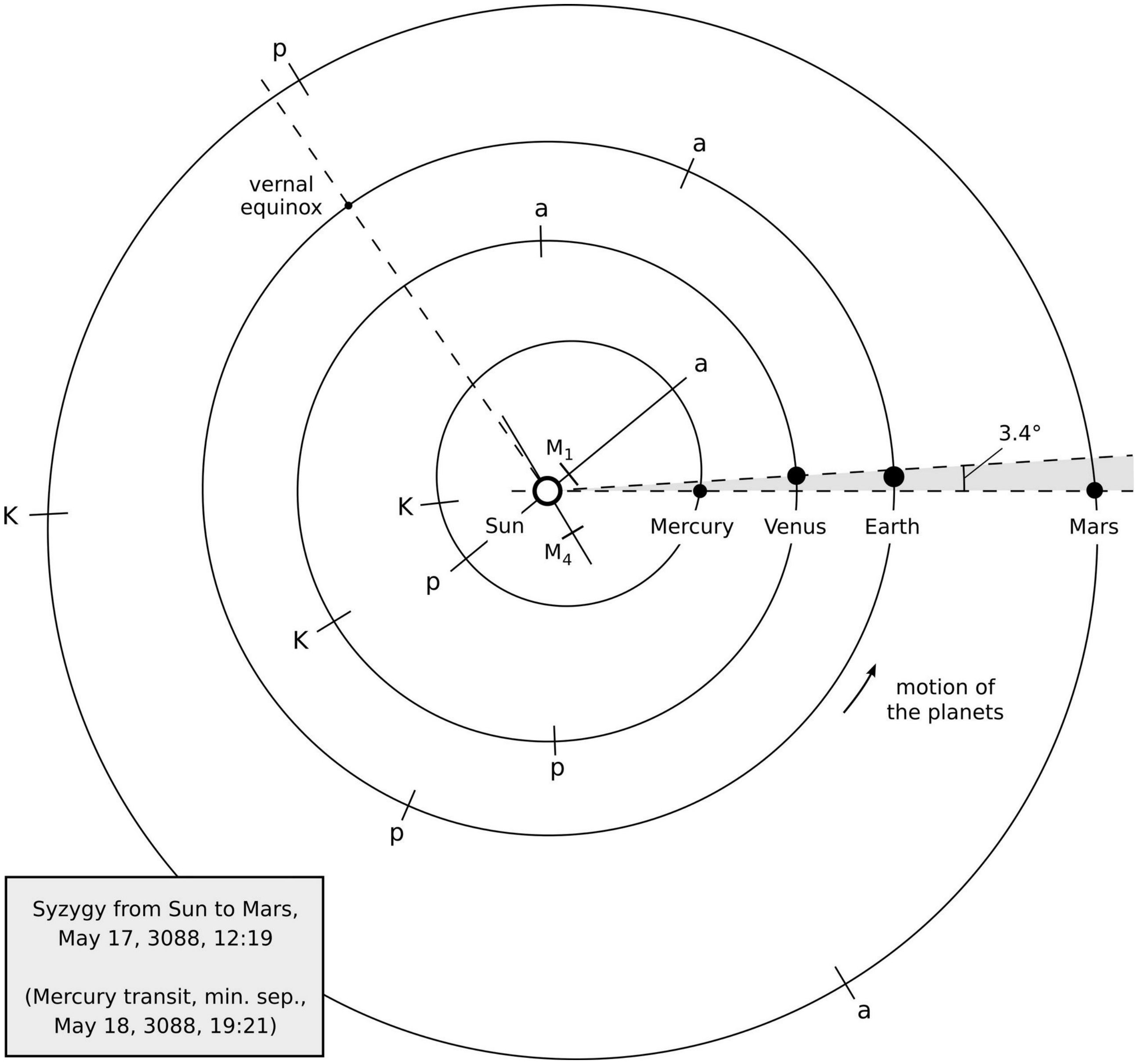


Figure 7: *Approximate true-to-scale representation of the orbits of Mercury, Venus, Earth, and Mars around the Sun. On May 17, 3088, the four planets and the Sun are positioned nearly in a straight line (syzygy), followed by a Mercury transit. The positions p, a, and K represent perihelion, aphelion, and ascending node, and M_1 and M_4 are the orbital centers of Mercury and Mars. For improved visibility, the planets and the Sun are drawn larger than they would normally be if they were drawn to scale. The dates are given in TT (Terrestrial Time). The range of ecliptic longitudes dL (or ΔL) is only 3.4°.*

The interesting point is that the two events – the four planets and the Sun in a straight line and a transit of Mercury or Venus – normally do not take place simultaneously. Within the given 30,000 years (from 13,000 BC to AD 17,000), this happens only six times according to the calculations if we fix the maximum angular range of the ecliptic longitudes to 5°. This means that the coincidence of the given syzygy and a transit of Mercury or Venus happens on average only every 5,000 years. This happens exactly between the two dates of the "chamber constellation" and the "pyramid constellation," which are separated by 44 days! Figures 6 and 7 show that within this period of time the four planets and the Sun form nearly a straight line. About one day later, Mercury passes the solar disc (for more details see [14]).

Option 11: `"Mercury tr  (11)"`

The contact dates of all Mercury transits are calculated for the years 3030 to 3300. Additionally, the minimum separation between Mercury and the Sun, the case of ascending or descending node, and the serial number are given. For the transit series of Mercury and Venus, see also section 4.7.4 and [25, pp. 7–13]. The period includes the year 3088, which is labeled automatically with the number 12. In the given time span, 35 Mercury transits are registered (section 3.4.7).

Option 12: `"Venus tr    (12)"`

All Venus transits with their four contact points (phases) and minimum separation are listed for the years 1500 to 4000. This time period is longer than for Mercury because Venus transits occur less frequently than Mercury transits. Between the years 1500 and 3000, and with a period of roughly 120 years, two Venus transits occur at a time, following each other with a time difference of eight years. (The time limits of this option are chosen in such a way that all results are displayed on one monitor screen.) If we do not have a full transit but a grazing transit, the corresponding line gets a v at the beginning (see also the first program output in section 3.1). The same is true for a grazing Mercury transit, but in this case the line gets an m. In the previous time period of option 11, there are, by chance, no grazing transits of Mercury.

Option 13: `"syzygy, 3 pl. (13)"`

This option yields linear constellations (syzygy) of the three planets Mercury, Venus, and Earth, together with the Sun. The condition for the syzygy is that the ecliptic longitudes of all three planets match within an angular range of dL = 5°. The investigated time period is 2900 to 3300. If a transit of Mercury or Venus also occurs during the syzygy event (within a few hours or a few days), the beginning of the line in the table gets an M or a V for a full transit of Mercury or Venus or an m or v for a corresponding grazing transit. It might also happen during such a linear constellation that both a Mercury and a Venus transit occur, so that the line is indicated with both letters, e.g., MV. This happens only three times between the years 13,000 BC and AD 17,000, assuming the ecliptic longitudes are within a range of 5°. Note that this does not necessarily mean a simultaneous transit, because both transits might be separated by a few hours or days. If a syzygy is near a known constellation within a certain time limit (10 orbital periods of Mercury ≈ 880 days), the corresponding line is marked with a small arrow `->` [14, Tables 25.A and 25.B]. However, for transits in the remote future or remote past, the decreasing precision of VSOP87 has to be considered (section 4.2.4).

Option 14: `"syzygy, 4 pl. (14)"`

Now, Mars is also included. This means that the program searches for linear constellations of Mercury, Venus, Earth, Mars, and the Sun. The condition is that the ecliptic longitudes of all four planets are again placed within the 5° angle, meaning a fourfold planetary conjunction (syzygy). This happens very rarely. For this, the whole time period from 13,000 BC to AD 17,000 is checked. As indicated above, the coincidence of the given syzygy together with a transit occurs only six times, which means an average of every 5000 years (see section 3.4.8 and [14, Tab. 26, app. 2]).

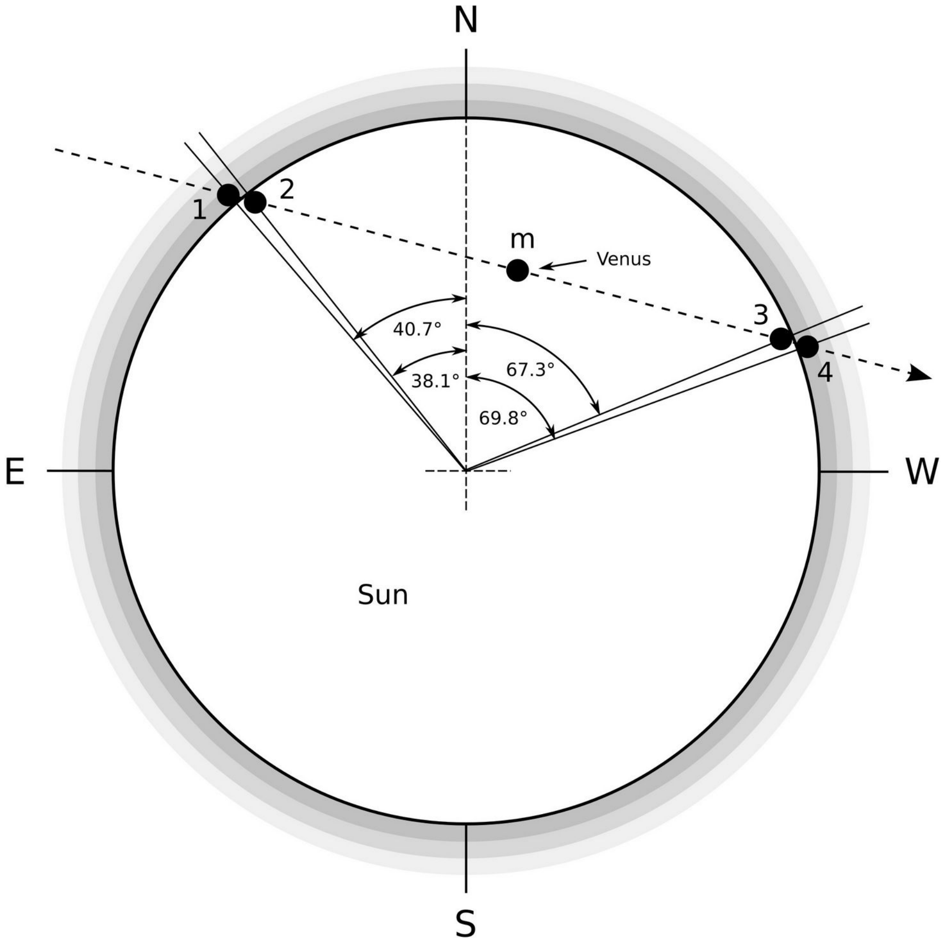


Figure 8: *Venus transit on June 5−6, 2012, as seen from the center of the Earth (geocentric phases). The positions 1 to 4 are the geocentric contact points (phases) and “m” represents the place of minimum separation between Venus and the center of the Sun. The size of Venus and the Sun are drawn to scale as seen from the Earth. N shows the direction of north on the celestial sphere. The direction from E (east) to W (west) is the direction of the apparent motion of the Sun due to the rotation of the Earth. The angles of the contact points were calculated with P5 (option 22, compare with Meeus [25, p. 48]).*

Option 15: `"TYMT test     (15)"`

This option is mainly a test to check the processing speed. TYMT stands for ten thousand years Mercury transit. The transits of Mercury (geocentric phases) are calculated for the years 3000 BC to AD 7000. Using an Intel® Core i5-3210M processor (2.5 GHz, 8 GB, dual channel), the TYMT test needs 49 seconds when using one thread and 20 seconds with four threads (two cores). During the 10,000 years, 31,520 passages of Mercury along the Sun are tested and 1,340 transits are found. The results are calculated with the full version of VSOP87. In contrast to version P4, which has two different source codes for single- and multi-thread hardware, only one P5 source code exists, and is applicable in both cases. About 35 years ago, without optimization of the software and using the computer hardware at that time, the TYMT test would have needed about one month of computation time. After upgrading Ubuntu and reinstalling GFortran, both versions run much faster – see section 4.2.6. (Option 21 calculates the same data including the position angles, and option 22 operates just as well for Venus. As an example, the recent Venus transit is illustrated in Fig. 8.)

The CPU time depends directly on the speed of the processor. More GHz means less CPU time. A criterion, which is more or less independent of the clock frequency and a better measure of the software efficiency, is the product of frequency and CPU time: 2.5 GHz · 49 s = 122.5 GHz·s. This is just a number and means the number of clock cycles necessary for the whole computation. We can call it 122.5 Gc (gigacycles) which is $122.5 \cdot 10^9$ cycles. In P5, CPU and run time are provided.

3.1.4 Planetary correlation of Teotihuacán

Resulting from a visit to Mexico in the year 2005, a new planetary correlation has been found with respect to the archaeological pyramid area of Teotihuacán. The first hint for this was given by the peculiar barriers on the Avenue of the Dead (see Fig. 9). As in Giza, this correlation has to do with the planets of our solar system, but the mathematical and astronomical implementation is completely different. Whereas at Giza, the inner planets from Mercury to Mars and the Sun are involved, at Teotihuacán, the Sun and all of the eight planets as well as the trans-Neptunian object Sedna are included. Figure 10 provides a graphical overview with the details of this correlation.

Figure 9: *Photographs taken in Teotihuacán from the top of the Pyramid of the Sun.* ***a)*** *Avenue of the Dead to the right with the Citadel in the background to the left.* ***b)*** *Close-up of four of the six barriers on the avenue.*

It seems that the Avenue of the Dead represents a logarithmic astronomical scale. Astronomical distances and the distances in Teotihuacán need physical length units, and the logarithm needs a defined logarithmic base. For the length unit of the astronomical distances, we take the solar radius, and for the distances in the archaeological area – measured from the center of the Pyramid of the "Moon" – we use the distance between the centers of the Plaza of the "Moon" and the Pyramid of the "Moon." (We place "Moon" in quotation marks because in chapter 5 it becomes clear that "Sun" is more appropriate.) The logarithmic base used is 3. However, the correlation, and especially the coefficient of determination, do not depend on the length units or the logarithmic base. The chosen values merely have the advantage that the resulting equation is simpler, without any arbitrary factors. More details are given in the references [26, 27] ([27] in German).

The astronomical distances can be obtained from astrophysical books [28], from the Internet, or they can be calculated using the VSOP theory of Bretagnon and Francou [1, 2]. In P5, the planetary distances are always calculated using the orbital elements, which were derived by Jean Meeus from VSOP82 [18, p. 200–204]. For the distances in Teotihuacán, different possibilities exist:

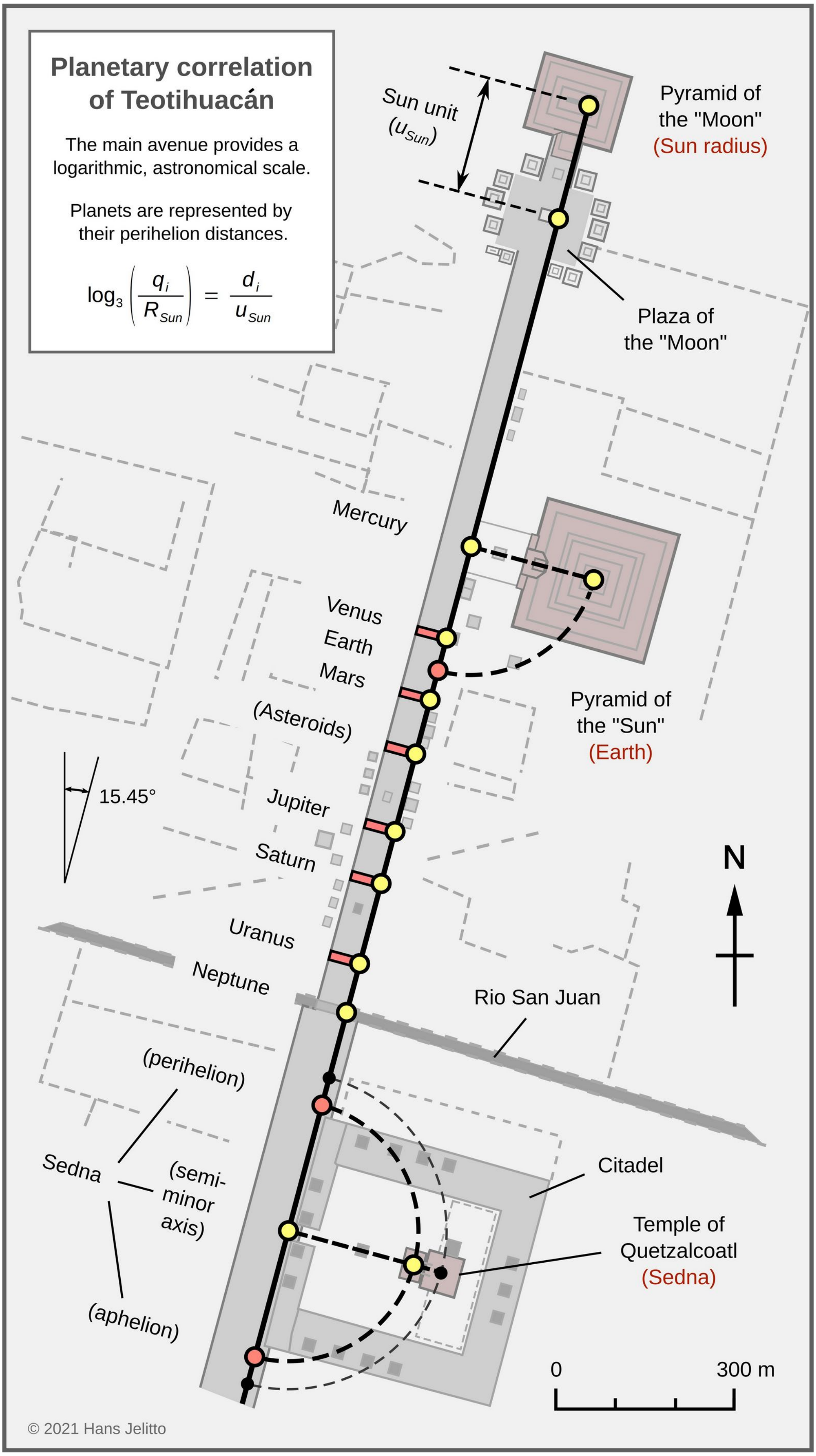


Figure 10: *True-to-scale overview of the planetary correlation of Teotihuacán. In the formula, d_i are the distances in Teotihuacán, measured from the Pyramid of the Moon. The drawing of the archaeological site is based on a satellite image (Google Maps); see Fig. 28 a).*

1. The relative distances can be obtained on a geographical map or on a satellite image on the computer monitor (Google Maps, etc.) by accurately using a ruler and measuring, e.g., in mm or cm.
2. The absolute distances can be calculated from the GPS data. These coordinates can be determined easily on Internet platforms (Google Maps, etc.), as mentioned before. The calculation of distances from the GPS data is not trivial – see the equations in section 4.6.4.
3. The GPS data can also be measured directly in the pyramid area with a GPS receiver, such as a smartphone with GPS function.
4. In principle, the distances can also be measured in Teotihuacán by using a measuring tape or any sort of range finder. This method would yield the most precise results. Nevertheless, this is not necessary because the precision obtained in methods 1 or 2 above is sufficiently accurate.

Option 16: `"GPS m km  (16)"`

The distances at the pyramid area are calculated in m from the geographical coordinates (GPS, section 4.6.4). The applied length unit for the astronomical distances is km and the logarithmic base is 10. A detailed description of the individual options and the possibility to use one's own input data is provided in sections 3.2.3 and 3.3.19–3.3.23.

Option 17: `"Map mm km (17)"`

Instead of the GPS data, the map distances in mm are used, precisely measured on a satellite image of Teotihuacán. The astronomical length unit is again km and the logarithmic base is 10. The "map data" are taken from the last column in the table of the input file inteoti.t (section 3.2.3).

Option 18: `"GPS log3  (18)"`

Based on the GPS data, the length unit in the pyramid area is the "Sun unit," given by the distance between the central platform on the Plaza of the "Moon" and the Pyramid of the "Moon" (approx. 197 m). The astronomical length unit is the solar radius [29] and the applied logarithmic base is 3.

Option 19: `"Map log3  (19)"`

This option is identical to option 18 with the exception that the "map data" rather than the GPS data are used. Thus, the "Sun unit" was also measured using a satellite image (computer monitor) in mm. This option yields a slightly better result ($R^2 = 99.990$ %) than the GPS data ($R^2 = 99.980$ %), although both coefficients of determination (R^2) are very close to 1 (section 3.4.9).

Option 20: `"24000 y.  (20)"`

Here, we have a time scan from 20,000 BC to AD 4000. The calculations (GPS (m), km and base 10) are done in steps of 1000 years in order to find the optimum correlation and maximum coefficient of determination, respectively (section 3.4.10). For the GPS data, the point in time with the maximum R^2 is found at approximately 9930 BC ($R^2 = 99.985$ %); for the map data, the corresponding year is 9570 BC ($R^2 = 99.994$ %).

Options 21 and 22 (not displayed in the start menu):

These options calculate the Mercury and Venus transits between 3000 BC and AD 7000, including the contact angles. For the output, a line width of 148 characters is needed. Apart from the contact angles and the semidiameters of the Sun and planets, option 21 corresponds to option 15.

3.2 Quick start options for the book tables

Most of the astronomical tables in the two books [5] and [14] can be reproduced by additional quick start options, called book options. These options are not shown in the main menu, but they can be found easily. All book options have three digits. The first two digits represent the number of the table and the last digit indicates the section of the table. For example, Table 39 in the first book [5, p. 319] consists of three parts, placed one above the other. These parts can be reproduced by the options 390, 391, and 392, meaning the digits 39 plus one digit 0, 1, or 2 for the different parts. If a table has only one part, e.g., Table 45 [5, p. 327], a zero has to be appended and the corresponding option is 450. Analogously, Tables 18.A and 18.B in [14], e.g., have the options 180 and 181.

3.2.1 Book 1 (*Pyramiden und Planeten*)

If the Tables 39 to 51 in [5] are reproduced with the P5 program, the program output is not always identical to the tables. In some cases, the program output is much larger, which means that in the book only the important quantities are printed.

In Table 50, the correlation between pyramids and planets is checked, in which Mercury is fixed to the aphelion of the orbit and the “Sun position” in the pyramid area is free in all 3 dimensions. The latter aspect is described in more detail in sections 3.3.10–3.3.12 and 4.6. There are three possibilities for defining the vertical position of a pyramid: the center of mass of the pyramid, the pyramid base, or the top of the pyramid. The first two cases are presented in Table 50 and can be calculated with the options 500 and 501. The constellations with the pyramid top as the vertical coordinate have been omitted in the table because there were no significant new results. Nevertheless, this case can be computed using the option 502.

In Table 51, the correlation between pyramids and planets is again investigated. Not only is the “Sun position” on the Giza plateau free in all 3 dimensions, but the date is also free, which means that the dates are not restricted to the aphelion passages of Mercury. In the first book, the search for the constellations was done using the short version of VSOP87. Afterwards, the relative error F''_{pos} was minimized by repeatedly starting the full VSOP87 version by hand using the P3 program. In P4 and P5, these results are calculated automatically with a fit subroutine and the VSOP87 full version (see also sections 3.4.14–3.4.16). Here, the results sometimes differ in the last digit from those in the book [5] because in [5] the relative error was minimized by manually adapting the point of time. The search routine from quick start option 5 uses the short version of VSOP87. This routine was also implemented later for the full version of VSOP87. If the reader wants to check the results in Table 51 with this different search method (full version), this can be done with the options 517, 518, and 519 (see also section 3.3.16).

3.2.2 Book 2 (in preparation)

Tables 17–38.B (except 29 and 34–36) in [14] can be reproduced using the corresponding quick start options described previously. Table 27.A, for instance, indicates the quick start option 270. The numbers of the tables in book 1 (39–51) have no overlap with those in book 2 (17–38.B). Thus, the options can be used without explicitly addressing book 1 or book 2.

3.2.3 Special test option (999)

Let us assume that a special parameter setting is used and several runs have to be done by changing only one parameter. In this case, it is not convenient to manually set all other parameters every time, as described in section 3.3. Instead, it is easier to use the input-output-file inedit.t. If the reader opens this file using an editor, they will see two sections: section 1 and section 2. An example of the content of inedit.t is provided below. Section 1 (big arrow) is read by the P5 program if the quick start option 999 is used and can be edited by hand. CAUTION: Not all combinations of parameters are possible and these parameters are not checked by the program when using option 999. The underlined parameters (see below) can be changed within their allowed values (section 3.3) without any problem. For other parameters, their modes of operation and interdependencies must be known and it is recommended to not change them if applying option 999.

Section 2 is always overwritten with the presently used parameter values when the program is started with options other than 999 and –804. Therefore, it is possible to copy the numbers of section 2 to section 1 and then to modify one or more numbers in section 1. In the lines above section 1, the text should not be changed or deleted because for the program to read the parameters, the number of lines must always be the same, and the original text may otherwise be lost.

Example of the content of inedit.t

```
-----------------------------------------------------------
(    User input and last input of P5 program, 4th Ed.     )
-----------------------------------------------------------
(    The input data in field 1) can be edited by the      )
(    user and are read by the program with the option     )
(    999. The input data in field 2) are written by the   )
(    program at each run and can be used for comparison.  )
(    The manual input by the user in field 1) allows      )
(    for the creation of input data to be copied into     )
(    the file inparm.t. Only CAPITAL LETTERS both refer   )
(    to Giza and Teotihuacan. The latter means IPLA=4.    )
(    Number and positions of the lines in this file       )
(    must not be changed!                                 )
-----------------------------------------------------------
 Parameter names of the values further below
-----------------------------------------------------------
      VVV              IPLA    ILIN    imod    imo4    ikomb
      VVV              lv      itran   ISEP    IUNIV   ICAL
      VVV              ika     iaph    iamax   STEP
    VVVVVVV            ison    ihi     irb     ijd
     VVVVV             ZMIN    ZMAX    ak      zjde1
      VVV              DWI     dwikomb dwi2    dwi3
       V               nurtr   iek     IO      IOUT
-----------------------------------------------------------
1)  Input to edit (999) - CAUTION: No check of parameters!
===========================================================
 3  1  1  0  1
 1  1  3  1  2
 0  1     0     0.00000
 5  0  1  15
  1900.00000   2100.00000      0.00000        0.00000
  0.000   0.000   0.000   0.000
 1  1  1  2
===========================================================
2)  Last used input (all options except 999,-804, and 111)
-----------------------------------------------------------
 2  4  3  0  0
 1  1  1  1  1
 1  2     0    24.00000
 1  0  1  15
-13000.00000  17000.00000      0.00000        0.00000
  1.850   0.000   0.000   0.000
 1  3  1  2
-----------------------------------------------------------
************************** END **************************
```

When applying the option 999, the parameters from field 1 are used by the program. The parameters in fields 1 and 2, provided here, are arbitrary. The parameter names beside the big arrow correspond to the numbers in fields 1 and 2. If the functionality of a given parameter is not known, see section 3.3. If the information in this section is insufficient, the Fortran source code p5.f95, listed in the appendix A1, provides more information. Unfortunately, most comments are in German.

Note: Generally, the parameters in inedit.t are not checked with respect to correct input. Thus, the user should follow the hints, given above.

If performing a calculation concerning Teotihuacán, the P5 program acquires the necessary geographical data from the file inteoti.t. The table in this file consists of four columns with numbers. The first two columns are the GPS coordinates, determined using Google Maps, or alternatively HERE WeGo. The third column shows the calculated distances according to the GPS data and in the last column the “map data” are listed, taken from a satellite image on the computer monitor. If this table is used, the entire data are plotted again at the beginning of the program output, even though not all of them are used. The options in section 3.3.19 determine which of the columns (GPS, “real distances, alternatively determined,” or “map data”) are used. The numbers in this file can be edited and changed by the user if they prefer to use numbers determined themselves. It does not matter which kind of length units and even which size of satellite image is used. In this case, it would be advisable to store a backup of the original inteoti.t file.

Table in the input file "inteoti.t"

```
' teot(i=0..17, k=1..4)          GPS lat.   GPS long.   dist./m   d/mm'
------------------------------ =====================================
'Moon Pyr.  (Sun)    ' ' ' ' '  19.699662  -98.843713     0.0      0.0
'Plaza de la Luna    ' ' ' ' '  19.697947  -98.844212   197.00    51.9
'Sun Pyr.   (Mercury)' ' ' ' '  19.692982  -98.845651   767.16   200.0
'barrier 1  (Venus)  ' ' ' ' '  19.691620  -98.846028   923.08   240.0
'Sun Pyr.   (Earth)  ' '*' '+'  19.692415  -98.843693   981.92   254.5
'barrier 2  (Mars)   ' ' ' ' '  19.690632  -98.846302  1036.20   270.2
'barrier 3  (Aster.) ' ' ' ' '  19.689801  -98.846546  1131.72   295.4
'barrier 4  (Jupiter)' ' ' ' '  19.688594  -98.846890  1270.16   331.0
'barrier 5  (Saturn) ' ' ' ' '  19.687797  -98.847053  1359.83   355.5
'barrier 6  (Uranus) ' ' ' ' '  19.686594  -98.847465  1499.71   391.4
'Rio San J. (Neptune)' ' ' ' '  19.685788  -98.847712  1592.64   415.5
'Q1a Feath. (Sedna)  ' '*' '+'  19.681881  -98.846180  1712.25   446.7
'Q1         (  "   ) ' '*' '+'  19.681952  -98.846438  1740.44   453.8
'Q1b Ados.  (  "   ) ' '*' '+'  19.682001  -98.846622  1760.48   458.4
'Q2         (  "   ) ' ' ' ' '  19.682515  -98.848481  1963.62   511.5
'Q3b Ados.  (  "   ) ' ' ' '+'   0.0         0.0       2166.75   564.6
'Q3         (  "   ) ' ' ' '+'   0.0         0.0       2186.80   569.2
'Q3a Feath. (  "   ) ' ' ' '+'   0.0         0.0       2214.98   576.3
------------------------------ =====================================
'                          *  ---> pyramid/temple position (off-axis)'
'                          +  ---> sum or difference of two distances'
```

3.3 Detailed options (0)

In contrast to the quick start options, the single parameters in the program can also be set individually, one after the other, each given by its own menu. In the main menu on page 8 (last line) we find `detailed options (0)`. The option to get into this mode is `0`. In a program run, not all combinations of the parameters are meaningful. Those that are not allowed are not presented. Sometimes a reduced menu is shown. If a number is typed that is not offered in the menu, an error message appears. Thus, the entire program start is controlled by the program and protected against any false input. In order to abort the start of program after specifying some parameters, type Ctrl + C (Strg + C on German keyboards). If the menu asks for a real number (with decimal point), an integer number is also accepted. If an integer number is required, it must be an integer. Below is a brief overview of the menus and options.

1. Planetary positions: pyramids, chambers (Great P.), linear constellations, Teotihuacán.
2. Linear constellations and transits: transits of Mercury/Venus, conjunction of 3 or 4 planets.
3. VSOP theory versions: short or full VSOP version or combination of both, planetary elements.
4. VSOP coordinate systems: ecliptic of epoch (VSOP87C, VSOP87D), J2000.0 (VSOP87A).
5. Transit options: equal ecliptic longitudes, nearest separation, transit phases, position angles.
6. Calendar systems: Julian/Gregorian calendar or only Gregorian calendar.
7. Time systems: Terrestrial Dynamical Time (TT, JDE), Universal Time (UT).
8. Mapping of planets and chambers: assignment of Mercury, Venus, Earth to the chambers.
9. Search method for the dates: Mercury passages at aphelion, perihelion, date not restricted.
10. “Sun position”: south of Mykerinos or Chefren Pyramid, “Sun position” free.
11. Computation of free “Sun position”: free in 2 or 3 dimensions, 3D calc. with SLE or FITEX.
12. Vertical coordinate of pyramid positions: pyramid base, center of mass, top of pyramid.
13. The z-coordinate of chamber positions: east wall, spatial middle, west wall of each chamber.
14. Datum plane for Earth's surface: projection on plane of Earth, Mercury, or Venus orbit (2D).
15. Specification of timing: number of constellation (1–14), k-number, years, Julian Day.
16. Tolerance in degree or percent: tolerance/angular range of ecliptic longitude, relative error.
17. Syzygy with simultaneous transit: all planetary conjunctions, only with simultaneous transit.
18. Polarity (orientation of planetary orbits): view from ecliptic north, south, or both options.
19. Distances in Teotihuacán: calculated from GPS data, real distances (m), map data (mm).
20. Time scan for Teotihuacán: year of beginning and end of scan, step width in time.
21. Length unit for Teotihuacán: as given (mm, cm, ...) or “Sun unit.”
22. Length unit for planetary distances: as given (km) or solar radius.
23. Logarithmic base: base 10, base 3, custom base.
24. Complexity of output: normal output, extended output.
25. Mode of program output: output only on monitor, monitor + file, special output, exit.

In the following sections the menus are described one by one in the order of their appearance during program start after the use of option `0`. Each menu is given at the beginning in blue. Note: Not all menus appear at program start, depending on the type of computation. On the right side of each menu, the corresponding internal parameter is given, e.g., ipla.

3.3.1 Planetary positions

```
>>>  Giza pyramids  (1), GP chambers (2),
     conj./transits (3), Teotihuacan (4)   :
```
(internal: ipla)

(1) planetary constellation of Mercury, Venus, Earth = positions of the three pyramids at Giza
(2) planetary constellation of Mercury, Venus, Earth = positions of the three chambers in Great P.
(3) linear constellations (syzygy, transit)
(4) calculation concerning Teotihuacán

There are four main categories. The positions of the planets are compared with (1) the positions of the three pyramids of Giza, or (2) the system of the three chambers in the Great Pyramid. Option (3) investigates when the planets build a planetary conjunction and linear constellation, respectively, or a Mercury or Venus transit. The origin of the coordinate system for option (1) is the middle of the base area of the Mykerinos Pyramid. The x-axis points to the north, the y-axis points to the west, and the z-axis points upward. For option (2), the origin is located on the middle axis in the east wall of the Queen's chamber on the level of the pyramid base. The x-axis points to the north, the y-axis points upward, and the z-axis points to the east. Option (4) refers to Teotihuacán.

3.3.2 Linear constellations and transits

```
Tr. Mer.(1), Ven.(2), 3-co.(3), 4-co.(4)   :
```
(internal: ilin)

(1) transits of Mercury
(2) transits of Venus
(3) triple conjunction of the planets Mercury, Venus, and Earth (min. range of ecliptic longitudes)
(4) fourfold conjunction of the four planets Mercury, Venus, Earth, and Mars (" " " " ")

The linear constellations (above option `conj./transits (3)`) are subdivided into the four given menu points. Options (1) and (2) are clear. Option (3) means a syzygy of the planets Mercury, Venus, Earth, and the Sun, and option (4) a syzygy of Mercury, Venus, Earth, Mars, and the Sun.

When running option (3), it becomes clear that Mercury and Earth always have the same ecliptic longitude. This seems reasonable, but it is not entered into the program as a boundary condition. With option (4), three different cases are observed. Either the ecliptic longitudes of Mercury and Mars, those of Mercury and Earth, or those of Venus and Mars are identical. In principle, there are other combinations of two of the four planets, but other solutions do not exist. If we think about the problem, it becomes clear why. (If p is the number of planets, the number of cases – different pairs of planets with equal longitude and minimum total angular range – is $N = (p–1)\cdot(p–2)/2$ with $p \geq 3$.)

3.3.3 VSOP theory versions

```
VSOP87      combi. (1), short version (2),
       Kepl. equ. (3), full version (4)   :
```
(internal: imod)

(1) combination of the short and full versions of the VSOP87 theory
(2) short version of the VSOP87 theory, Meeus [18, pp. 381 ff.]
(3) planetary elements, polynomials of third degree [18, pp. 200 ff.] and solving Kepler's equation
(4) full version of the VSOP87 theory, Bretagnon and Francou [1, 2]

Option (3) (solving Kepler's equation) is the fastest algorithm and has the lowest accuracy. Option (2) (short VSOP87 version) is not as fast, but it has a higher precision. Option (4) (full VSOP87 version) has the highest precision, but also the longest computation time. Option (1) (combination of short and full VSOP87 version) is fast and yields the same high precision as (4). Thus, the recommendation is option (1) for longer time periods and option (4) for single constellations.

3.3.4 VSOP coordinate systems

```
System   ecl. of epoch (1),  J2000.0 (2)  :
```
(internal: lv)

(1) ecliptic of epoch (dynamical equinox)
(2) standard system J2000.0 (ecliptic of Jan 1, 2000, 12:00, TT, or *JDE* = 2,451,545.0)

The two options are the two applied coordinate systems for the VSOP87 theory. The short VSOP87 version is provided only with the ecliptic of epoch (1). The VSOP87 full version and the orbital elements for solving Kepler's equation are given for both systems.

3.3.5 Transit options

```
Date  equ.L.(1), nearest (2), phases (3)
          phases and position angles (4)  :
```
(internal: isep)

(1) transit check at equal ecliptic longitudes (planet, Earth), finite speed of light not considered
(2) transit check at minimum separation (nearest approach), finite speed of light not considered
(3) geocentric transit phases, as seen from Earth
(4) geocentric transit phases and position angles (output requires line width of 148 characters)

Options (1) and (2) are each calculated for a fixed moment. Thus, the travel time of light is not taken into account. These options were written during an early stage of the program development and now serve for testing purposes. Option (3) yields the true geocentric transit phases 1 to 4 as well as the minimum separation by considering the finite speed of light. In option (4), the position angles on the solar disk and the semidiameters of the Sun and planet are also calculated. In addition, central transits (minimum separation < semidiameter of planet) are labeled with C (geocentric central transit) and c (central transit, seen from some place on Earth). Only option (4) and the quick start options 21 and 22 require a line width of 148 rather than 80 characters on the monitor.

3.3.6 Calendar systems

```
Calendar  only Greg. (1), Jul./Greg. (2)  :
```
(internal: ical)

(1) Gregorian calendar for all times
(2) Automatic choice of Julian and Gregorian calendar

Option (1) means that only the Gregorian calendar is used for all times. In the second option, the Julian calendar is used for the years from 4712 BC to AD 1582 and the Gregorian calendar for all other times. It makes no sense to use the historical Julian calendar before 4712 BC because in this distant past the calendar becomes more and more unreliable and there are no historical events to apply this calendar. In contrast, the Gregorian calendar in these past times is in much better agreement with the seasons. The calendar menu is also presented when no calendar dates are calculated. The reason is that the decimal year, displayed in all outputs, is slightly different for both calendars. (Special and detailed calculations are possible with the separate program DATUM-2.)

3.3.7 Time systems

```
Time system          JDE/TT (1),  UT (2)  :
```
(internal: iuniv)

(1) JDE (Julian Ephemeris Day, equal to JD) and TT (Terrestrial Time), respectively
(2) UT (Universal Time)

JDE and TT are identical time scales with a constant length of days. UT takes into account the deceleration of the Earth's rotation due to tidal friction, so that from time to time a leap second is introduced (in UTC). Because the slowing down of the Earth's rotation cannot be predicted precisely, TT is the accurate measure. With option (2), TT can be transformed to UT by using the equations for $\Delta T = TT - UT$ of F. Espenak and J. Meeus [30, 31] (see section 4.8).

3.3.8 Mapping of planets and chambers

```
Planets  E-V-M (1), E-M-V (2), V-E-M (3),
         V-M-E (4), M-E-V (5), M-V-E (6)  :
```
(internal: ika)

(1) Earth – Venus – Mercury
(2) Earth – Mercury – Venus
(3) Venus – Earth – Mercury
(4) Venus – Mercury – Earth
(5) Mercury – Earth – Venus
(6) Mercury – Venus – Earth

The three planets each correspond in the given sequence to the King's chamber, the Queen's chamber, and the subterranean chamber (rock chamber) in the Cheops Pyramid. Option (1) is the case that actually makes sense. The other options are added as a test and for the sake of completeness.

3.3.9 Search method for the dates

```
Passage aph./per. area of aph./per. free
        (1)  (2)            (3)  (4)   (5)  :
```
(internal: iaph)

For options (3) and (4) it follows

```
Steps per Mercury passage :
Step width  (hours, real) :
```
(internal: iamax)
(internal: step)

(1) date: Mercury passage at aphelion
(2) date: Mercury passage at perihelion
(3) time interval around aphelion passage of Mercury
(4) time interval around perihelion passage of Mercury
(5) date completely free (within a given epoch)

Note: There are similar input menus in which not all of the options (1) to (5) are given, but the meaning of the numbers is always the same. In option (1), only the dates are tested, when Mercury passes the aphelion. Option (2) means the same for perihelion. In options (3) and (4), those constellations are tested that are in a given time interval around the aphelion and perihelion passage of Mercury, respectively. This could be, for example, an interval starting seven days before and ending seven days after each aphelion passage with equal time steps of (for instance) 12 hours. In this case, it means that $14 \cdot 2 + 1 = 29$ dates are checked for each aphelion passage. In option (5), the date is totally free. Thus, during the search, the time increases in automatically chosen time steps, and if a promising constellation is found, the relative error is minimized by an automated fit procedure.

For options (3) and (4), two additional input lines (see above) ask for a specific time interval for each aphelion or perihelion passage. First, this requires the number of steps per Mercury passage; and second, the step width in hours is required. In the given example, the number of steps would be 28 and the step width would be 12 (hours). When searching for linear constellations (syzygies) with the short VSOP87 version or the planetary elements version (Kepler's equation), the following input line allows for a search with fixed time steps:

```
Step width [hrs] (min.-search 0.) (real)  :
```
(internal: step)

Thus, in the case where the overall interval dL of ecliptic longitudes of the corresponding planets falls below a certain limit (e.g., below 5°) the program calculates all of the following constellations in the given steps (e.g., in 1-hour steps) until dL again exceeds the previously given limit. If the input 0.0 is given as the step width, the time steps are automatically chosen, and if dL decreases below the given limit (e.g., 5°), the date is optimized by minimizing dL. This case of automatically minimizing dL is always used in the combined search with the short and full VSOP87 versions.

3.3.10 "Sun position"

```
Sun pos.  Myk.(1),  Chefr.(2),  free (3)  :
```
(internal: ison)

(1) "Sun position" fixed 726 m south of the center of the Mykerinos Pyramid
(2) "Sun position" fixed 963 m south of the center of the Chefren Pyramid
(3) "Sun position" free

In options (1) and (2), the "Sun position" south of Mykerinos Pyramid and south of Chefren Pyramid means that the "Sun position" is placed exactly on the north−south middle axis of the corresponding pyramid. The given distances were determined geometrically on the basis of, e.g., Fig. 1 or 11 (yielding the angles δ_1 and δ_2 in Fig. 11).

In option (3), the "Sun position" at the Giza plateau is not fixed and its calculation has to be further specified by the menu below. Note that not fixed does not mean not defined. The "Sun position" at the Giza plateau is defined exactly by the positions of the three planets Mercury, Venus, and Earth if mathematically considering all 3 dimensions.

3.3.11 Computation of free "Sun position"

```
Sun      2D (1), 3D/SLE (2), 3D/FITEX (3)  :
```
(internal: ison2)

(1) "Sun position" free in the 2 horizontal dimensions (meaning restricted to the Earth's surface)
(2) "Sun position" free in 3 dimensions, calculation with a system of linear equations (SLE)
(3) "Sun position" free in 3 dimensions, calculation with coordinate transformation and FITEX

When the planetary and pyramid constellations are adapted to each other by comparing the coordinates or by coordinate transformation, the "Sun position" can be predefined or not. Predefined means that it is restricted in the vertical dimension. This option (1) was applied mainly for the constellations 1 to 11. Options (2) and (3) are two different ways of calculating the "Sun position" in 3 dimensions on the Giza plateau south of the pyramids and also in the Great Pyramid (for details, see section 4.6 and [5, app. A16]). In the case of a rather small relative error between pyramid and planet configuration, both mathematical methods yield the same result and the same "Sun position," respectively. In the case of the chambers, the option `2D (1)` does not exist.

3.3.12 Vertical coordinate of pyramid positions

```
z-coord.    base (1),   C-M (2),   top (3)  :
```
(internal: ihi)

(1) z-coordinate at level of pyramid base
(2) z-coordinate at level of center of mass of the pyramid
(3) z-coordinate at top of pyramid

When fixing the pyramid positions in 3 dimensions, it is necessary to determine the height of the positions. Three alternatives are given. The center of mass of a pyramid (option (2)) can be shown to be located at a quarter of the pyramid height. Options (1) and (2) create mostly the same or similar results, whereas option (3) also generates other constellations.

3.3.13 The z-coordinate of chamber positions

```
Wall      east (1), middle (2), west (3)  :
```
(internal: ihi)

(1) center of east wall of each chamber
(2) spatial middle of each chamber
(3) center of west wall of each chamber

When fixing the chamber positions in 3 dimensions, it is necessary to fix the east−west location (z-coordinate) of the positions. Because only the east walls of all three chambers are located in the same vertical plane, but not the west walls, three alternatives are also given here.

3.3.14 Datum plane for Earth's surface

```
Coord.   ecl.(1),   Mer.(2-4),   Ven.(5)  :
```
(internal: irb)

(1) projection plane is ecliptic plane (plane of Earth’s orbit)
(2−4) projection plane is plane of Mercury’s orbit
(5) projection plane is plane of Venus’s orbit

For the 2-dimensional calculation (section 3.3.11, option (1)), the planetary positions are projected vertically into one plane. (In the same way, the pyramid positions are projected vertically onto the Earth's surface.) Three different planes can be tested, defined by the orbits of Earth, Mercury, and Venus, respectively. The change from the Earth's orbit (heliocentric coordinate system, VSOP87C) to a system based on the Mercury or Venus orbit is performed with rotational matrices (see section 4.5 and [5, app. A15, pp. 328 ff.]). For Mercury, three different combinations of matrices are available (options (2–4)), all yielding the same result. These were used for test purposes during the development of the program.

3.3.15 Specification of timing

```
Constell. (1..14),  k-No. (15),  JDE (0)  :
```
(internal: ijd) or

```
Constell. (1..14),  years (15),  JDE (0)  :
```
(internal: ijd)

(1–14) dates of the constellations 1 to 14 as given in [5, p. 315, Tab. 38]
(15) *k*-number (integer number of Mercury’s passages through aphelion or perihelion) or
(15) time period in years, specified in the menu lines following below
(0) JDE (Julian Ephemeris Day or Julian Day)

Options (1) to (14) belong to the dates of the planetary constellations 1 to 14. With these options, only one constellation is calculated. The *k*-number in option (15) counts the passages of Mercury through its aphelion or perihelion (see section 3.3.9, options (1) and (2)). The numbers start with zero after the beginning of the year 2000. Before this date, the numbers are negative. For the calculation of *JDE* by means of *k*, see Eq. (12) in section 4.3. The format of the *k*-number is real (i.e., number with a decimal point). Normally, the number (not the format) is an integer, but in the given menu it does not need to be an integer. The latter case means that the date is not the passage through aphelion or perihelion, but somewhere between both moments. The option `years (15)` allows us to check the dates in a given time period. Here, the result normally consists of several planetary constellations. The last option, `JDE (0)`, enables us to directly specify a JDE number so that the constellation at this moment is calculated. Next, we have to specify:

	`k (real):`	(internal: ak)
or	`from  year (real):` `until year (real):`	(internal: zmin) (internal: zmax)
or	`JDE (real):`	(internal: zjde1)

The Julian day JDE can be any date, just like the *k*-number, which implies that the relative error between the alignment of pyramids (chambers) and planets can be very large.

3.3.16 Tolerance in degree or percent

```
Tolerance ecl. long. Venus, Earth (real)  :
Max. F-pos at aphelion/ per.  (real) [%]  :
Tolerance ecl. long. VSOP short   (real)  :
    "       "     "  VSOP full    (real)  :
Max. F-pos     VSOP short ver. (real) [%] :
 "     "       VSOP  full ver. (real) [%] :
Max. F-pos, VSOP short, start fitmin [%]  :
 "     "   VSOP short, final range   [%]  :
Ang. range of eclipt. longitude   (real)  :
Ecl. angular range, VSOP short v. (real)  :
 "      "      "  , VSOP  full v. (real)  :
```

(internal: dwi)	or
(internal: dwi)	or
(internal: dwi) (internal: dwikomb)	or
(internal: dwi) (internal: dwikomb)	or
(internal: dwi) (internal: dwikomb)	or
(internal: dwi)	or
(internal: dwi) (internal: dwikomb)	

The accuracy between the theoretical arrangement of the planets – given by the positions of the pyramids, by the positions of chambers, or by a linear constellation – and the current positions of the planets is either measured in degree (difference in ecliptic longitude) or in percent (relative error F_{pos}, F'_{pos}, F''_{pos}, see sections 4.3.1, 4.3.2 and also [5]). The upper limit of these values has to be inserted as real number. A higher upper limit yields a larger number of detected constellations.

When the option “time interval around aphelion or perihelion” (section 3.3.9, options (3) and (4)) combined with a (large) time period is used, like 2000 BC to AD 4000, then this indicates a “special search.” At first, only passages through aphelion or perihelion with a maximum allowed error, dwi, are printed. If the current relative error is below another threshold, dwi2, a time interval of a few hours or days around this aphelion (perihelion) is also tested by scanning the interval in small time steps. Then, only constellations beyond the aphelion or perihelion passage are printed if their relative error is below a third threshold, dwi3. Therefore, the corresponding input line (provided again) is followed by two additional lines:

```
Max. F-pos at aphelion/ per.  (real) [%]  :     (internal: dwi)
 "     "   consider without printing [%]  :     (internal: dwi2)
 "     "   print beyond aphelion/per.[%]  :     (internal: dwi3)
```

Thus, when checking a constellation with this search, there are three limits (relative errors), and the last limit is normally small compared to the others. A typical parameter set is: dwi = 3 %, dwi2 = 5 %, and dwi3 = 0.2 % (see the quick start options 5, 517, 518, and 519). In combination with the VSOP87 short version, the latter search was used to create Table 51 in [5] with the previous program version (P3). A fine adjustment was manually performed using the full VSOP87 version. Now, the search with quick start option 5 can be performed with the VSOP87 full version, too. Furthermore, the fine adjustment is automatically performed when using an automated minimum search with respect to dwi (section 3.3.9 with step width 0.0; see also examples in sections 3.4.14–3.4.16).

3.3.17 Syzygy with simultaneous transit

`All conjunctions (1),  only transits (2)  :` (internal: nurtr)

(1) all linear constellations (syzygies)

(2) only linear constellations with associated Mercury or Venus transit

Option (1): When searching for linear constellations with three or four planets, all constellations are printed that meet the given condition, meaning that the range of ecliptic longitudes is smaller than or equal to a given angle d*L* (e.g., 5°). Constellations that are accompanied by a Mercury or Venus transit within a few hours or a few days are marked with an M or a V. If the transit is a grazing transit, the line is marked with m or v. In the second option, the detected linear constellations are printed only when there is an associated transit of Mercury or Venus. This reduces the size of the output.

3.3.18 Polarity (orientation of planetary orbits)

`View from ecliptic  North (1), South (2)  :` (internal: iek) or
`View from eclipt.  N (1), S (2), N/S (3)  :` (internal: iek)

(1) looking from ecliptic north
(2) looking from ecliptic south
(3) looking from ecliptic north and south

When comparing the pyramid and planetary positions in 2 dimensions, the pyramid positions are projected onto the Earth's surface and the planetary positions, for example, onto the plane of the Earth's orbit. In this case, there are two possibilities when looking down onto the planets. We can look from the ecliptic north (option (1)) or from the ecliptic south (option (2)). One figure is the mirror-inverted configuration of the other. Thus, when comparing with the pyramid positions, the perspective used to look down onto the planets makes a difference. Finally, option (3) simply combines both options (1) and (2). Thus, all constellations found in (1) and (2) are given in (3).

3.3.19 Distances in Teotihuacán

`Distances   GPS (1), meters (2), Map (3)  :` (internal: ilin)

(1) using the GPS-coordinates
(2) real distances, for example measured in m
(3) map data, measured with a ruler on a satellite photo (e.g., in mm on a computer monitor)

With option (1) the distances are calculated on the basis of the GPS coordinates using the equations in section 4.6.4. These distances are printed in the corresponding column in the program output, meaning that the “real distances” in the input data (from inteoti.t) are overwritten by the calculated data. Note that the corresponding column in the file inteoti.t is not replaced. If using option (2) the real distances are taken directly from the column dist./m in the file inteoti.t (without applying the GPS data). Option (3) means that the data in the last column of the table in inteoti.t are applied. All of these numbers in the table can be modified.

3.3.20 Time scan for Teotihuacán

```
from  the year      (real):
until the year      (real):
Step width in years (real):
```

(internal: zmin)
(internal: zmax)
(internal: step)

This (sub)menu allows a time span to be included in the calculations with the step width in years. The results change over the years because the semi-major axis and the eccentricity of the planetary orbit vary throughout time for most of our planets. This information is given by the orbital elements, calculated by J. Meeus [18] from the VSOP theory. If only one point in time is to be considered, the years for the beginning and the end of the time span should be identical.

3.3.21 Length unit for Teotihuacán

`Teotih. unit    as given (1), "luna" (2)  :` (internal: isep)

(1) unit of length as given in the input file inteoti.t (GPS (m), m, mm, ...)
(2) unit of length defined by the "Sun unit" u_{Sun} (Plaza of the Moon)

Option (1) means that the unit of length is taken from the table in the input file, being the GPS data, meters in column dist./m, mm, or something else. With option (2) the unit of length is given by the distance between the centers of the Plaza of the "Moon" and the Pyramid of the "Moon."

3.3.22 Length unit for planetary distances

`Planetary unit, kilometer (1), R-Sun (2)  :` (internal: iuniv)

(1) astronomical length unit is km
(2) astronomical length unit is the solar radius

In option (1), the planetary distances are given in km, whereas option (2) refers to the solar radius (695,508 km [29]).

3.3.23 Logarithmic base

`Logar. base    10 (1), 3 (3), custom (4)  :` (internal: ical)

(1) logarithmic base 10
(3) logarithmic base 3
(4) any other logarithmic base

Here, we have two predefined bases, namely 10 and 3. However, any other real base greater than 1 and less than or equal to 1000 can be used with option (4), specified in the next input line (internal parameter: dwi). Note that option (2) is missing.

3.3.24 Complexity of output

`Output          normal (1),  extended (2)  :` (internal: io)

(1) one line for each detected constellation
(2) several lines for each detected constellation

In option (2), the size of the output for each date depends on other parameters. The orbital elements of all planets can be obtained only by also using the option: `Kepl. equ. (3)`.

3.3.25 Mode of program output

```
Mon.(1), file (2), special (3), exit (4)  :        (internal: iout)    or
Monitor (1),  mon. + file (2),  exit (4)  :        (internal: iout)
```

(1) output only on monitor
(2) output on monitor and written into the file out.txt
(3) as for (2) but with some output quantities being replaced (special output for constellation 12)
(4) cancels program start

With option (1), the results are written only onto the computer monitor. When using option (2), all results are additionally written into the file out.txt. This file is overwritten after each program run if option (2) or (3) is used. Thus, in order to save the latest results, the created file out.txt must be renamed. Option (3) means that some parameters, i.e.,

`JDE`	Julian Ephemeris Day
`e`	error code when calculating the “Sun position” with FITEX (0 means no error)
`it`	number of iterations when using FITEX

are replaced by

`dt [days]`	time difference to the closest aphelion (perihelion) passage
`X5`	tilt angle between the Earth's surface and the transformed ecliptic plane
`M`	scaling factor between alignment of planets and pyramids (chambers).

The latter values allow for the reproduction of some tables in [14]. For constellation 12, in combination with some certain parameter settings, option (3) calculates all planetary positions in the Giza area as provided in Figs. 6 and 15 (book options 330–335). When typing the parameters individually for the “special” output of constellation 12, only the internal parameters ipla, imod, lv, and ihi can be varied. With option (4), the program start is canceled. The program start and the running program can be terminated at any time by typing Ctrl + C (Strg + C on German keyboards).

Detailed technical information about the subroutines VSOP87Z, FITEX, and all other program parts can be found in the FORTRAN source code p5.f95 (appendix A1) and to some extent in chapter 4. Additional mathematical details of the astronomical calculations are provided in [5, app. A14–A16].

3.4 Some program outputs

Some basic information concerning the program output has already been provided in section 3.1. With the following selected options and corresponding results, most of the expressions and parameters in the table head of the program output are explained. The output is always printed in blue and calculated with the multi-thread version of P5. Section 3.4.17 provides a compilation of all quick start options. Note that the following program outputs not only are examples but also provide the main results of the planetary correlations.

3.4.1 Option 0

When starting the program with the option `0`, the parameters can be determined individually in several menus, described previously and appearing one after the other. In the following example, the transits of Mercury in front of the Sun are calculated for the years between 1930 and 2200. The input numbers are underlined.

Input:
```
>>>  Giza pyramids  (1), GP chambers (2),
     conj./transits (3), Teotihuacan (4)  : 3
Tr. Mer.(1), Ven.(2), 3-co.(3), 4-co.(4)  : 1
VSOP87-version  full v.(1),  short v.(2)  : 1
```

```
Date  equ.L.(1), nearest (2), phases (3)
          phases and position angles (4)   : 3
Calendar  only Greg. (1), Jul./Greg. (2)   : 2
Time system        JDE/TT (1),  UT (2)     : 1
   from  year (real): 1930
   until year (real): 2200
Output         normal (1),  extended (2)   : 1
Monitor (1),  mon. + file (2),  exit (4)   : 2
```

Output:

```
                             TRANSITS OF MERCURY
                 (geocentric transit phases, terrestrial time TT)
                              < P5-option   0 >

 VSOP87C, comb. search,     ecliptic of date,            all Mercury transits
      Period   (years) from   1930.00  to   2200.00,  Jul./Greg. calendar

co/p    date/  time:    I         II       nearest      III       IV    sep["]a  S
===============================================================================
  m 11. May  1937    --         --       8:59:41      --        --    -955.5   5
    11. Nov. 1940 20:49:22  20:51:11  23:21:31   1:51:56   1:53:45  368.5/  4
    14. Nov. 1953 15:37:44  15:41:23  16:54:17  18: 7:11  18:10:50  861.8/  2
     6. May  1957 23:59:53   0: 9:54   1:14:46   2:19:36   2:29:37  907.3   9
     7. Nov. 1960 14:34:31  14:36:32  16:53:27  19:10:25  19:12:27 -527.9/  8
     9. May  1970  4:20:11   4:23:13   8:16:50  12:10:20  12:13:22 -114.1   7
    10. Nov. 1973  7:48:13   7:49:54  10:32:58  13:16: 7  13:17:49  -26.4/  6
    13. Nov. 1986  1:44: 5   1:46: 0   4: 7:57   6:29:57   6:31:52  470.5/  4
     6. Nov. 1993  3: 7:14   3:13:10   3:57:31   4:41:53   4:47:49 -926.7/ 10
  m 15. Nov. 1999 21:16:48  21:32:36  21:41:57  21:51:19  22: 7: 7  963.0/  2
     7. May  2003  5:14:15   5:18:44   7:53:28  10:28: 9  10:32:37  708.3   9
     8. Nov. 2006 19:13:17  19:15:10  21:42: 9   0: 9:13   0:11: 6 -422.9/  8
     9. May  2016 11:13:36  11:16:49  14:58:33  18:40:11  18:43:22 -318.5   7
    11. Nov. 2019 12:36:43  12:38:24  15:20:57  18: 3:35  18: 5:16   75.9/  6
    13. Nov. 2032  6:42:26   6:44:30   8:55:22  11: 6:17  11: 8:22  572.1/  4
     7. Nov. 2039  7:19:30   7:22:44   8:48: 4  10:13:25  10:16:39 -822.3/ 10
     7. May  2049 11: 5:22  11: 8:54  14:25:43  17:42:26  17:45:58  511.8   9
     9. Nov. 2052 23:55:24  23:57:12   2:31:31   5: 5:55   5: 7:42 -318.7/  8
    10. May  2062 18:18:36  18:22:13  21:38:53   0:55:27   0:59: 4 -520.5   7
    11. Nov. 2065 17:26:32  17:28:15  20: 8:20  22:48:30  22:50:13  180.7/  6
    14. Nov. 2078 11:45: 6  11:47:26  13:43:36  15:39:48  15:42: 8  674.3/  4
     7. Nov. 2085 11:45:39  11:48:12  13:37:22  15:26:34  15:29: 7 -718.5/ 10
     8. May  2095 17:24: 2  17:27:12  21: 8:40   0:49:59   0:53: 9  309.8   9
    10. Nov. 2098  4:38:48   4:40:32   7:19:52   9:59:17  10: 1: 1 -214.7/  8
    12. May  2108  1:43:45   1:48:26   4:19:33   6:50:38   6:55:18 -724.7   7
    14. Nov. 2111 22:19:23  22:21: 8   0:56:51   3:32:38   3:34:24  283.3/  6
    15. Nov. 2124 16:54: 5  16:56:54  18:32:40  20: 8:28  20:11:17  778.9/  4
     9. Nov. 2131 16:18:41  16:20:53  18:27: 7  20:33:24  20:35:36 -614.4/ 10
    10. May  2141 23:51:40  23:54:41   3:47:48   7:40:46   7:43:47  108.1   9
    11. Nov. 2144  9:22:53   9:24:35  12: 6:50  14:49:10  14:50:52 -112.7/  8
    13. May  2154 10: 9: 4  10:24:30  11: 3:26  11:42:22  11:57:47 -930.6   7
    14. Nov. 2157  3:14: 6   3:15:56   5:45: 5   8:14:17   8:16: 7  386.9/  6
    16. Nov. 2170 22:11:11  22:15:11  23:21:26   0:27:41   0:31:41  880.4/  4
     8. May  2174  2:30:42   2:43:27   3:31:41   4:19:56   4:32:40  924.4  11
     9. Nov. 2177 20:54: 2  20:56: 2  23:15: 2   1:34: 7   1:36: 7 -509.8/ 10
    11. May  2187  6:33:51   6:36:52  10:30:32  14:24: 4  14:27: 5  -96.0   9
  c 12. Nov. 2190 14: 9:54  14:11:35  16:54:52  19:38:14  19:39:56   -9.1/  8
===============================================================================
 Computed constellations:     11582               ("/" means ascending node)
 Tested planet. passages:       851
 Detected transits      :        37
 Centr./grazing transits:    1 /  2      CPU time  0: 0: 1.194
                                         run time  0: 0: 0.328 -- end of run.
```

More details about the output format are provided in section 3.4.7. This is one example of fixing the parameters individually. The results of some quick start options are presented below.

3.4.2 Quick start option 1

Output:

```
                 PLANETS IN ALIGNMENT WITH THE PYRAMIDS OF GIZA
                            (Mercury at aphelion)
                               < P5-option   1 >

  VSOP87C, comb. search,     ecliptic of date,        "Sun" free 3D, C-M, FITEX
  Ecl. N and S, years -13000.00 to  17000.00 (c1), tolerance F <  1.50/  1.00 %

 con   k       year    Lm-Lv   Lm-Le   e  it  x-Sun  y-Sun  z-Sun   dr  P   F[%]
================================================================================
    -59934-12435.214   9.349  13.126  0  85 -588.5  370.0  310.5   5.2 *  0.434
    -59768-12395.233  13.603  20.167  0  87 -570.7  227.1  397.3   8.1    0.716
    -55865-11455.188  12.728  18.358  0  89 -552.5  298.7  394.0   8.0 *  0.697
    -35839 -6631.888 -10.931 -15.051  0  63 -411.5  557.6 -273.4  10.5    0.889
    -31770 -5651.861  -7.629  -9.869  0  82 -482.4  571.5 -169.5   8.1    0.668
    -27867 -4711.816  -8.557 -11.707  0  75 -457.9  570.4 -209.9   4.6    0.379
    -23798 -3731.790  -5.250  -6.516  0  65 -526.9  564.6  -99.3   8.6    0.697
      -712  1828.517 -19.311 -31.682  0 111 -606.7 -146.1 -182.7   8.1 *  0.803
       450  2108.387  10.444  17.871  0  76 -676.8  153.5  282.4   7.9    0.677
      3191  2768.562 -20.247 -33.537  0 114 -596.5 -184.6 -143.6   6.5 *  0.662
 12   4519  3088.413  13.779  23.191  0  46 -667.5   21.3  272.4   0.8    0.069
      7260  3748.588 -16.937 -28.312  0 129 -648.7 -110.6 -136.6   6.7 *  0.642
     11163  4688.633 -17.875 -30.163  0 117 -636.4 -148.3 -107.7   5.5 *  0.540
     12491  5008.485  16.189  26.748  0  72 -660.7  -70.6  221.9   9.3    0.866
================================================================================
  Computed constellations:    124559      (P: polarity, * view from ecl. south)
  Detected constellations:        14       CPU time  0: 0: 2.643
                                           run time  0: 0: 0.695 -- end of run.
```

After `tolerance F`, the two given values belong to the VSOP87 short and full versions. After the limiting year dates, the parameter `(c1)` means that the Gregorian calendar as well as the corresponding decimal years are applied for all times; `(c2)` means that both the Julian and the Gregorian calendar (years) are used within their time periods. The parameters in the header line just above the table are as follows:

`con` number of constellation `1`–`14` or arrow `->`, if date is not far from a known constellation
`k` number of Mercury aphelion (or perihelion) passage (see section 3.3.15)
`year` decimal year (astronomical counting, which means that the year 0 exists; see section 4.9.1)
`Lm-Lv` difference of heliocentric longitudes of Mercury and Venus
`Lm-Le` difference of heliocentric longitudes of Mercury and Earth
`e` error code from FITEX, `0` means no error, more information in the source code (app. A1)
`it` number of iterations when using FITEX for calculating the “Sun position” in 3D
`x-Sun` x-coordinate of the “Sun position” in m at the Giza plateau (`y-Sun`, `z-Sun` analog)
`dr` accuracy of “Sun position” in the pyramid area in m (see section 4.9.2)
`P` polarity: the star `*` represents the view from the ecliptic south (no star = ecliptic north)
`F[%]` relative accuracy of comparing pyramid (chamber) positions with planetary positions

In the subroutine FITEX, the error code *e* is named KE. In [5] and [14], the relative deviation *F* is also named F_{pos}, F'_{pos}, or F''_{pos}, depending on the method of calculation. These quantities are always the relative error when comparing the pyramid positions with the planetary positions. The options 1 and 500 are identical (compare with [5, Tab. 50]).

3.4.3 Quick start option 3

Output:

```
              PLANETS IN ALIGNMENT WITH THE PYRAMIDS OF GIZA
                          (Mercury at aphelion)
                            < P5-option   3 >

 VSOP87C (2005) full ver.,  ecliptic of date,        "Sun" free 3D, C-M, FITEX
 Ecl. N and S,  constellation 12,   JDE =  2849079.76330, year =  3088.41 (c2)
                       date (Gregor.,TT) = 31. May  3088,  6:19: 9,  Thursday

===============================================================================
con   k      year    Lm-Lv   Lm-Le   e  it  x-Sun  y-Sun  z-Sun   dr  P   F[%]
-------------------------------------------------------------------------------
  Lm       Bm       Rm       Lv       Bv       Rv       Le       Be       Re
   xm      ym      zm        xv       yv      zv        xe       ye       ze
 xv-xm    xe-xm    yv-ym    ye-ym    zv-zm    ze-zm             rel. deviation
===============================================================================
12   4519  3088.413  13.779  23.191  0  46 -667.5   21.3  272.4   0.8    0.069
-------------------------------------------------------------------------------
 274.2350 -3.8355 0.466784 260.4560  0.3611 0.725141 251.0441  0.0001 1.010140
 0.465739 0.000000-0.031224 .704258-.172709 .004569 0.928519-0.397789 0.000001
 0.238520 0.462780-0.172709-0.397789 0.035794 0.031226            0.06939575 %
-------------------------------------------------------------------------------
 ascending node (M/V/E/Ma):    61.262371   86.535685      ---      57.966374
 inclination i  (M/V/E/Ma):     7.022736    3.405473    0.000000    1.844689
 perihelion pi  (M/V/E/Ma):    94.431801  146.691325  121.707611  356.114290
 transl. X1, X2, X3; del-t:    -0.465804   -0.000042    0.031160    0.000 days
 Euler angl. X4, X5, X6; M:   -45.993868   24.468218   43.897290    97644154.
===============================================================================

===============================================================================
pla.   x[AU]     y[AU]     z[AU]      L        B       r[AU]      Lm-L     dev.
-------------------------------------------------------------------------------
Mer  0.034394 -0.464467 -0.031224 274.2350 -3.8355  0.466784   0.0000   0.0000
Ven -0.120230 -0.715090  0.004569 260.4560  0.3611  0.725141  13.7790   1.5890
Ear -0.328133 -0.955359  0.000001 251.0441  0.0001  1.010140  23.1909   1.7809
Mar -0.742601 -1.384620 -0.003376 241.7944 -0.1231  1.571191  32.4406    ---
Jup  3.659951 -3.600495 -0.044977 315.4692 -0.5019  5.134280 -41.2342  -6.4502
Sat  6.950958  6.246050 -0.394537  41.9425 -2.4175  9.353321-127.7075  87.2925
Ura 14.561780-13.283778 -0.228606 317.6278 -0.6645 19.711836 -43.3928    ---
Nep-30.177931  0.905335  0.497313 178.2816  0.9437 30.195604  95.9534    ---
===============================================================================
 Celestial pos. in Giza    body            x[m]      y[m]      z[m]    dr[m]
-------------------------------------------------------------------------------
 Local coordinates         Sun            -667.49     21.30    272.36    0.77
 of the "planets"          Mercury          -0.12     -0.09     16.24    0.15 <
 (pyramid positions)       Venus           385.58   -239.89     33.43    0.30 <
                           Earth           739.06   -574.51     23.93    0.14 <
                           Mars           1373.30  -1232.84     34.00    0.96
                           Jupiter        4545.10   4889.86  -3044.47    5.07
                           Saturn       -10104.25  10738.27   -928.40   10.57
                           Uranus        18521.60  19497.37 -12553.34   20.57
                           Neptune       -1354.80 -43610.38  15633.42   31.99
===============================================================================
             ("<" exact deviation dr)     CPU time  0: 0: 0.145
                                          run time  0: 0: 0.099 -- end of run.
```

Information above the first solid line of the tables:

`VSOP87C (2005) full ver., ecliptic of date`	describes the VSOP version
`"Sun" free`	"Sun position" at the Giza plateau not predefined
`3D`	calculation of "Sun position" in 3 dimensions
`C-M`	vertical coordinate z of pyramid positions at the center of mass of each pyramid
`FITEX`	calculation of "Sun position" by coordinate transformation and fit program FITEX
`Ecl. N and S`	view on ecliptic plane not fixed because of 3-dimensional calculation

The parameters below the first solid line (double line) are identical to those in section 3.4.2. The quantities below the first dashed line mean the following:

`Lm Bm Rm`	heliocentric spherical coordinates of Mercury
`Lv Bv Rv Le Be Re`	heliocentric spherical coordinates of Venus and Earth
`xm ym zm`	Cartesian coordinates of Mercury; x-axis through Sun and Mercury aphelion
`xv yv zv xe ye ze`	analog Cartesian (rectangular) coordinates for Venus and Earth
`xv-xm ...`	difference of Cartesian coordinates for comparison with pyramid positions
`rel. deviation`	the relative accuracy or error F (resp. F_{pos}, F'_{pos}, or F''_{pos})
`ascending node`	ecl. longitude when the planet moves through ecl. plane from south to north
`inclination i`	tilt angle between planes of planetary orbit and Earth's orbit
`perihelion pi`	ecl. longitude for location of shortest distance between planet and the Sun
`(M/V/E/Ma)`	Mercury, Venus, Earth, and Mars
`transl. X1, X2, X3`	translation coordinates of planetary positions in 3D when using FITEX
`del-t`	time difference betw. current date and closest aphelion/perihelion passage
`Euler ang. X4, X5, X6`	the Eulerian angles for rotation of planetary configuration (FITEX)
`M`	scale factor, calculated with $M = 1$ AU/X_7 (AU = astronomical unit)

The next table in the output contains the astronomical positions of all planets from Mercury to Neptune.

`x[AU] y[AU] z[AU]`	Cartesian coordinates in AU
`L, B, r[AU]`	spherical coordinates, r (radius) = distance from the Sun in AU

`Lm–L` is the difference in ecliptic longitude between Mercury and the corresponding planet, which can be used for a comparison with the accordant angle in the pyramid area. The quantity `dev.` is the deviation of `Lm–L` in degree from the angles given by the pyramid positions. The first three angles in the column `dev.`, belonging to the positions of Mercury, Venus, and Earth, are quite clear (compare with Fig. 11). The deviations for Jupiter and Saturn are based on the positions of the pyramid at Abu Rawash (Jupiter) and the pyramid area in Abusir (Saturn) [5, Fig. 70, p. 150]. These pyramid locations are very near to the (transformed) orbits of Jupiter and Saturn, after coordinate transformation of all planetary positions in the solar system with respect to the pyramids of Giza.

The last table shows the local coordinates of the Sun and all planets after coordinate transformation to the pyramid area at Giza. The origin of the coordinate system is located in the center of the base area of the Mykerinos Pyramid. The x-axis points to the north, the y-axis points to the west, and the z-axis points upward. The quantity `dr[m]` in the last column is the accuracy of the calculated "Sun position" and the "planetary positions." For the calculation of `dr` and for the meaning of `<`, see section 4.9.2 and also [14, Tab. 24]. The remarkable "Sun position" and "Mars position" are highlighted.

3.4.4 Quick start option 4

Output:

```
                  PLANETS IN ALIGNMENT WITH THE PYRAMIDS OF GIZA
                             (Mercury near aphelion)
                                < P5-option   4 >

  VSOP87C (2005) full ver., ecliptic of date,        "Sun" free 3D, C-M, FITEX
  Ecl. N and S,  constellation 12,   JDE =  2849079.76330, year =  3088.41 (c2)
  Special search (interval), step number =    36,  step width =   1.000 hour(s)

 con   k       year     Lm-Lv   Lm-Le   e  it  x-Sun  y-Sun  z-Sun    dr  P    F[%]
    (  JDE        dt[h]    X5   M/10^7  h-Sun    "      "      "       "   "      "  )
===============================================================================
 12   4519  3088.413  13.779  23.191  0  46 -667.5   21.3  272.4   0.8      0.069
  2849079.01330  -18.0  25.67   9.649  20.71 -674.6   50.3  272.1   1.6      0.145
  2849079.05497  -17.0  25.61   9.656  20.73 -674.3   48.7  272.2   1.5      0.136
  2849079.09664  -16.0  25.55   9.662  20.75 -673.9   47.0  272.3   1.4      0.126
  2849079.13830  -15.0  25.48   9.668  20.77 -673.5   45.4  272.3   1.3      0.117
  2849079.17997  -14.0  25.42   9.674  20.79 -673.1   43.8  272.4   1.2      0.108
  2849079.22164  -13.0  25.35   9.681  20.81 -672.7   42.2  272.5   1.1      0.098
  2849079.26330  -12.0  25.29   9.687  20.83 -672.3   40.6  272.5   1.0      0.089
  2849079.30497  -11.0  25.22   9.693  20.84 -671.9   39.0  272.5   0.9      0.081
  2849079.34664  -10.0  25.16   9.700  20.86 -671.6   37.4  272.6   0.8      0.072
  2849079.38830   -9.0  25.09   9.706  20.87 -671.2   35.8  272.6   0.7      0.064
  2849079.42997   -8.0  25.02   9.712  20.89 -670.8   34.1  272.6   0.6      0.057
  2849079.47164   -7.0  24.96   9.719  20.90 -670.4   32.5  272.6   0.6      0.051
  2849079.51330   -6.0  24.89   9.725  20.92 -670.0   30.9  272.6   0.5      0.047
  2849079.55497   -5.0  24.82   9.732  20.93 -669.5   29.3  272.6   0.5      0.045
  2849079.59664   -4.0  24.75   9.738  20.94 -669.1   27.7  272.5   0.5      0.046
  2849079.63830   -3.0  24.68   9.745  20.95 -668.7   26.1  272.5   0.5      0.049
  2849079.67997   -2.0  24.61   9.751  20.96 -668.3   24.5  272.5   0.6      0.054
  2849079.72164   -1.0  24.54   9.758  20.97 -667.9   22.9  272.4   0.7      0.061
  2849079.76330    0.0  24.47   9.764  20.98 -667.5   21.3  272.4   0.8      0.069
  2849079.80497    1.0  24.40   9.771  20.99 -667.1   19.7  272.3   0.9      0.078
  2849079.84664    2.0  24.33   9.778  20.99 -666.7   18.1  272.2   1.0      0.088
  2849079.88830    3.0  24.25   9.784  21.00 -666.2   16.5  272.1   1.1      0.098
  2849079.92997    4.0  24.18   9.791  21.01 -665.8   14.9  272.0   1.2      0.108
  2849079.97164    5.0  24.11   9.798  21.01 -665.4   13.3  271.9   1.3      0.119
  2849080.01330    6.0  24.03   9.804  21.02 -664.9   11.7  271.8   1.4      0.130
  2849080.05497    7.0  23.96   9.811  21.02 -664.5   10.1  271.7   1.6      0.141
  2849080.09664    8.0  23.89   9.818  21.03 -664.1    8.5  271.6   1.7      0.152
  2849080.13830    9.0  23.81   9.824  21.03 -663.6    6.9  271.4   1.8      0.164
  2849080.17997   10.0  23.74   9.831  21.03 -663.2    5.3  271.3   1.9      0.175
  2849080.22164   11.0  23.66   9.838  21.03 -662.8    3.7  271.1   2.1      0.187
  2849080.26330   12.0  23.58   9.845  21.03 -662.3    2.2  271.0   2.2      0.199
  2849080.30497   13.0  23.51   9.852  21.03 -661.9    0.6  270.8   2.3      0.211
  2849080.34664   14.0  23.43   9.859  21.03 -661.4   -1.0  270.6   2.4      0.223
  2849080.38830   15.0  23.35   9.865  21.03 -661.0   -2.6  270.4   2.6      0.235
  2849080.42997   16.0  23.28   9.872  21.03 -660.5   -4.2  270.2   2.7      0.247
  2849080.47164   17.0  23.20   9.879  21.03 -660.1   -5.8  270.0   2.8      0.259
  2849080.51330   18.0  23.12   9.886  21.02 -659.6   -7.3  269.8   3.0      0.272
===============================================================================
                                              CPU time  0: 0: 0.141
                                              run time  0: 0: 0.061 -- end of run.
```

This is a time scan around the aphelion passage of Mercury in the year AD 3088 – see also [14, Tab. 22.B]. Theoretical and (almost) ideal values are highlighted (see explanations in [14]). The new parameters are described as follows:

`Special search (interval)`	search with Mercury near to the aphelion position
`step number`	number of time steps in the time interval for each aphelion passage

`step width` width of time steps in hours

The first of the two rows just above the solid line at the beginning of the table is identical to that in section 3.4.2. It belongs to the very first row of numbers in the table, which gives some quantities for the date of the aphelion passage. The second of the two header rows belongs to all other rows in the table. It contains some new parameters:

`dt[h]` time difference in hours to the middle of the time interval (aphelion passage)
`X5` tilt angle X_5 between Earth's surface and the transformed plane of the Earth's orbit
`M/10^7` scale factor between positions of planets and pyramids (divided by 10^7)
`h-Sun` height of the transformed "Sun position" above the southern horizon in degrees, as seen from the "Mercury position" at the Giza plateau (see Figs. 2, 21, and [5, Fig. 151])

3.4.5 Quick start option 7

Output:

```
          PLANETS IN ALIGNMENT WITH THE CHAMBERS OF THE CHEOPS PYRAMID
                            (Mercury at perihelion)
                               < P5-option    7 >

  "Keplers equation",        ecliptic of date, E-V-M, "Sun" south of sub. cham.
  Ecl. N and S, years -13000.00 to  17000.00 (c1)    angular range:  1.8500 deg

 con   k         JDE           year       Lm      Lm-Lv   Lm-Le    del1    del2  P
===============================================================================
    -56861 -2550434.93231 -11694.956  230.093  98.696 114.324   1.736  -1.186 *
    -54691 -2359541.44361 -11172.307  237.794 -98.170-115.408  -1.210   0.102
    -51783 -2103726.57489 -10471.910  248.141  95.639 116.330  -1.321   0.820 *
    -42816 -1314905.41676  -8312.191  280.237 -96.743-113.723   0.217   1.787
  1 -38913  -971561.04515  -7372.146  294.297 -97.721-115.558  -0.761  -0.048
    -36005  -715746.17643  -6671.749  304.808  95.922 116.021  -1.038   0.511 *
    -27038    73074.98170  -4512.030  337.411 -96.272-113.791   0.688   1.719
  2 -23135   416419.35331  -3571.985  351.692 -97.244-115.611  -0.284  -0.101
    -20227   672234.22203  -2871.587    2.367  96.199 115.701  -0.761   0.191 *
    -16324  1015578.59364  -1931.542   16.743  95.214 113.840  -1.746  -1.670 *
    -11260  1461055.38017   -711.868   35.477 -95.807-113.768   1.153   1.742
  7  -8352  1716870.24889    -11.471   46.276  97.477 117.232   0.517   1.722 *
  3  -7357  1804399.75177    228.176   49.978 -96.781-115.572   0.179  -0.062
     -4449  2060214.62050    928.574   60.818  96.494 115.370  -0.466  -0.140 *
 12   4518  2849035.77863   3088.293   94.434 -95.370-113.661   1.590   1.849
  8   7426  3104850.64735   3788.690  105.397  97.792 116.894   0.832   1.384 *
  4   8421  3192380.15023   4028.338  109.155 -96.348-115.453   0.612   0.057
     11329  3448195.01896   4728.735  120.159  96.809 115.031  -0.151  -0.479 *
     12324  3535724.52184   4968.383  123.931 -97.326-117.242  -0.366  -1.732
  9  23204  4492831.04581   7588.851  165.409  98.118 116.555   1.158   1.045 *
  5  24199  4580360.54870   7828.499  169.223 -95.949-115.263   1.011   0.247
     27107  4836175.41742   8528.896  180.391  97.132 114.689   0.172  -0.821 *
     28102  4923704.92030   8768.544  184.219 -96.933-117.041   0.027  -1.531
 10  38982  5880811.44428  11389.013  226.310  98.444 116.222   1.484   0.712 *
     39977  5968340.94716  11628.660  230.180 -95.586-115.017   1.374   0.493
     42885  6224155.81588  12329.058  241.512  97.455 114.354   0.495  -1.156 *
     43880  6311685.31876  12568.705  245.396 -96.577-116.788   0.383  -1.278
     54760  7268791.84274  15189.174  288.099  98.766 115.911   1.806   0.401 *
     55755  7356321.34562  15428.822  292.026 -95.262-114.737   1.698   0.773
     58663  7612136.21434  16129.219  303.521  97.771 114.045   0.811  -1.465 *
     59658  7699665.71723  16368.866  307.461 -96.255-116.505   0.705  -0.995
===============================================================================
  Computed constellations:    124558      (P: polarity, resp. view on ecliptic)
  Detected constellations:        31       CPU time  0: 0: 0.097
                                           run time  0: 0: 0.098 -- end of run.
```

New terms and parameters:

"Keplers equation"	calculation with orbital elements and solving Kepler's equation
E-V-M	mapping of Earth, Venus, Mercury to King's, Queen's, and subt. chamber
"Sun" south of sub. cham.	“Sun position” fixed, south of the subterranean chamber
angular range	limit for angular deviations in degree when comparing the positions
del1 del2	angular deviations δ_1 and δ_2 in degrees between angles of planetary positions ($L_m - L_v$ and $L_m - L_e$) and of chamber positions in Cheops Pyramid

This run uses the orbital elements given as polynomials of the third degree and solves Kepler's equation numerically. It needs less than 0.2 seconds to check more than 124,000 constellations. The results are not as precise as those calculated with the short and full versions of VSOP87, but this is a good test of the outcome calculated with the other VSOP87 versions. The main constellations are found, although the approach is different.

3.4.6 Quick start option 10

Output: PLANETS IN ALIGNMENT WITH THE CHAMBERS OF THE CHEOPS PYRAMID
(time not restricted, F minimized)
< P5-option 10 >

VSOP87C, comb. search, ecliptic of date, E-V-M, "Sun" free 3D east, FITEX
Ecl. N and S, years 2500.00 to 3900.00 (c1), tolerance F < 0.50/ 0.40 %

con	k	year	dt[days]	X5	M/10^9	x-Sun	y-Sun	z-Sun	P	F[%]
	2680	2645.600	-2.608	162.762	2.3653	-20.77	-17.28	5.02	*	0.170
	2965	2714.278	10.189	-166.581	1.6921	21.53	-35.52	10.35	*	0.146
	3204	2771.809	-1.709	22.674	1.6588	19.11	-38.47	8.37		0.133
	3323	2800.510	12.973	5.499	2.4645	-21.49	-14.76	-3.36		0.052
	3350	2806.890	-31.923	6.128	2.5040	-25.91	-11.27	0.35		0.315
	3357	2808.665	0.625	-159.854	2.3558	-20.22	-17.40	6.35	*	0.320
	3516	2846.857	-37.343	8.668	2.5668	-25.21	-10.30	-0.83		0.222
	3642	2877.345	14.092	-172.401	1.7168	24.28	-33.94	7.57	*	0.388
	3668	2883.633	23.533	13.472	2.5226	-23.30	-12.60	-5.11		0.392
	3669	2883.717	-33.660	164.422	1.7139	31.19	-29.83	-14.57	*	0.093
	3834	2923.605	20.102	12.243	2.5249	-22.18	-13.03	-4.84		0.173
	3888	2936.493	-23.191	4.270	1.7203	29.82	-32.17	3.08		0.234
	4153	3000.440	21.167	176.776	1.7354	28.69	-31.85	1.24	*	0.344
	4180	3006.827	-21.232	169.511	1.6595	28.35	-34.27	-10.70	*	0.228
	4207	3013.325	-22.855	167.267	2.4925	-23.17	-12.69	-6.01	*	0.042
	4359	3050.098	36.851	163.581	2.5330	-24.78	-10.32	-5.85	*	0.268
	4372	3053.211	30.150	23.100	1.6627	30.73	-31.18	-15.95		0.030
12	4518	3088.293	0.000	-13.957	2.3202	-21.76	-17.44	2.68		0.253
->	4525	3090.065	31.601	166.455	2.4825	-25.63	-11.24	-4.45	*	0.353
	4704	3133.046	-16.472	7.973	2.3641	-24.11	-14.83	4.35		0.298
	4718	3136.422	-14.922	177.462	2.4425	-22.81	-14.69	-2.24	*	0.329
	4724	3137.917	3.071	-14.184	1.6077	20.72	-40.43	2.84		0.376
	4857	3169.903	-14.246	175.779	1.6144	25.96	-37.96	-5.37	*	0.001
	4877	3174.774	5.534	-161.858	2.3298	-21.09	-17.38	6.84	*	0.060
	5229	3259.516	-8.376	-171.270	2.4277	-21.15	-15.85	1.65	*	0.287
	5906	3422.581	-5.117	-165.411	2.3976	-20.71	-16.63	3.82	*	0.176
	6191	3491.258	7.135	-163.527	1.6650	19.84	-37.02	11.30	*	0.048
	6224	3499.163	-8.502	-15.215	2.2996	-22.73	-16.97	5.10		0.228
	6244	3504.029	9.277	3.015	1.5900	23.22	-39.96	-4.86		0.194
	6377	3536.018	-6.695	-171.699	1.6048	22.01	-39.73	3.39	*	0.348
	6397	3540.887	12.128	-171.005	2.3615	-22.81	-15.81	4.66	*	0.166
	6583	3585.647	-1.951	-161.254	2.3781	-20.15	-17.07	5.49	*	0.375
	7226	3740.558	13.924	7.663	2.4527	-22.00	-14.80	-3.97		0.335
->	7419	3786.906	-35.983	6.901	2.5539	-25.38	-10.49	-0.05		0.042

```
     7545   3817.393  15.006 -174.669   1.6924   25.40 -34.81   6.27  *  0.086
     7585   3826.873 -41.181    9.416   2.5973  -24.79  -9.92  -1.23     0.374
     7737   3863.653  21.181   13.715   2.5188  -22.61 -12.95  -5.17     0.112
     7738   3863.734 -37.204  164.229   1.7411   31.36 -28.47 -14.44  *  0.167
     7744   3865.273  -2.874  -15.892   2.3052  -21.90 -17.50   3.77     0.101
     7791   3876.540 -22.176    3.348   1.7360   28.82 -31.77   5.08     0.107
================================================================================
 Computed constellations:    114751      (P: polarity, resp. view on ecliptic)
 Detected constellations:        40       CPU time  0: 0: 3.659
                                          run time  0: 0: 0.964 -- end of run.
```

This time scan refers to the chambers of the Great Pyramid. The parameter `dt` is the time interval to the closest perihelion passage; "`->`" means near to a known constellation. Note that in the year 3088, we have d*t* = 0.000 days (5.7 seconds).

3.4.7 Quick start option 11

Output:

```
                             TRANSITS OF MERCURY
                 (geocentric transit phases, terrestrial time TT)
                               < P5-option  11 >

 VSOP87C, comb. search,     ecliptic of date,            all Mercury transits
      Period   (years) from   3030.00  to   3300.00,  Jul./Greg. calendar

co/p     date/  time:   I         II       nearest      III        IV    sep["]a  S
================================================================================
    18. Nov. 3032   6:15:14   6:23:43   6:53: 7   7:22:32   7:31: 1 -945.6/ 19
    28. Nov. 3038   0:31: 3   0:43:41   0:59:11   1:14:41   1:27:19  958.8/ 12
    21. Nov. 3045  22:13: 3  22:14:57   0:41:37   3: 8:21   3:10:15 -436.5/ 17
    21. May  3055  15:25:25  15:28:23  19:21:16  23:13:59  23:16:56  -14.3   18
    23. Nov. 3058  15:42: 5  15:43:47  18:27:22  21:11: 1  21:12:43   66.7/ 16
    26. Nov. 3071   9:56:42   9:58:47  12:10:45  14:22:47  14:24:52  567.6/ 14
    19. Nov. 3078  10:18:37  10:22: 5  11:42:28  13: 2:52  13: 6:20 -839.5/ 19
12  18. May  3088  17:10:47  17:16: 8  19:20:59  21:25:48  21:31: 8  796.5   20
    22. Nov. 3091   2:54:52   2:56:41   5:31: 3   8: 5:30   8: 7:19 -332.3/ 17
    23. May  3101  22: 4:47  22: 7:50   1:54:59   5:41:59   5:45: 1 -212.9   18
    24. Nov. 3104  20:34:48  20:36:32  23:17:46   1:59: 4   2: 0:48  172.4/ 16
    27. Nov. 3117  14:59:22  15: 1:43  16:58:56  18:56:12  18:58:32  670.4/ 14
    20. Nov. 3124  14:42:24  14:45: 3  16:31:23  18:17:46  18:20:25 -735.1/ 19
    21. May  3134  22:49:57  22:53:44   1:52:34   4:51:19   4:55: 6  601.2   20
    23. Nov. 3137   7:38:45   7:40:30  10:20:16  13: 0: 8  13: 1:53 -226.8/ 17
    24. May  3147   4:59: 9   5: 2:27   8:32:13  12: 1:53  12: 5:11 -414.1   18
    26. Nov. 3150   1:28:23   1:30: 9   4: 7: 4   6:44: 4   6:45:50  276.0/ 16
    28. Nov. 3163  20: 6:36  20: 9:25  21:46:36  23:23:49  23:26:37  773.6/ 14
    21. Nov. 3170  19:14:30  19:16:46  21:21:19  23:25:55  23:28:11 -630.3/ 19
    21. May  3180   4:52: 1   4:55:16   8:24:42  11:54: 1  11:57:15  405.9   20
    24. Nov. 3183  12:26:11  12:27:54  15:10:53  17:53:57  17:55:40 -123.0/ 17
    24. May  3193  12: 2:16  12: 6:10  15: 4: 4  18: 1:52  18: 5:45 -612.6   18
    26. Nov. 3196   6:22:28   6:24:19   8:54:45  11:25:16  11:27: 6  379.7/ 16
    29. Nov. 3209   1:21:57   1:25:55   2:33:34   3:41:14   3:45:12  876.1/ 14
    22. Nov. 3216  23:51:17  23:53:19   2:11:38   4:30: 2   4:32: 4 -523.2/ 19
    22. May  3226  11:10:54  11:13:55  15: 0:30  18:46:56  18:49:57  205.4   20
    24. Nov. 3229  17:14:40  17:16:22  20: 0:34  22:44:52  22:46:34  -18.3/ 17
    25. May  3239  19:39:27  19:45:15  21:43:15  23:41:13  23:47: 1 -815.9   18
    27. Nov. 3242  11:20: 7  11:22: 4  13:43:40  16: 5:19  16: 7:16  482.5/ 16
    20. Nov. 3249  12:21: 4  12:27:14  13:10:33  13:53:52  14: 0: 2 -927.1/ 21
    23. Nov. 3262   4:32:28   4:34:21   7: 2:45   9:31:13   9:33: 7 -418.1/ 19
  c 22. May  3272  17:34: 3  17:37: 0  21:29:21   1:21:32   1:24:29   10.9   20
    26. Nov. 3275  22: 5:44  22: 7:26   0:50:54   3:34:27   3:36: 9   87.9/ 17
    27. Nov. 3288  16:20: 1  16:22: 9  18:31:46  20:41:26  20:43:34  587.0/ 16
    21. Nov. 3295  16:32: 9  16:35:26  18: 0:50  19:26:15  19:29:32 -822.7/ 21
================================================================================
```

```
  Computed constellations:     11474                ("/" means ascending node)
  Tested planet. passages:       851
  Detected transits      :        35
  Centr./grazing transits:     1 /  0        CPU time  0: 0: 1.126
                                             run time  0: 0: 0.312 -- end of run.
```

In the header, the expression `comb. search` means combination search: For each transit, the search starts with the VSOP87C short version and continues with the full version. No grazing transit and only one central transit appears during this time period. The constellation number (12) at the beginning of the line is automatically generated by the program (subroutine konst). This program run is similar to the book option 271 [14, Tab. 27.B].

The parameters in the last header line are as follows:

`co`	number of constellation (e.g., 12)
`p`	partial transit: `m` Mercury (not given here), (`v` Venus); `c` or `C` central transit
`date`	calendar date of transit, more precisely: date of minimum separation
`I II III IV`	times of inner and outer contact points, transit phases (see Fig. 8)
`nearest`	moment of nearest approach (min. sep.) between planet and center of the Sun
`sep["]`	minimum separation between planet and the Sun in arc seconds
`a`	ascending node: slash (descending node: no slash)
`S`	serial number of transit

In our epoch, the passage of Mercury through the ascending node always takes place in November and the passage through the descending node in May. However, the ascending and descending nodes for Mercury and Venus are not determined by the given months but independently on the basis of geometrical considerations.

Each transit of one series is labeled with the same number. In contrast, the choice of the serial numbers is arbitrary. Jean Meeus, for example, did not name each series with a number but with the letters A, B, C, ... [25, pp. 42 ff.]. Here, we take the numbers used on the NASA Goddard Space Flight Center website (webmaster, Fred Espenak): Mercury transits (URL 9), Venus transits (URL 10). (Note: In the given URLs, the transit phases are provided in Universal Time, UT.) In order to always get the same serial numbers *S*, independent of the starting date of the chosen time period, the first numbers are taken from the file inserie.t. Thus, this file is used only at the beginning of each run. All other serial numbers are determined during the run time of the program.

3.4.8 Quick start option 14

Output:

```
                            PLANETS IN A LINE (SYZYGY)
              (angular range of eclipt. longitudes dL minimized, JDE)
                                 < P5-option  14 >

  VSOP87C, comb. search,     ecliptic of date,       linear c. Mercury to Mars
  Period   (years) -13000.00  to  17000.00 (c1),   angular r.:  6.00/  5.00 deg

 co tr    k          JDE           year    dt[days]  Lm-Lv    Lm-Le   Lm-Lma   dLmin
================================================================================
       -62144 -3015259.12387 -12967.601 -38.133   1.757   0.0      0.105   1.757
       -61116 -2924752.04882 -12719.801  36.451  -2.558  -3.025   0.0      3.025
       -56018 -2476304.86414 -11491.995  15.891  -1.871  -3.100   0.0      3.100
       -55699 -2448270.17542 -11415.238 -11.643   2.577   3.490   0.0      3.490
       -54830 -2371781.88140 -11205.820  31.286   1.469   0.0      0.900    1.469
       -51975 -2120688.28715 -10518.350 -27.612  -1.048  -0.384   0.0      1.048
    M  -50946 -2030182.60335 -10270.553 -42.389  -0.500  -0.988   0.0      0.988
       -48544 -1818813.77667  -9691.845  24.059   4.074   0.0      1.955    4.074
```

```
       -48225 -1790778.08803  -9615.086  -2.474   3.782   3.234   0.0     3.782
       -44501 -1463199.29860  -8718.206 -21.543   2.831  -0.189   2.831   3.019
     V -40777 -1135613.28929  -7821.306 -33.392  -2.288  -2.121   0.0     2.288
       -39749 -1045106.26361  -7573.506  41.143  -4.656  -3.657   0.0     4.656
       -33463  -492135.45751  -6059.523  36.617  -0.732  -0.497   0.0     0.732
       -29579  -150537.15060  -5124.259 -38.030  -3.780  -2.775   0.0     3.780
       -28046   -15654.33423  -4754.962 -12.227   4.108   3.948   0.0     4.108
       -27177    60834.13961  -4545.544  30.882   1.499   0.0     0.676   1.499
       -24322   311928.64773  -3858.071 -27.103   2.726   3.541   0.0     3.541
       -23293   402434.40305  -3610.274 -41.808   3.463   3.984   0.0     3.984
       -20891   613802.18908  -3031.569  23.600   4.661   0.0     3.870   4.661
       -20572   641837.19908  -2954.812  -3.613   4.509   2.495   0.0     4.509
       -19384   746366.77815  -2668.619  18.379   0.226   4.109   0.0     4.109
       -16848   969416.60704  -2057.930 -22.063   4.094   0.469   0.0     4.094
       -13124  1297003.80471  -1161.026 -32.723   2.174   3.048   0.0     3.048
    M  -12096  1387510.46824   -913.228  41.449  -2.172  -0.337   0.0     2.172
        -5810  1940480.90518    600.754  36.554   0.079   0.707   0.0     0.707
        -5650  1954490.74930    639.112 -28.698   4.457   0.0     0.403   4.457
        -4621  2044997.03231    886.909 -42.875   4.093   0.0     1.425   4.093
        -2955  2191576.37670   1288.230 -20.467  -4.154   0.0    -0.771   4.154
        -1926  2282079.86037   1536.020 -37.445   0.453   2.400   0.0     2.400
          476  2493450.15754   2114.732  30.475   1.507  -0.232   1.507   1.739
          795  2521489.41998   2191.501   7.515  -3.907   0.0    -1.484   3.907
 12 M    4519  2849066.01327   3088.376 -13.750  -3.397  -2.605   0.0     3.397
         5548  2939566.30702   3336.157 -33.917   3.882   0.0     0.569   3.882
         8243  3176650.26922   3985.271 -27.352  -3.312   0.0    -0.379   3.312
         8269  3178981.57686   3991.654  16.752  -0.800   1.467  -0.800   2.267
         9272  3267156.02956   4233.067 -42.053  -3.820  -0.574   0.0     3.820
        15557  3820126.65275   5747.049  41.208  -2.124   0.0    -1.952   2.124
        15717  3834138.13217   5785.411 -22.408  -2.064  -3.418   0.0     3.419
        15743  3836472.36564   5791.802  24.622   4.659   4.389   0.0     4.659
        16746  3924642.67733   6033.204 -38.324   2.520   0.0     2.339   2.520
        19441  4161725.56802   6682.315 -32.830  -3.781   0.0    -0.412   3.781
        20974  4296612.30826   7051.623  -3.103  -4.188   0.0    -3.368   4.188
    M   21843  4373097.00488   7261.031  36.229  -0.135   0.380  -0.135   0.515
        24698  4624193.65495   7948.510 -19.615  -0.632   4.214  -0.632   4.846
        26915  4819212.51626   8482.453 -28.801  -1.437  -3.049   0.0     3.049
        28129  4926065.39990   8775.007  29.292  -0.821  -3.563   0.0     3.563
        28448  4954105.08050   8851.777   6.750  -2.902   0.0    -1.249   2.902
        32172  5281682.80510   9748.654 -13.384  -0.969   0.0    -0.897   0.969
        35922  5611596.79305  10651.928  15.543  -0.971   0.043  -0.971   1.013
        36925  5699772.17611  10893.344 -42.331  -3.639   0.0    -2.630   3.639
        38113  5804287.97146  11179.498 -34.123  -2.641  -3.546   0.0     3.546
        39646  5939173.53429  11548.803  -5.573  -1.199  -3.007   0.0     3.007
        43210  6252743.27610  12407.327  41.406   0.134   2.572   0.0     2.572
    M   43370  6266755.55351  12445.692 -21.412   1.706   1.352   0.0     1.706
        43396  6269087.74498  12452.077  23.576   3.799   2.388   0.0     3.799
        44399  6357258.98958  12693.482 -38.437   3.641   1.774   0.0     3.641
        49496  6805712.91680  13921.307  35.715  -0.904  -0.924   0.0     0.924
        50844  6924245.65229  14245.838 -14.233   3.579  -0.162   3.579   3.742
        54568  7251829.78723  15142.733 -27.956   2.517   2.077   0.0     2.517
        56101  7386721.26013  15512.054   6.504  -1.307   1.289  -1.307   2.596
================================================================================
  Computed constellations:    150720
  Number of syzygies     :        60      CPU time  0: 0: 6.711
                                          run time  0: 0: 1.747 -- end of run.
```

New expressions and parameters:

`linear c.`	linear constellation, syzygy
`angular r.`	max. angular range, first value: short version, second value: full version of VSOP87
`tr`	transit, `M`, `V`: full transit, `m`, `v`: grazing transit (within a few hours or days)
`dLmin`	minimum angular range dL_{min} of ecliptic longitudes of all participating planets

The moment of minimum angular range for the ecliptic longitudes of all participating planets does not need to occur within the period of the planetary transit, but can occur shortly before or after the transit. Thus, the time difference between the moment of minimum angular range and transit can be a few hours or days. The angular range (`angular r.`) of 6° and 5° in the top lines belong to the short and the full version of VSOP87, respectively. The first number should be larger than the second (see also [14, Tab. 26]), otherwise, one or a few constellations can be lost.

3.4.9 Quick start option 19

In the following output concerning the archaeological area of Teotihuacán, the entire input data (input file inteoti.t) are listed, although only the map data in the last column are used. The distances in the column `dist. [m]` are calculated using the GPS coordinates. Nevertheless, they can be manually changed in the file inteoti.t – in cases where the reader has other numbers – and used as the input data.

Output:

```
                     Planetary Correlation
                  of the Pyramids at Teotihuacan
                      < P5-option  19 >

   1. INPUT DATA

   Position                  GPS lat.     GPS long.    dist. [m]     d [mm]
  ==============================================================================
   Moon Pyr.   (Sun)        19.699662    -98.843713       0.00        0.0
   Plaza de la Luna         19.697947    -98.844212     197.00       51.9
   Sun Pyr.    (Mercury)    19.692982    -98.845651     767.16      200.0
   barrier 1   (Venus)      19.691620    -98.846028     923.08      240.0
   Sun Pyr.    (Earth)      19.692415 *  -98.843693 *   981.92 +    254.5 +
   barrier 2   (Mars)       19.690632    -98.846302    1036.20      270.2
   barrier 3   (Aster.)     19.689801    -98.846546    1131.72      295.4
   barrier 4   (Jupiter)    19.688594    -98.846890    1270.16      331.0
   barrier 5   (Saturn)     19.687797    -98.847053    1359.83      355.5
   barrier 6   (Uranus)     19.686594    -98.847465    1499.71      391.4
   Rio San J. (Neptune)     19.685788    -98.847712    1592.64      415.5
  ------------------------------------------------------------------------------
   Q1a Feath. (Sedna)       19.681881 *  -98.846180 *  1712.25 +    446.7 +
   Q1          (  "  )      19.681952 *  -98.846438 *  1740.44 +    453.8 +
   Q1b Ados.   (  "  )      19.682001 *  -98.846622 *  1760.48 +    458.4 +
   Q2          (  "  )      19.682515    -98.848481    1963.62      511.5
   Q3b Ados.   (  "  )        ---           ---        2166.75 +    564.6 +
   Q3          (  "  )        ---           ---        2186.80 +    569.2 +
   Q3a Feath. (  "  )         ---           ---        2214.98 +    576.3 +
  ==============================================================================
                                        (* pyramid/temple position - off-axis)
                                        (+ sum or difference of two distances)

   2. CALCULATED DATA

   Teotihuacan, length unit:   Sun unit (Plaza de la Luna)
   astronomical length unit:   Sun radius
   logarithmic base (astr.):   3.0000

   Body          Map distance       log(per./Rs)   log(a/Rs)     log(aph./Rs)
  ==============================================================================
   Sun               0.0000            0.0000        0.0000         0.0000
   Mercury           3.8536            3.8160        4.0251         4.1950
   Venus             4.6243            4.5871        4.5941         4.6011
   Earth             4.9037            4.8730        4.8890         4.9047
```

```
   Mars                 5.2062              5.1847         5.2723         5.3522
   Jupiter              6.3776              6.3478         6.3901         6.4305
   Saturn               6.8497              6.8856         6.9434         6.9978
   Uranus               7.5414              7.5359         7.5795         7.6211
   Neptune              8.0058              7.9801         7.9882         7.9962
--------------------------------------------------------------------------------
  (Phaeton)             5.6917              5.6751         5.7270         5.7733
--------------------------------------------------------------------------------
   linear fit, f(x)=ux+v          u:    1.00063042     0.99957822     0.99945632
                                  v:   -0.02022494     0.03770845     0.08467909
--------------------------------------------------------------------------------
   Julian year:      200.00     R^2:    0.99990419     0.99926631     0.99766170
                           adj. R^2:    0.99989050     0.99916150     0.99732766
================================================================================
                                            CPU time   0: 0: 0.002
                                            run time   0: 0: 0.002 -- end of run.
```

New expressions and parameters:

GPS	GPS data, determined with Google Maps or HERE WeGo
dist. [m]	distances in Teotihuacán calculated with the GPS data or manually edited in inteoti.t
d [mm]	distances measured with a ruler on a satellite image of Teotihuacán
Q1a...Q3a	positions at and nearby the temple of Quetzalcoatl and the Citadel
(Phaeton)	values according to the barrier of the asteroids belt (hypothetical former planet)
per.	perihelion distance of the planet
aph.	aphelion distance of the planet
log(a/Rs)	the distance *a* indicates the semi-major axis of the planetary orbit.
Rs	solar radius
u, v	coefficients of the linear fit, their calculation is given in section 5.2.1.
R^2	coefficient of determination (includes the eight planets and the Sun)
adj. R^2	adjusted coefficient of determination (")

3.4.10 Quick start option 20

Output:

```
                              Planetary Correlation
                         of the Pyramids at Teotihuacan
                                < P5-option   20 >

   2. CALCULATED DATA

   Teotihuacan, length unit:    normal (mm or m)
   astronomical length unit:    normal (km)
   logarithmic base (astr.):    10.0000

                          ------------------- R^2 (GPS) ------------------
       Julian year        (per. distance)         (a)        (aph. distance)
================================================================================
        -20000.00          0.9998013205     0.9991004937     0.9974834971
        -19000.00          0.9998121659     0.9991005003     0.9974700638
        -18000.00          0.9998213907     0.9991005070     0.9974580673
        -17000.00          0.9998291829     0.9991005137     0.9974475417
        -16000.00          0.9998356957     0.9991005204     0.9974384922
        -15000.00          0.9998410511     0.9991005272     0.9974308991
        -14000.00          0.9998453425     0.9991005339     0.9974247216
        -13000.00          0.9998486383     0.9991005406     0.9974199017
        -12000.00          0.9998509849     0.9991005473     0.9974163669
        -11000.00          0.9998524104     0.9991005541     0.9974140334
```

```
        -10000.00         0.9998529276     0.9991005608     0.9974128081
 -------------------------------------------------------------------------------
         -9990.00         0.9998529282     0.9991005609     0.9974128011
         -9980.00         0.9998529287     0.9991005609     0.9974127942
         -9970.00         0.9998529291     0.9991005610     0.9974127875
         -9960.00         0.9998529294     0.9991005611     0.9974127808
         -9950.00         0.9998529296     0.9991005611     0.9974127742
         -9940.00         0.9998529298     0.9991005612     0.9974127677
         -9930.00         0.9998529298     0.9991005613     0.9974127613
         -9920.00         0.9998529298     0.9991005614     0.9974127550
         -9910.00         0.9998529297     0.9991005614     0.9974127488
         -9900.00         0.9998529295     0.9991005615     0.9974127428
 -------------------------------------------------------------------------------
         -9000.00         0.9998525376     0.9991005676     0.9974125916
         -8000.00         0.9998512329     0.9991005743     0.9974132796
         -7000.00         0.9998490004     0.9991005811     0.9974147651
         -6000.00         0.9998458242     0.9991005879     0.9974169399
         -5000.00         0.9998416885     0.9991005946     0.9974196964
         -4000.00         0.9998365798     0.9991006014     0.9974229286
         -3000.00         0.9998304891     0.9991006082     0.9974265335
         -2000.00         0.9998234142     0.9991006150     0.9974304120
         -1000.00         0.9998153608     0.9991006218     0.9974344701
             0.00         0.9998063443     0.9991006286     0.9974386199
          1000.00         0.9997963907     0.9991006354     0.9974427795
          2000.00         0.9997855375     0.9991006422     0.9974468747
          3000.00         0.9997738341     0.9991006491     0.9974508387
          4000.00         0.9997613421     0.9991006559     0.9974546128
 ===============================================================================
                                          CPU time  0: 0: 0.003
                                          run time  0: 0: 0.003 -- end of run.
```

In this output of the time scan from 20,000 BC to AD 4000 (GPS data), the input data from inteoti.t are omitted because they are identical to those of option 19. Between the years 10,000 and 9900 BC, the data of a special run (input parameters: 0, 4, 1, -10 000, -9900, 10, 1, 1, 1, 1, 2) are inserted in order to specify more exactly the point in time of the maximum R^2 for the perihelion distances. However, this maximum probably has an uncertainty of a few hundred years because, when applying the map data, the maximum R^2 is found at 9570 BC – see Fig. 35 in section 5.2.1.

3.4.11 Book option 230 (“Sun position 2”)

This table [14, Tab. 23] represents all important data when the planets stand in a constellation according to the chamber arrangement (east walls) in the Cheops Pyramid. This book option is identical to option 8. Mercury is placed at its perihelion 44 days before the pyramid constellation (option 3). Two significant locations in the Cheops Pyramid – secret chambers (?) – are highlighted.

Output:

```
          PLANETS IN ALIGNMENT WITH THE CHAMBERS OF THE CHEOPS PYRAMID
                            (Mercury at perihelion)
                              < P5-option 230 >

  VSOP87C (2005) full ver.,  ecliptic of date, E-V-M, "Sun" free 3D east, FITEX
  Ecl. N and S,  constellation 12,   JDE =  2849035.77863, year =  3088.29 (c2)
                        date (Gregor.,TT) = 17. Apr. 3088,  6:41:13,   Tuesday
 ===============================================================================
 con   k      year    Lm-Lv   Lm-Le   e  it  x-Sun  y-Sun  z-Sun   dr  P   F[%]
 -------------------------------------------------------------------------------
   Lm       Bm       Rm       Lv       Bv       Rv       Le       Be       Re
```

```
   xm       ym       zm        xv      yv      zv       xe        ye      ze
 xv-xm    xe-xm    yv-ym    ye-ym   zv-zm   ze-zm                rel. deviation
================================================================================
12   4518  3088.293 -95.595-113.868  0  58 -21.76 -17.44   2.68  0.09    0.253
--------------------------------------------------------------------------------
  94.2332  3.8355 0.307417 189.8280  3.3144 0.720012 208.1012 -0.0001 0.998755
 0.306728 0.000000 0.020564-.070079 .715384 .041627-0.404127 0.913342-0.000002
-0.376807-0.710856 0.715384 0.913342 0.021064-0.020566            0.25271687 %
--------------------------------------------------------------------------------
 ascending node (M/V/E/Ma):    61.260938   86.534589     ---      57.965443
 inclination i  (M/V/E/Ma):     7.022735    3.405472   0.000000    1.844689
 perihelion pi  (M/V/E/Ma):    94.429919  146.689667  121.705528  356.112069
 transl. X1, X2, X3; del-t:    -0.307159    0.000430   -0.020593    0.000 days
 Euler angl. X4, X5, X6; M:   123.793585  -13.956844  -90.108532  2320248812.
================================================================================

================================================================================
pla.   x[AU]     y[AU]     z[AU]       L       B       r[AU]     Lm-L     dev.
--------------------------------------------------------------------------------
Mer -0.022641  0.305891  0.020564  94.2332  3.8355  0.307417   0.0000   0.0000
Ven -0.708259 -0.122694  0.041627 189.8280  3.3144  0.720012 -95.5948   1.3652
Ear -0.881019 -0.470443 -0.000002 208.1012 -0.0001  0.998755-113.8680   1.6420
Mar -1.223696 -1.059317  0.015315 220.8818  0.5421  1.618585-126.6486    ---
Jup  3.428235 -3.843492 -0.038355 311.7316 -0.4267  5.150408 142.5015 177.2855
Sat  7.124162  6.065578 -0.396527  40.4114 -2.4267  9.364943  53.8218 -91.1782
Ura 14.442448-13.403693 -0.227280 317.1363 -0.6609 19.705201 137.0968    ---
Nep-30.172472  1.043277  0.493994 178.0197  0.9374 30.194544 -83.7865    ---
================================================================================
 Celestial pos. in Giza     body            x[m]      y[m]       z[m]     dr[m]
--------------------------------------------------------------------------------
 Local coordinates          Sun            -21.76    -17.44       2.68     0.09
 of the "planets"           Mercury         -5.47    -28.41      -0.00     0.04 <
 (chamber positions)        Venus            0.06     23.44       0.00     0.12 <
                            Earth          -11.11     46.01      -0.00     0.08 <
                            Mars           -26.76     86.77       4.89     0.19
                            Jupiter       -343.90    -39.51      80.20     0.89
                            Saturn          -2.42   -620.15     -28.19     1.60
                            Uranus       -1242.26   -220.89     291.00     3.27
                            Neptune       1197.54   1475.84    -268.24     4.83
================================================================================
             ("<" exact deviation dr)     CPU time  0: 0: 0.146
                                          run time  0: 0: 0.098 -- end of run.
```

For the description of the parameters, see the parameters of the pyramid constellation (option 3, section 3.4.3). The "Sun position" is now located in the Great Pyramid, or, more precisely, below the Great Pyramid, and the "Mars position" can be found about 40 m above the King's chamber (see Figs. 6, 22, and "Sun position 2" in Figs. 23, 25). The local coordinates are listed at the bottom of the output. In the headlines of the output, `east` means that the relevant position is the center of the east wall of each chamber. (Note: The "Jupiter position" is ca. 229 m south of the Cheops Pyramid at 29° 58.56568' N, 31° 8.10636' E, and approx. 39.5 m below the level of the pyramid base.)

3.4.12 Book option 231 ("Sun position 1")

Option 230 corresponds to the "Sun position 2" (see Figs. 23, 25) and was probably intended by the master builders. The original "Sun position 1" (option 231) is provided as an alternative. The difference between both options is that the positions are defined by the east wall and the spatial middle (`mid.`) of the chambers, respectively. If going from position 1 to 2, the positions of the Sun and the planets are shifted by a few meters to the east. For the Sun, this shift is 11.44 m to the east, which means in the z-direction. The x- and y-coordinates are almost unchanged.

```
Output:      PLANETS IN ALIGNMENT WITH THE CHAMBERS OF THE CHEOPS PYRAMID
                               (Mercury at perihelion)
                                  < P5-option 231 >

  VSOP87C (2005) full ver.,  ecliptic of date, E-V-M, "Sun" free 3D mid., FITEX
  Ecl. N and S,  constellation 12,   JDE =  2849035.77863, year =  3088.29 (c2)
                       date (Gregor.,TT) = 17. Apr. 3088,  6:41:13,   Tuesday

 ===============================================================================
 con    k       year     Lm-Lv    Lm-Le    e  it  x-Sun  y-Sun  z-Sun   dr  P    F[%]
 -------------------------------------------------------------------------------
   Lm        Bm        Rm        Lv        Bv        Rv        Le        Be        Re
    xm        ym       zm         xv        yv       zv        xe        ye        ze
  xv-xm     xe-xm     yv-ym     ye-ym     zv-zm     ze-zm              rel. deviation
 ===============================================================================
 12    4518   3088.293 -95.595-113.868  0  96 -21.78 -17.38  -8.76  0.20    0.570
 -------------------------------------------------------------------------------
   94.2332   3.8355 0.307417 189.8280  3.3144 0.720012 208.1012 -0.0001 0.998755
  0.306728 0.000000 0.020564-.070079 .715384 .041627-0.404127 0.913342-0.000002
 -0.376807-0.710856 0.715384 0.913342 0.021064-0.020566            0.56966279 %
 -------------------------------------------------------------------------------
  ascending node (M/V/E/Ma):    61.260938   86.534589      ---       57.965443
  inclination i  (M/V/E/Ma):     7.022735    3.405472    0.000000     1.844689
  perihelion pi  (M/V/E/Ma):    94.429919  146.689667  121.705528  356.112069
  transl. X1, X2, X3; del-t:    -0.305872    0.001024   -0.020245    0.000 days
  Euler angl. X4, X5, X6; M:   -16.990377   -4.182911   50.631415  2316903299.
 ===============================================================================

 ===============================================================================
 pla.   x[AU]     y[AU]     z[AU]      L        B       r[AU]     Lm-L     dev.
 -------------------------------------------------------------------------------
 Mer -0.022641  0.305891  0.020564  94.2332  3.8355  0.307417   0.0000   0.0000
 Ven -0.708259 -0.122694  0.041627 189.8280  3.3144  0.720012 -95.5948   1.3652
 Ear -0.881019 -0.470443 -0.000002 208.1012 -0.0001  0.998755-113.8680   1.6420
 Mar -1.223696 -1.059317  0.015315 220.8818  0.5421  1.618585-126.6486    ---
 Jup  3.428235 -3.843492 -0.038355 311.7316 -0.4267  5.150408 142.5015 177.2855
 Sat  7.124162  6.065578 -0.396527  40.4114 -2.4267  9.364943  53.8218 -91.1782
 Ura 14.442448-13.403693 -0.227280 317.1363 -0.6609 19.705201 137.0968    ---
 Nep-30.172472  1.043277  0.493994 178.0197  0.9374 30.194544 -83.7865    ---
 ===============================================================================
  Celestial pos. in Giza     body            x[m]      y[m]      z[m]     dr[m]
 -------------------------------------------------------------------------------
  Local coordinates          Sun            -21.78    -17.38     -8.76     0.20
  of the "planets"           Mercury         -5.38    -28.43     -7.01     0.09 <
  (chamber positions)        Venus           -0.20     23.38     -2.95     0.26 <
                             Earth          -10.93     46.09     -5.20     0.18 <
                             Mars           -27.45     86.83     -3.25     0.44
                             Jupiter       -352.87    -39.18    -30.97     2.01
                             Saturn           7.15   -619.13    -60.71     3.62
                             Uranus       -1274.34   -219.61   -103.67     7.37
                             Neptune       1221.69   1474.36    162.73    10.91
 ===============================================================================
            ("<" exact deviation dr)     CPU time   0: 0: 0.147
                                         run time   0: 0: 0.098 -- end of run.
```

3.4.13 Book option 334

Here, we get additional planetary positions on the Giza plateau for the four main dates in AD 3088 (see Fig. 15). The calculation of the coordinates is described in sections 4.6.3 and 4.6.4. Analog positions inside the Cheops Pyramid (Figs. 22, 23) can be computed with options 330–332. The data of the “Sun position” appear only once because they do not vary in time. The average accuracy of the GPS data for the four planets Mercury to Mars, according to the average of the errors `dr[m]`, is approximately ± 0.00034 arc minutes.

Output:

PLANETS IN ALIGNMENT WITH THE PYRAMIDS OF GIZA
(more positions - coordinate system of pyramids)
< P5-option 334 >

VSOP87A (2005) full ver., standard J2000.0, "Sun" free 3D, C-M, FITEX
Ecl. N and S, constellation 12, JDE = 2849079.76330, year = 3088.41 (c2)
date (Gregor.,TT) = 31. May 3088, 6:19: 9, Thursday

con	k	year	X5	M/10^7	h-Sun	x-Sun	y-Sun	z-Sun	dr	P	F[%]
12	4519	3088.413	24.58	9.764	20.98	-667.5	21.3	272.4	0.8		0.069

Celestial positions in Giza

body	x[m]	y[m]	z[m]	dr[m]	latitude N	longitude E
date of chambers: JDE = 2849035.77863						
Sun	-667.49	21.30	272.36	0.77	29 57.99045	31 7.68134
Mercury	-1107.01	35.38	441.03	1.09	29 57.75256	31 7.67258
Venus	-459.18	-1006.75	613.80	0.87	29 58.10320	31 8.32044
Earth	35.24	-1320.33	490.56	0.83	29 58.37081	31 8.51540
Mars	910.94	-1885.15	425.24	1.21	29 58.84480	31 8.86658
Jupiter	4907.69	4525.51	-3028.25	5.05	30 1.00808	31 4.88054
Saturn	-9843.02	10967.26	-1089.26	10.57	29 53.02405	31 0.87950
Uranus	18699.19	19309.96	-12545.69	20.55	30 8.47278	30 55.67982
Neptune	-1557.45	-43587.10	15684.14	31.99	29 57.50875	31 34.78986
date of syzygy: JDE = 2849066.01327						
Mercury	-130.71	-387.90	246.95	0.39	29 58.28099	31 7.93573
Venus	251.70	-595.71	218.68	0.28	29 58.48798	31 8.06493
Earth	589.42	-867.01	167.06	0.45	29 58.67077	31 8.23360
Mars	1257.17	-1460.49	155.25	1.03	29 59.03220	31 8.60259
Jupiter	4660.88	4778.10	-3040.90	5.06	30 0.87449	31 4.72351
Saturn	-10023.29	10810.67	-978.79	10.57	29 52.92648	31 0.97686
Uranus	18577.27	19438.94	-12551.05	20.56	30 8.40679	30 55.59964
Neptune	-1418.15	-43603.20	15649.31	31.99	29 57.58415	31 34.80003
date of transit: JDE = 2849067.30624						
Mercury	-102.82	-358.64	222.36	0.36	29 58.29609	31 7.91754
Venus	270.45	-565.35	200.49	0.25	29 58.49812	31 8.04605
Earth	606.56	-841.32	153.23	0.44	29 58.68005	31 8.21763
Mars	1269.28	-1439.92	143.75	1.02	29 59.03875	31 8.58980
Jupiter	4650.09	4788.69	-3041.29	5.07	30 0.86865	31 4.71693
Saturn	-10030.93	10803.89	-974.05	10.57	29 52.92234	31 0.98107
Uranus	18572.04	19444.44	-12551.27	20.56	30 8.40396	30 55.59622
Neptune	-1412.19	-43603.88	15647.82	31.99	29 57.58737	31 34.80046
date of pyramids: JDE = 2849079.76330						
Mercury	-0.12	-0.09	16.24	0.32	29 58.35168	31 7.69464
Venus	385.58	-239.89	33.43	0.11	29 58.56044	31 7.84372
Earth	739.06	-574.51	23.93	0.33	29 58.75177	31 8.05175
Mars	1373.30	-1232.84	34.00	0.96	29 59.09506	31 8.46106
Jupiter	4545.10	4889.86	-3044.47	5.07	30 0.81182	31 4.65403
Saturn	-10104.25	10738.27	-928.40	10.57	29 52.88265	31 1.02187
Uranus	18521.60	19497.37	-12553.34	20.57	30 8.37666	30 55.56331
Neptune	-1354.80	-43610.38	15633.42	31.99	29 57.61844	31 34.80457

CPU time 0: 0: 0.135
run time 0: 0: 0.085 -- end of run.

Compared to the previous version, program P5 was modified in such a way that, in addition to the positions for Mercury to Mars, those for Jupiter to Neptune are also provided.[2] When examining these locations at Giza, it seems that there are certain priorities. The two positions of Sun and Mars, marked in terms of color, are the most important. Next, the positions of Mercury to Mars on the four dates are significant, followed by the positions of Jupiter to Neptune on the pyramid date. Nevertheless, the vertical coordinates, `z[m]`, for the outer planets are very high or low, respectively, i.e., not close to the Earth's surface. In principle, the calculations can also be performed with the center of the base area of the pyramids (option 333) or with the top of the pyramids (option 335). These cases have the drawback that the uncertainties, `dr[m]`, are, in most cases, very large.

3.4.14 Book option 511

In this run, the date is completely free when comparing the positions of pyramids and planets. The position of Mercury is not restricted to the aphelion or perihelion, but can be anywhere on the orbit. Originally, the search for such events took place with constant time steps around each aphelion passage. As shown in [5], Mercury must always be located close to the aphelion, otherwise no solution exists. In P3, the short version of VSOP87 was used for this search, which can be reproduced in P5 with the quick start option 5. If a constellation was found, the exact date was originally optimized by manually minimizing the relative error *F* with the VSOP87 full version.

Now, it is possible to find all these optimized constellations calculated with the VSOP87 full version within one program run. In the program output provided, the middle section of Table 51 [5] is reproduced with the book option 511, where the centers of mass in each pyramid form the basis of the calculations. This option is a good check for the correctness of Table 51. The search criterion using the error *F* is: $F \leq 0.1$ %.

Output:

```
                PLANETS IN ALIGNMENT WITH THE PYRAMIDS OF GIZA
                      (time not restricted, F minimized)
                              < P5-option 511 >

  VSOP87C, comb. search,     ecliptic of date,       "Sun" free 3D, C-M, FITEX
  Ecl. N and S, years -13000.00 to  17000.00 (c2), tolerance F <  0.50/  0.10 %

 con   k        year   dt[days]    X5      M/10^7    x-Sun  y-Sun  z-Sun  P   F[%]
===============================================================================
    -55865 -11455.192  -1.594   85.176    9.3060  -572.8  339.8  352.1  *  0.059
    -39921  -7615.072  -9.846   99.961    9.0800  -605.0  402.1  184.3  *  0.081
    -23632  -3691.743  -5.645   45.801    9.0661  -483.1  559.7 -168.4     0.092
      3191   2768.567   1.918  168.702   10.5076  -627.8 -134.4 -152.6  *  0.083
 12   4519   3088.413  -0.198   24.801    9.7334  -669.4   28.9  272.6     0.045
     11163   4688.662  10.522  174.347    9.3062  -670.6  232.1 -136.7  *  0.065
     19301   6648.673  -4.921  176.513   10.3385  -654.2 -124.6  -65.2  *  0.015
     31176   9508.827   9.112 -167.460    9.0020  -617.6  406.0 -147.7  *  0.055
 13  39314  11468.834  -7.615  167.389    9.6831  -709.1   -1.4    4.6  *  0.060
     55258  15308.976  -8.037 -159.439    9.2144  -731.5  122.0   -1.2  *  0.025
===============================================================================
  Computed constellations:    313698      (P: polarity, * view from ecl. south)
  Detected constellations:        10       CPU time  0: 0: 7.086
                                           run time  0: 0: 1.805 -- end of run.
```

2 This was proposed by Thorsten Sander, who was interested in the planetary positions at Giza belonging to the outer planets of our solar system. Later, Patrick Wackenhut independently gave me a similar hint.

When comparing this output with the middle columns of Table 51 in [5, p. 347], there are sometimes slight differences in the last digit of `dt`, X_5, and M. These are small deviations due to the manual fit procedure used in [5]. Actually, this option for reproducing Table 51 [5] required a major programming effort, because while the algorithm should be fast, no planetary constellation should be lost. The automatic minimization of `F[%]` with this quick start option is normally more precise than the data in Table 51; however, the numerical differences are negligibly small. Also, the (decimal) year exhibits some small differences from the data in [5]. Generally, the reasons for such deviations are described in section 4.9.

In the above program output, it can be seen that the time interval to the next aphelion passage for constellation 12 is only 0.198 days, which is less than 5 hours. Compared to the other constellations, this is a relatively short time span and confirms the significance of constellation 12.

Another interesting number is the scale factor $M = 9.7334 \cdot 10^7$ connecting the arrangements of planets and pyramids. In [5], a formula was found yielding a theoretical factor by using the "ancient meter" $m_A = 1.003964$ m (see [5, pp. 295 ff.]) and the light-year, ly. The factor is $M_{th} = \sqrt{(1\,\text{ly}/1\,\text{m}_A)} = 9.7073 \cdot 10^7$ [5, pp. 146 ff.]. By applying the present-day meter definition, we alternatively obtain: $M_{th} = \sqrt{(1\,\text{ly}/1\,\text{m})} = 9.7266 \cdot 10^7$. Nonetheless, this correspondence seems to have only low priority.

The approach – center of mass, position free in 3 dimensions, point of time free, and full VSOP87 version – can also be investigated with another special search routine by performing an equidistant scan (in time) around each aphelion passage. The respective quick start option is 518, as mentioned in section 3.2.1. Analog calculations with the vertical coordinate being the top and the base of the pyramid can be performed with the options 517 and 519, respectively.

3.4.15 Book option 512

For the sake of completeness, the corresponding search results for the base and the top of the pyramids are provided in this and the next section.

Output:

```
             PLANETS IN ALIGNMENT WITH THE PYRAMIDS OF GIZA
                   (time not restricted, F minimized)
                           < P5-option 512 >

 VSOP87C, comb. search,     ecliptic of date,       "Sun" free 3D base, FITEX
 Ecl. N and S, years -13000.00 to  17000.00 (c2), tolerance F <  0.50/  0.10 %

con   k       year   dt[days]    X5      M/10^7   x-Sun  y-Sun  z-Sun  P   F[%]
===============================================================================
   -27867  -4711.713   0.506   66.461   9.0734  -500.5  540.1 -219.9      0.027
    -7522    188.293  -8.689   48.948   8.8886  -594.4  485.3  -19.3      0.010
    -3619   1128.353   3.880   43.593   9.5705  -610.8  172.1  353.0      0.088
      450   2108.392   1.963   40.049   9.6579  -619.1  118.7  352.2      0.090
     3191   2768.570   2.792  177.403  10.3604  -657.2 -127.0  -63.4  *   0.056
     7260   3748.586  -0.974  176.875  10.4899  -646.8 -152.7  -36.3  *   0.099
 9  23204   7588.757   9.693 -160.437   9.0651  -654.8  345.4 -115.3  *   0.002
    31342   9548.765  -6.880  161.116   9.8640  -693.6  -37.7   73.0  *   0.064
===============================================================================
 Computed constellations:    312383      (P: polarity, * view from ecl. south)
 Detected constellations:         8       CPU time  0: 0: 7.059
                                          run time  0: 0: 1.794 -- end of run.
```

This option does not yield any significant new information except for constellation 9, which is described in detail in [5, pp. 136 ff. and appendix A14, pp. 314 ff.].

3.4.16 Book option 510

If the tops of the pyramids are applied, constellation 14 in the year 2876 BC (historical counting of years, in which the year AD 0 is missing) catches the eye because of some aspects: 1. The year is close to the time of pyramid construction. 2. The angle X_5 is almost 90°, implying that after coordinate transformation the planetary orbits are aligned more or less perpendicular to the Earth's surface (see also [5, Fig. 152, p. 345]). 3. The z-coordinate of the Sun position is around 30 m, meaning 30 m above the base area of the Mykerinos Pyramid and defining a position close to the Earth's surface (GPS data: 29° 58.01590' N, 31° 7.44308' E). 4. Finally, the error of 0.008 % is rather small. This seems interesting, but the significance is nevertheless still an open question, because until now no other aspects – such as relations to chamber positions, aphelion position, planetary conjunction, or transit – have been found.

Output:

```
                 PLANETS IN ALIGNMENT WITH THE PYRAMIDS OF GIZA
                       (time not restricted, F minimized)
                               < P5-option 510 >

 VSOP87C, comb. search,     ecliptic of date,       "Sun" free 3D, top, FITEX
 Ecl. N and S, years -13000.00 to  17000.00 (c2), tolerance F <  0.50/  0.10 %

con    k        year   dt[days]    X5      M/10^7    x-Sun  y-Sun  z-Sun  P   F[%]
================================================================================
    -52460 -10635.121 -12.524  110.580  10.5759  -454.1   57.2 -364.5  *   0.056
    -40087  -7655.032  -2.068   72.056   9.2056  -632.2  326.3  320.4  *   0.068
    -31272  -5531.943  -9.538    7.890   9.0508  -484.6  556.8  -65.5      0.071
 14 -20240  -2874.807 -11.985   89.600   9.0096  -620.5  404.6   29.5  *   0.008
    -19231  -2631.791 -11.421    6.349   8.9938  -564.6  474.7  -45.2      0.065
    -11425   -751.692   5.961    1.346  10.2126  -668.3  -96.5   98.9      0.095
      3025   2728.585   1.536  144.062  11.3045  -458.4 -161.0 -315.5  *   0.052
      8754   4108.390 -11.259   10.825   9.2711  -705.7  139.3  -26.4      0.048
      8754   4108.413  -2.931   11.157  10.3665  -646.5 -167.4   -9.7      0.096
     19135   6608.690  -5.374  150.647  11.1460  -500.5 -173.4 -259.8  *   0.057
     20476   6931.702   5.369  -56.758   8.9355  -628.0  428.4  216.9      0.041
     20476   6931.723  13.003   59.936   9.3351  -659.1  227.7  -74.4      0.050
     28448   8851.766   2.747   60.701   8.8966  -629.6  430.9  244.1      0.074
     31176   9508.780  -8.038  156.056  10.7402  -553.2 -140.7 -220.3  *   0.084
     36420  10771.831   0.679   64.028   8.8568  -640.9  421.9  248.2      0.093
     39148  11428.850  -8.672  159.103  10.4978  -581.4 -116.2 -203.0  *   0.053
     44392  12691.898  -1.074  -66.295   8.8128  -655.1  410.5  238.0      0.006
     47120  13348.920  -9.247  161.575  10.2507  -607.1  -84.5 -190.1  *   0.045
     48295  13631.980  12.610   66.232   9.5849  -608.3  149.9 -232.6      0.025
================================================================================
 Computed constellations:    326471      (P: polarity, * view from ecl. south)
 Detected constellations:        19       CPU time  0: 0: 7.790
                                          run time  0: 0: 1.987 -- end of run.
```

3.4.17 List of quick start options

In this section, all quick start options are summarized in Table 2. With the special quick start option 999, the input parameters are taken from the input file inedit.t. Note that in this case, the program does not check whether the combination of the edited parameters is allowed or not. The quick start option –804 is a very special one. In the case of Mercury or Venus transits, the serial number of the first detected transit is taken from the list in the file inserie.t. If this file should be modified and created again, it can be done with the option –804. Note that in this case, the original file is not overwritten. Instead, the new data are stored in the file inser-2.t. If necessary, this file can replace the original. Therefore, the original file must be deleted and the new file renamed inserie.t.

Table 2: *Summary of all quick start options for the P5 program with a brief description (key words) of each. More information is provided in the header lines of each output after running the program. The book options allow the results in the book tables to be reproduced and provide a few supplementary tables not given in the books. Options that have been changed or added compared to P4 are 6, 8–10, 16–22 and most of the book 2 options. The main changes have to do with the chamber positions and with Teotihuacán. "Spatial middle" is often replaced by "middle of the east walls." The P5 program comprises all options of the P4 program – sometimes renumbered. (***** These options do not refer to a book table.)*

Option	Brief description
Quick start options	
1	Pyramid positions, Mercury at aphelion, 13,000 BC–AD 17,000, 3D-calc., F_{pos} < 1 %
2	Pyramid positions, Mercury at aphelion, 13,000 BC–AD 17,000, 2D-calc., F_{pos} < 1.5 %
3	Pyramid positions (center of mass), constellation 12 (May 31, 3088) with relevant information, celestial positions at Giza
4	Pyramid positions, time scan around Mercury passage through aphelion, constellation 12 (May 31, 3088), time span 1.5 days
5	Pyramid positions, special search around aphelion passage of Mercury, 13,000 BC–AD 17,000, VSOP87 short version, 3D-calc., F_{pos} (at aphelion) < 3 %, F_{pos} (beyond aphelion) < 0.2 %
6	Positions are the east walls of the chambers in Great Pyramid, Mercury at perihelion, 13,000 BC–AD 17,000, 3D-calc., F_{pos} < 0.8 %
7	Chamber positions, Mercury at perihelion, 13,000 BC–AD 17,000, calculation by solving Kepler's equation, maximum angular deviation 1.85°
8	Chamber positions (east walls), constellation 12 (April 17, 3088) with relevant information, "celestial positions" in Great Pyramid
9	Chamber positions (east walls), time scan around Mercury passage through perihelion, constellation 12 (April 17, 3088), time span 1.5 days
10	Chamber positions (east walls), time not restricted, F_{pos} minimized, AD 2500–3900, F_{pos} < 0.4 %
11	Mercury transits in front of the Sun, geocentric phases, AD 3030–3300 (includes constell. 12)
12	Venus transits in front of the Sun, geocentric phases, AD 1500–4000
13	Triple conjunction, planets Mercury, Venus, and Earth in a line, AD 2900–3300, almost equal ecliptic longitudes, dL < 5° (includes constellation 12)
14	Fourfold conjunction, planets Mercury, Venus, Earth, and Mars in a line, 13,000 BC–AD 17,000, almost equal ecliptic longitudes, dL < 5°
15	TYMT test (test of program performance), Mercury transits, 3000 BC–AD 7000
16	Planetary corr. in Teotihuacán (GPS), units: m, km, logarithmic base: 10, Julian year: AD 200
17	" " " " (map), units: mm, km, " " 10, " " "
18	" " " " (GPS), units: Sun unit, Sun radius, " " 3, " " "
19	" " " " (map), units: Sun unit, Sun radius, " " 3, " " "
20	Teotihuacán, time scan 20,000 BC–AD 4000 (GPS), units: m, km, logarithmic base: 10
21–22	Mercury/Venus transits, 3000 BC–AD 7000, with position angles; the output needs 148 characters per line; these and the following options (except 0 and 111) are not listed in the start menu.
Quick start options for book 2 [14]	
170–171*	Chamber position, Mercury at aphelion, 13,000 BC–AD 17,000, 3D-calc., F_{pos} < 1 %, **170**: east walls, **171***: spatial middle of chambers
180–181	Chamber positions, Mercury at perihelion, **180**: identical to option **6**, except: F_{pos} < 1 %; 13,000 BC–AD 17,000, 3D-calc., **180**: east walls, **181**: middle of chambers
190–195	Alternative mapping of chambers and planets, **190–192**: east walls, 3D-calc., **190**: Mercury at aphelion, mapping of planets: E-M-V, F_{pos} < 1 %, **191**: perihelion, V-M-E, F_{pos} < 1.2 %, **192**: aphelion, V-M-E, F_{pos} < 1.2 %, **193–195**: similar to **190–192**, but middle of chambers and F_{pos} < 1 %
200–202*	**200**: identical to option **10**, except time period; **200**: east walls, **201**: middle, **202***: west walls
210–213	Similar to opt. **9**, east walls, changes in **210**: time span 24 days, time step 12 hours, **211**: 2 days, 1 hour, **212–213**: identical to options **210–211**, except middle of chambers instead of east walls

Table 2: – *continue* –

220–221	Similar to option **4**, positions of pyramids, changes in **220**: time span 24 days, time step 12 hours, **221**: 2 days, 1 hour
230–232*	**230**: identical to option **8** (east walls), **231***: middle of chambers, **232***: west walls of chambers
240–242*	**240**: identical to option **3** (center of mass), **241***: base of pyramids, **242***: top of pyramids
250–251	Identical to option **13**, except time period, **250**: AD 2800–3300, **251:** AD 11,000–11,700
260	Identical to option **14**
270–271	Mercury transits, geocentric phases, **270**: AD 1900–2300, **271**: AD 2900–3300 (constell. 12)
280–281	Venus transits, geocentric phases, **280**: 4000 BC–AD 0, **281**: AD 0–4000
300–301	**300**: four planets in a line, May 17, 3088; **301**: Mercury transit, min. separation, May 18, 3088
310–311	**310**: Venus transit, minimum sep., Dec. 18, 3089; **311**: Three planets in a line, Dec. 23, 3089
320	Search for “shadow-constellations,” pyramids, time not restricted, 3000 BC–AD 7000, $F_{pos} < 1$ %
321	Search for “shadow-constellations,” chambers, time not restricted, 2900 BC–AD 3500, $F_{pos} < 1$ %
322*–323*	“Shadow-const.” (12), pyramids, May 22, 3088, **322***: ecliptic of date, **323***: J2000.0
330–332*	Special output, chambers, const. 12 (Figs. 6, 22, 23), **330**: east wall, **331**: middle, **332***: west wall
333*–335*	Special output, pyr., const. 12, more positions at Giza (Fig. 15), **333***: base, **334**: c-m, **335***: top
338*	Special output, elements of all of the planetary orbits for the year AD 0
370–373	Teotihuacán, **370**–**373** identical to options **16**–**19**
380–381	Teotihuacán, **380** identical to option **20**, option **381**: id. to 380 except map data instead of GPS
Quick start options for book 1 [5]	
390–392	Pyramid positions, Mercury at aphelion, comparison of angles, view from ecl. north, 10,000 BC–AD 10,000, max. angular deviation 1.2°… 1.4°: VSOP87A; VSOP87C; Kepler's equation
400–402	Similar to **390**–**392**, except: view from ecliptic south, 5000 BC–AD 15,000
410–419	Pyramid positions, constellations: 2–4 and 8–14, VSOP98C, spherical heliocentric coordinates
420–429	Identical to **410**–**419**, except: rectangular heliocentric coordinates
430–432	Pyramid positions, constellations 3, 9, and 13, planets Mercury to Neptune, VSOP87A (J2000.0)
440–442	Identical to **430**–**432**, except: VSOP87C (ecliptic of date)
450	Identical to option **2**
460–461	Pyramid positions, reference Mercury orbit, parameters ω, i, and τ, constellations 1–5 and 6–10
470–471	Identical to **460**–**461**, rectangular heliocentric coordinates in Table 47 [5]
480–481	Pyramid positions, reference Venus orbit. Only F_{pos} is given for constellations 1–5 and 6–10.
490–492	Identical to **417**–**419**, pyramid positions. In Table 49 [5], only the parameters X_1–X_7 of the coordinate transformation are given for the constellations 12, 13, and 14.
500–502*	Identical or similar to option **1**, **500**: center of mass, **501**: pyramid base, **502***: top of pyramids
510–512	Pyramid positions, time not restricted, F_{pos} minimized, 13,000 BC–AD 17,000, $F_{pos} < 0.1$ %, **510**: positions are top of pyramids, **511**: center of mass, **512**: center of pyramid base
517*	Special search, similar to option **5**, except: VSOP87C full version, top of pyramids, F_{pos} (at aphelion) < 3.8 %, F_{pos} (outside aphelion) < 0.1 %
518*	Similar to option **517***, except: center of mass of pyramids, F_{pos} (at aphelion) < 3.0 %
519*	Similar to option **517***, except: center of pyramid base, F_{pos} (at aphelion) < 2.1 %
Special options	
0	Individual setting of parameters with separate menus, protected against false input
111	General information: authors, copyrights, and basis of calculations
999	Start with parameters from section 1) in the input file inedit.t, which can be edited manually
–804	Calculates start times of the transit series and creates the file inser-2.t to replace inserie.t

4. Technical and theoretical basis (Giza)

This chapter is a brief description of the archaeological and astronomical basis of the P5 program concerning Giza. For details and further background, see the corresponding references and the source code in appendix A1. When proceeding from the program version P3 to P4 and P5, respectively, some details of geometrical aspects and time systems are slightly changed (see section 4.9).

4.1 Positions on the Giza plateau

Finding the exact coordinates of the pyramid positions and the chamber positions in the Cheops Pyramid is necessary for an accurate comparison with the planetary positions. This is summarized here. For more details see [5, 14].

4.1.1 Positions of pyramids

The positions of the Giza pyramids were measured very precisely within a geodetic network by Sir W. M. F. Petrie [6]. The main distances are provided on the right half of Fig. 11. The "Sun position" was determined here by graphically comparing the pyramid positions with the planetary orbits of Mercury, Venus, and Earth [5, pp. 121 ff.]. From the distance between the "Sun position" and the Mykerinos Pyramid we obtain the angles δ_1 and δ_2. The first search for planetary constellations was done by comparing these angles with the difference in ecliptic longitude of Mercury and Venus, as well as of Mercury and Earth. Later, a 2-dimensional search was performed without a predefined "Sun position." For the 3-dimensional search, the pyramid positions in height were required. The relative level of the pyramid base for the Cheops Pyramid is 0.0 m, for the Chefren Pyramid 10.11 m, and for the Mykerinos Pyramid 12.68 m [9, part IV, map 1]. To obtain the coordinates of the pyramid positions, a coordinate system is defined with its origin in the center of the base area of the Mykerinos Pyramid. As mentioned before, the x-axis points to the north, the y-axis points to the west, and the z-axis points upward (see Fig. 2 and cover page).

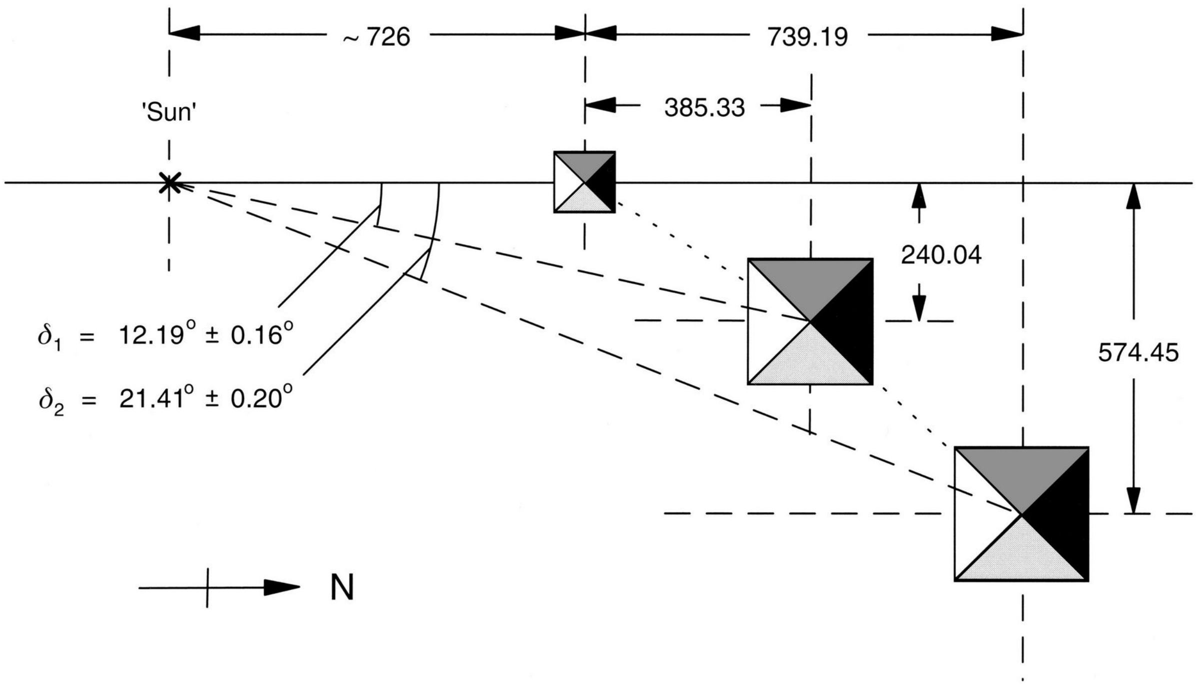


Figure 11: *Geometric relations on the pyramid plateau at Giza. The distances between the pyramids, given in m, were measured by Petrie [6, p. 125] (see also [5, pp. 130 ff.]). For the angular errors, see [5, p. 128]. The distance of 726 m belongs to a simple geometric 1D approach. The 3D calculation (section 3.4.3) yields 667.5 m.*

For the height positions of the pyramids, three different levels were tested to compare with the planetary positions: the ground base, the center of mass, and the top of each pyramid. It can be mathematically shown that the center of mass of a pyramid is located at a quarter of the pyramid height [5, p. 314]. With the coordinate system defined previously, the three pyramid positions have the coordinates listed in Table 3. The original heights of the pyramids are 146.59 m (Cheops Pyramid), 143.70 m (Chefren Pyramid), and 65.14 m (Mykerinos Pyramid), calculated with the base lengths and the pyramid angles from [6] (see also [5, p. 257]).

Table 3: *Coordinates of the centers of the three pyramids at Giza (in m) [6] according to the defined coordinate system. For the z-component, three options are given: the level of the pyramid base, the center of mass, and the top of the pyramid.*

pyramids	***x* [m]**	***y* [m]**	**z_b [m]** (base)	**z_{cm} [m]** (c-m)	**z_t [m]** (top)
Cheops Pyramid	739.19	–574.45	–12.68	23.968	133.91
Chefren Pyramid	385.33	–240.04	–2.57	33.355	141.13
Mykerinos Pyramid	0	0	0	16.285	65.14

4.1.2 Positions of chambers

The exact positions of the chambers in the Cheops Pyramid were taken from the drawings of V. Maragioglio and C. Rinaldi [9]. They used length and angular data from the measurements and publications of Piazzi Smyth [32, 33], John and Morton Edgar [34], Howard Vyse [35], J. S. Perring [36], and Sir W. M. F. Petrie [6].[3] The coordinates of the chamber positions, derived from the given data, are summarized in Table 4. The origin of the coordinate system is placed at the middle axis of the east wall of the Queen's chamber on the ground level of the pyramid (see Fig. 12). The x-axis points to the north, the y-axis points upward, and the z-axis points to the east, which is out of the drawing plane. (Remark: Arithmetically, the shift of the Queen's chamber to $x = 0.103$ m and $y = 23.385$ m yields perfect agreement in AD 3088.)

Table 4: *Coordinates of the three chambers in the Cheops Pyramid (in m, taken from [9]) according to the coordinate system in Fig. 12. Concerning the z-component, three options are given: the middle of the east wall, the spatial middle, and the middle of the west wall of each chamber.*

chambers	***x* [m]**	***y* [m]**	**z_E [m]** (east wall)	**z_M [m]** (middle)	**z_W [m]** (west wall)
King's chamber	–11.05	45.95	0	–5.24	–10.48
Queen's chamber	0	23.54	0	–2.88	–5.76
Subterranean cham.	–5.46	–28.45	0	–7.035	–14.07

The essential (and additional) data are shown in Fig. 12. The numbers in italic are calculated here from the given data. The numbers are not always consistent because in the charts of Maragioglio and Rinaldi the data are based on various measurements performed by different researchers. If different data exist, the deviations are in the range of one or a few centimeters and are not relevant for the astronomical comparison. In one case, the number has not been taken from the reference: The horizontal distance from the northern baseline of the pyramid to the end of the descending corridor is given as 107.39 m [9, Part IV, Map 3]. This distance could not be measured directly, but

3 The full texts of several of these old references, available on the Internet, are extremely interesting. The references [32–36] are examples for each author. It was not checked whether all the data in Fig. 12 is found therein.

had to be calculated from other distances and angles. In Fig. 12, we obtain this length by the sum of 94.27 m + 13.33 m = 107.60 m. Thus, we rely primarily on data that can be measured directly, e.g., the length of the descending corridor, and thus avoid possible inaccuracies from the references. At this point, it must be said that the work of Maragioglio and Rinaldi shows outstanding quality because their encyclopedic drawings provide a huge amount of measured data and valuable detail.

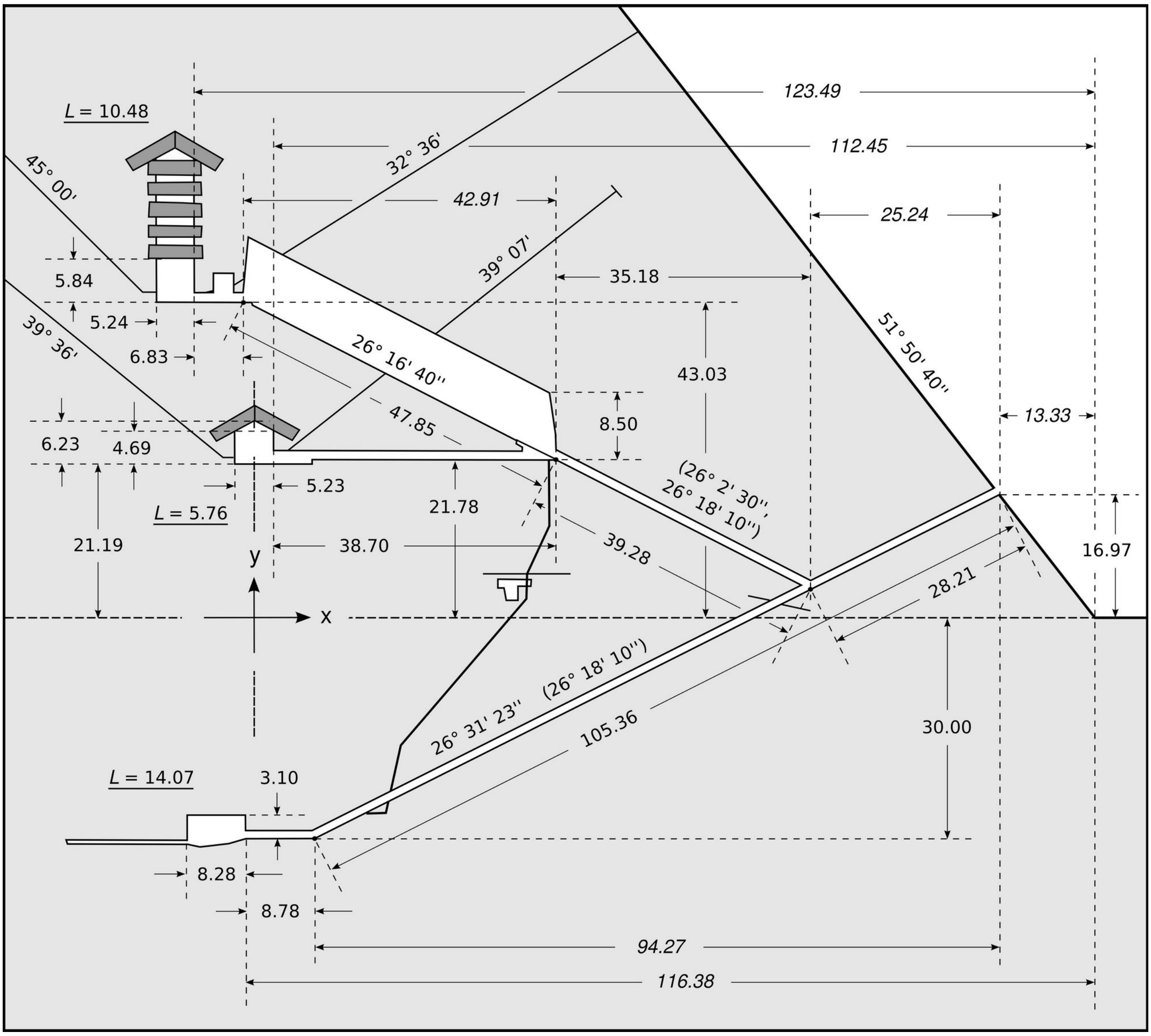


Figure 12: *Inner construction of the Cheops Pyramid as seen from the east, with the linear measurement data given in m. The technical data are taken from detailed drawings of V. Maragioglio and C. Rinaldi [9, part IV, maps 3–7]. The numbers in italic are additionally calculated from the other data. The underlined quantities L are the lengths of the chambers, extending horizontally from the east walls of the corridors to the west (into the depth of the drawing).*

4.2 VSOP – planetary positions

The VSOP87 planetary theory was developed by P. Bretagnon and G. Francou (Bureau des Longitudes, Paris, today the IMCCE, Institut de mécanique céleste et de calcul des éphémérides) [1, 2]. VSOP stands for Varations Séculaires des Orbites Planétaires and 87 is the year (1987) of publication. As noted in the introduction, the files for the VSOP87 theory can be downloaded from the FTP server on the IMCCE website (URL 3).

VSOP87 allows for the calculation of the positions of the planets of our solar system (Mercury to Neptune) with a very high precision as a function of time. The theory includes all gravitational perturbations between the planets and relativistic effects. It is valid for a time period ranging across several thousand years into both the past and future. Although the theory was further improved (VSOP2000, VSOP2002 by Fienga and Simon [37], VSOP2013 by Simon et al. [38], and INPOP19a by Fienga et al. [39]), the accuracy of VSOP87 is perfectly sufficient for our purposes. The available six VSOP87 versions, which differ in the type of coordinate system used, are listed in the following table.

Table 5: *The six VSOP87 versions. The full versions VSOP87A and VSOP87C and a short version of VSOP87D [18, pp. 381 ff.] are used here.*

version	kind of coordinates	coordinate system
VSOP87	Heliocentric ecliptic orbital elements (elliptical coords.)	equinox J2000.0
VSOP87A	Heliocentric ecliptic rectangular coordinates	equinox J2000.0
VSOP87B	Heliocentric ecliptic spherical coordinates	equinox J2000.0
VSOP87C	Heliocentric ecliptic rectangular coordinates	equinox of the date
VSOP87D	Heliocentric ecliptic spherical coordinates	equinox of the date
VSOP87E	Barycentric ecliptic rectangular coordinates	equinox J2000.0

4.2.1 VSOP87 full version

For technical information, a brief summary is quoted from [1]: *"The VSOP82 solution is made of the perturbations developed up to the third order of the masses for all the planets. Perturbations up to the sixth order obtained by an iterative method complete the theory of the four outer planets. It also contains the perturbations of the Moon onto the Earth-Moon barycenter and the relativistic perturbations expressed in isotropic and standard coordinates. The integration constants are determined by adjustment to the numerical integration DE200."* The planetary and lunar ephemerides DE200 are based on numerical integration and interpolation (JPL, Jet Propulsion Laboratory, E. M. Standish et al. [40, 41]).

The next VSOP version, VSOP87, was improved in such a way that the planetary positions no longer need be calculated from the orbital elements based on elliptical coordinates. Instead, the positions are given directly in rectangular variables X, Y, Z, and in spherical variables L, B, r, being the longitude, latitude, and distance of a planet to the Sun (r = radius). Two different kinds of coordinate systems are used: the standard equinox J2000.0 and the dynamical equinox (equinox of the date). The reference J2000.0 is a fixed system and is directly linked to the reference frame of DE200. The conversion from the standard system J2000.0 to the equinox of the date is performed with a precession matrix as a function of time. This matrix, taking into account the precession of the Earth's axis, is valid for several thousand years in the past and in the future. Although further developments lead to the new versions DE405 and DE406, we guess that the modifications to DE200 are slight. Comprehensive information about further developments is provided in *JPL Planetary and Lunar Ephemerides* (URL 11) and, e.g., in a recent publication by Park et al. [42].

Here, the full versions VSOP87A and VSOP87C are applied using rectangular coordinates. All theoretical input of the VSOP87 theory is finally expressed in analytical expressions of rectangular coordinates in terms of periodic series and Poisson series. These sums contain up to several thousand parameters $A_{\alpha n}$, $B_{\alpha n}$, and $C_{\alpha n}$, where the index α runs from 0 to a maximum of 5 and n from 1 to a maximum of 2,047 (for Saturn). As an example, the analytical expression for the X-coordinate of a planet is given as a function of the time τ with $\tau = (JDE - 2{,}451{,}545.0) / 365{,}250.0$:

$$X(\tau) = \sum_{\alpha=0}^{\alpha(\max)} \sum_{n=1}^{N(\alpha)} \tau^{\alpha} \cdot A_{\alpha n} \cdot \cos(B_{\alpha n} + C_{\alpha n} \tau) \quad (4)$$

N becomes smaller as α increases. The expressions for the *Y*- and *Z*-variables are analog. The conversion to appropriate spherical coordinates and other rectangular coordinates is performed separately in the P5 program. From Eq. (4) and the corresponding equations for *Y* and *Z,* it is easy to obtain the current velocity of the planet by calculating the derivatives with respect to time (τ). Thus, we obtain, for example, the x-component of the velocity by:

$$v_X(\tau) = \sum_{\alpha=0}^{\alpha(\max)} \sum_{n=1}^{N(\alpha)} \left(\alpha \tau^{\alpha-1} \cdot A_{\alpha n} \cos(B_{\alpha n} + C_{\alpha n} \tau) - \tau^{\alpha} \cdot A_{\alpha n} C_{\alpha n} \sin(B_{\alpha n} + C_{\alpha n} \tau) \right) \quad (5)$$

Nevertheless, velocity is not required in P5; thus Eq. (5) is provided here because the calculation is quite simple. (The relevant program lines in the VSOP subroutine were converted to comment lines.) In addition to other parameters, the coefficients $A_{\alpha n}$, $B_{\alpha n}$, and $C_{\alpha n}$ are stored for each planet in one file, e.g., VSOP87A.mer for Mercury and the standard equinox J2000.0. To improve the speed of computation, these coefficients are read only once from the file (from hard disk or solid-state drive) and are stored for all coordinates *X*, *Y*, *Z*, and for all planets in a single five-dimensional array for direct access. The subroutine VSOP87 has been adapted accordingly and renamed VSOP87Z. Details about application of the gravitational theory and the derivation of the analytical results, respectively, can be found in [1, 2]. More technical information is provided in the files README-vsop87 and vsop87.doc (Tab. 1), and in the source code in the appendix A1.

4.2.2 VSOP87 short version

In *Astronomical Algorithms* by Jean Meeus, the most important periodic terms of the VSOP87D version (spherical coordinates) are compiled in table form [18, app. II, pp. 381–422]. The tables contain different coefficients, *A*, *B*, and *C*, also belonging to Eqs. (4) and (5), but the series are shortened by more than 95 % and the number of decimals is also reduced. After the appropriate conversion to rectangular coordinates, the results of the VSOP87D short version can be compared directly with the VSOP87C full version because both alternatives are based on the mean equinox of the date. The accuracy of the short version is not much lower than that of the VSOP87C full version. For the year 3088, the difference in the ecliptic longitudes and latitudes between the short and the full version of VSOP87 is approximately 0.0001°, which is less than 1 arc second.

4.2.3 Orbital elements and Kepler's equation

In an alternative method, we use the orbital elements listed in [18] as polynomials of third degree as a function of time in the following form:

$$a_0 + a_1 T + a_2 T^2 + a_3 T^3 \quad (6)$$

These were derived from VSOP82 [1]. The coefficients a_0 to a_3 are given for the mean equinox of the date and also for the standard equinox J2000.0 ([18, pp. 200–204] and invsop3.t in Tab. 1). The time *T* is measured in Julian centuries:

$$T = \frac{JDE - 2451545.0}{36525} \quad (7)$$

The six orbital elements for each planet are [18, pp. 197 ff.]:

L_c = mean longitude of the planet
a = semi-major axis of the orbit
e = eccentricity of the orbit
i = inclination on the plane of the ecliptic
Ω = longitude of the ascending node
π = longitude of the perihelion

The mean longitude L_c is the longitude of a body if its orbit were circular; therefore, we use the subscript *c*. To obtain the real position of the planet, we must solve Kepler's equation [18, pp. 184 ff.] with respect to the eccentric anomaly *E*:

$$E = M + e \cdot \sin E \tag{8}$$

The mean anomaly *M* is given by $M = L_c - \pi$. Because Kepler's equation is a transcendental equation, it can only be solved numerically. Different iterative methods exist to find the roots of a function *f*, as follows:

$$f(E) = M + e \cdot \sin E - E = 0 \tag{9}$$

The following three methods are included in the P5 program: the Newton-Raphson method, the fixed point method, and the secant method. Only the first method is actually used. (The other methods can be activated by changing the value of the parameter “meth” in the P5 source code.) The Newton-Raphson method is fairly rapid and is appropriate because the derivative $f'(E) = \partial f(E)/\partial E$ can be determined analytically. In our case the corresponding equation is:

$$E_{n+1} = E_n - \frac{f(E_n)}{f'(E_n)} = E_n + \frac{M + e \cdot \sin E_n - E_n}{1 - e \cdot \cos E_n} \tag{10}$$

An iterative application of Eq. (10) yields the solution *E*, satisfying Eq. (8). The index *n* refers to the *n*th iteration. By using

$$\tan\frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}} \cdot \tan\frac{E}{2} \tag{11}$$

from [43, p. 36] we get the true anomaly ν. The inclinations of the orbits of Mercury and Venus are only a few degrees and thus the true anomaly of a planet is nearly identical to its ecliptic longitude. The first search for planetary constellations was performed by comparing differences in ecliptic longitude of Mercury, Venus, and Earth with the corresponding angles δ_1 and δ_2 in the pyramid area (see Fig. 11). Therefore, a good test of the results was to compare differences of the true anomalies with the same angles. Using Kepler's equation, all of the main constellations are found, although a 3-dimensional search is not performed here. The ecliptic latitude *B* and radius *r* (distance to the Sun) are not calculated in this search option. The computation time is much shorter than with the other VSOP87 versions.

4.2.4 Accuracy of the theory

The accuracy of VSOP87 for Mercury, Venus, the Earth-Moon barycenter, and Mars is better than 1” (1 arc second) within the time period of 2000 BC to AD 6000 and about 1” at both ends of this time span [2]. For Jupiter and Saturn, the same precision is valid for the years AD 0 to 4000 and for Uranus and Neptune from 4000 BC to AD 8000 [2]. For our purposes, which are a comparison with the positions at Giza, 1 arc second is much better than required; and for the important year AD 3088, the precision is even better. The question is: What precision do we have for years further into the past or future, for instance for the year AD 15,000?

An accurate answer to this question is not easy as no additional information was found in [1, 2]. The deviation of the theory does not increase linearly with time, but more strongly. A vague possibility is to compare the full VSOP87 version with the short VSOP87 version [18]. If we calculate the planetary positions for the beginning of the years 2000 BC and AD 6000, the differences in ecliptic longitudes and latitudes between both theory versions are also around 1 arc second for the planets Mercury to Mars. This is the same value as the accuracy of the full version alone. If we do the same for the years 13,000 BC and AD 17,000 for angles like ($L_M - L_E$), the corresponding differences of the theory versions have a magnitude of 0.1° or 0.2°. Thus, even for these deviations, the precision is good enough for comparison with the pyramid positions, the chamber positions, and also for checking the planetary conjunctions (dL < 5°.) On the other hand, a deviation of 0.2° or even 0.1° is not precise enough to determine the exact transit data of Mercury and Venus. In this case, the errors of the corresponding position angles have an order of magnitude of 45°. Therefore, the transit calculations are valid primarily from 2000 BC to AD 6000, and because the year AD 3088 is well in this range, the relevant calculations are unproblematic. Another possibility (not concerning precision) is comparing VSOP87A and VSOP87C. The angular difference $L_M - L$ should be nearly the same and the distances r should be identical. (The results are very similar to the previous test.) Nevertheless, the time span allowed for applying VSOP87 is limited to the range 13,000 BC to AD 17,000. In addition, when computing planetary transits more than 4000 years into the past or the future, the user should be aware of this increasing uncertainty.

In order to check the results, the calculations were also performed using the orbital elements of the planets given by Meeus [18, pp. 200−204] and by solving Kepler's equation (subroutine vsop3, see, e.g., quick start option 7). In the present age, its accuracy is a bit lower than that of the other VSOP87 subroutines. However, when using years further in the past or future, the deviations do not increase as strongly as with the other routines, especially if the differences of ecliptic longitudes are computed. Therefore, this routine can be used over a longer time period and the time limits are set to 30,000 BC and AD 30,000 (see Fig. 13 and last paragraph of section 4.9.1).

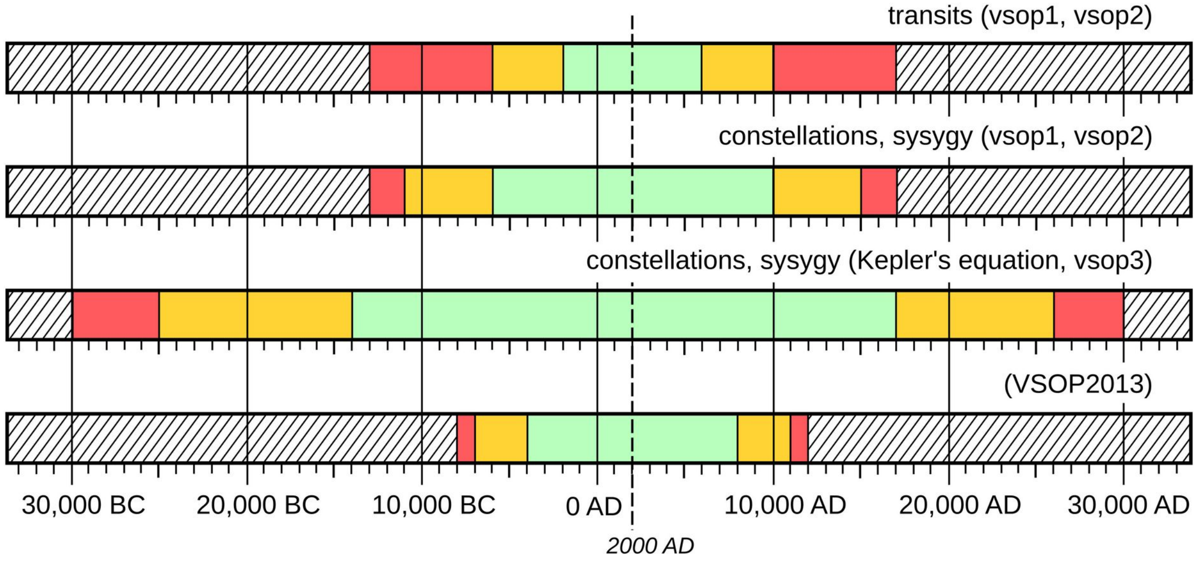


Figure 13: *Estimated time periods with different precision of the astronomical calculations. The colors represent the following: light green: relatively high to very high precision; yellow: precision acceptable, care should be taken; red: larger deviations and errors possible; hatched area: years are out of range, error message. Three different subroutines exist in the P5 program, which are based on the VSOP theory: vsop1 (VSOP87 short version), vsop2 (VSOP87 full version), and vsop3 (orbital elements according to VSOP82, Kepler's equation). Additionally, the valid range of the new version, VSOP2013, is shown, but this version is used neither in the P4 nor in the P5 program.*

The theory versions do not have the same basic precision of validity in their ranges and different precisions are required depending on their application. This means that in Fig. 13 the transitions from green to yellow and from yellow to red for different versions do not always imply the same accuracy. Thus, the colors within one version have “relative” character and the diagram should be seen as a qualitative overview.

Meanwhile, the new version VSOP2013 [38] (Fig. 13) has been tested in more detail. Its accuracy is a few 0.1 arc seconds in the range 4000 BC to AD 8000. After several runs of VSOP2013 with known dates of the planetary correlation, some technical aspects arose, which suggests that for the given purpose VSOP87 is sufficient and even more suitable.

4.2.5 VSOP2013

Some years ago, the new version VSOP2013 was published by J.-L. Simon, G. Francou, A. Fienga, and H. Manche [38]. Generally, this version is a huge improvement concerning the exact determination of the planetary positions. It can be downloaded from the FTP server of the IMCCE and, in its range of validity, it is approximately five to ten times more precise than VSOP87. In principle, it would be nice to have it included in the P5 program. Nevertheless, for the comparison of positions between planets and pyramids, as in P3, P4, and P5, the precision of VSOP87 is better than actually needed. However, apart from this and concerning the comparison of planetary and pyramid positions, there are some other technical aspects that are described below.

1. *Range of validity:* The planetary coordinates are given analytically in a series of trigonometric functions and powers of time t (e.g., t, t^2, t^3, …). In VSOP87, the time is included up to the 5th power, whereas in VSOP2013 the powers go up to 10 and more. As a consequence, the uncertainty of the calculated positions *outside* the valid time period increases much faster in VSOP2013 than in VSOP87 (see also Fig. 13). Thus, for our purposes, the period of acceptable (lower) accuracy is larger for VSOP87 than for VSOP2013. Or, in other words, the “runaway properties” are less pronounced in VSOP87.

   But how can we check this? The answer is given by the planets themselves. In addition to the possibilities for VSOP87, described in the last section, another method exists. After one (sidereal) year, the Earth is located again at the same point on its orbit. If we wait 1000 years and assume the sidereal year to be almost constant, the Earth should deviate only little from this place 1000 years before. And this is true for all times. More precisely, the sidereal orbital period of a planet changes only very little and not drastically over 1000 years. With the VSOP2013 program, it is easy to perform calculations for a series of dates. One millennium is about 365,256 days. We use 365,256.4587 days and the starting point J2000 and concentrate on the x-coordinate of Earth. Now, a program run is started beginning at AD 2000 (*JDE* = 2,451,545.0) going in 12 steps of 1000 years each into the future and into the past. The coordinates of Earth should change very slowly and more or less monotonically. If the changes become more drastic and “irregular,” it is obvious that the calculations are no longer valid. The estimated range of validity of VSOP2013 can be seen in Fig. 13. Correspondingly, the reader can also check this with other planets.

2. *Transit calculations:* In principle, the higher accuracy of VSOP2013 between 4000 BC and AD 8000 would allow for a better prediction of geocentric Mercury and Venus transits. The problem is that in VSOP2013 the Earth is not included as a body but rather as the Earth-Moon system. Its barycenter is located inside the Earth about 4,670 km away from the Earth’s center. For Venus, this would mean a maximum angular shift of approximately 23 arc seconds, which is

too much to obtain precise transit data. If comparing planetary constellations and pyramid positions, this effect can be neglected, but not if calculating transits.

3. *Processing speed:* The amount of data for the periodic series is approximately 50 times larger in VSOP2013 than in VSOP87. Thus, we expect the computation time to also be 50 times longer for VSOP2013. This was tested with the given application program for VSOP2013. The result was, in fact, a factor of 100. (The time of reading the data from disk was not considered.) It follows that, rather than a few seconds, the computations would last some minutes. This would be acceptable, but another problem arises.

4. *Program modification:* For rapidity of computation, Bretagnon and Francou proposed reading all of the data once for all into memory for direct access. In P4 and P5, the data for all of the eight planets exist simultaneously in the RAM memory. In the attached VSOP2013 program, this is true only for one single planet at the same time. When using another planet, the data of the preceding planet in the memory are overwritten. Thus, every time a new planet is used, VSOP2013 must read the data again from the hard disk or SSD, even if this planet has been used before. (Remark: For the published VSOP2013 version, this seems reasonable, because storage of the entire, huge amount of data in the RAM memory should be adapted to the given hardware and to the kind of application.) Therefore, if directly implementing the VSOP2013 subroutines into P5, this would slow down the computation even more. In order to handle this, the VSOP2013 program has to be considerably modified. Nevertheless, if this is done, the factors of 50 or 100 from point 3 above still remain.

5. *Estimated improvement:* The approximate effect of using VSOP2013 can be estimated geometrically. We assume that the planet Earth is implemented in addition to the Earth-Moon system, which in principle is possible in the future. Let us say that the results of VSOP87 and VSOP2013 differ by 1 arc second. The longest applicable distance in the Great Pyramid for the comparison of chamber and planetary positions is the distance between King's chamber and the subterranean chamber. Along this length of 74.6 m (calculated according to Tab. 4), 1 arc second implies a position shift of 0.36 mm. For the pyramid positions at Giza (Fig. 1), we accordingly have a shift of 4.5 mm. Both values are not sufficient to have any significant effect on the calculated results. For shorter distances, the effect becomes even smaller.

Hence, VSOP2013 is not implemented in P4/P5. Once more, it should be stressed that VSOP2013 is one of the most accurate theories worldwide concerning planetary positions, but for our subject, it would not yield much improvement compared to VSOP87. It might possibly be useful to include a future upgrade of VSOP. In this case, it would probably make sense to again parallelize the P5 program. Instead of trying to improve P5 at the present time, it seems more reasonable – as previously noted – to check the general results of P5 by writing an independent, new program. Another programming language could be used and, if possible, a theory other than VSOP.

4.2.6 Single- and multi-thread versions of P5

In order to obtain a higher processing speed, some "hot spots" in the second and third editions of the program (P4, P5) were parallelized with the application programming interface (API) OpenMP. The modified subroutine VSOP87X was renamed VSOP87Y in P4 and VSOP87Z in P5. In P4, the subroutine has been adapted according to four threads because the processor used has two cores with hyper-threading. Therefore, the corresponding file names belonging to P4 have an additional 4. In P5, the parallelization was implemented in a more general way, which enables to use any number of (multiple) threads and, thus, the P5 file names have an additional m. The compilation of the source code p5.f95 with the GNU Fortran compiler was performed with the command: `gfortran -fopenmp -static-libgfortran -O3 -Wall p5.f95 ↩`. The single thread version of P5 was obtained by: `gfortran -static -O3 -Wall p5.f95 ↵`, as previously mentioned. While P4 has two slightly different source codes (p4.f95 and p4-4.f95) for single and multiple threads, P5 has only one version, p5.f95, for both.

Now, the calculated *combined* CPU time is longer than the run time of P5. Thus, the execution time, especially of the TYMT test (64-bit version, see page 18), is determined not only with the subroutine CPU_time but also with date_and_time. The run time decreases from 49 s to about 22 s, and with a small terminal window of three lines to 20 s.

After upgrading Ubuntu from 16.04 LTS to 20.04 LTS (later 24.04 LTS) and reinstalling GFortran, a strange phenomenon appeared. The single-thread version of the TYMT test now runs four times faster and the multi-thread version two times faster than before. Now, both versions need a run time of around 10 seconds. The huge speedup is possibly based on a better utilization of the 64-bit hardware by the GFortran compiler. The reason that the multi-thread version is not faster than the single-thread version, as before, could be a limited stack size, a limited memory bandwidth, or something else. (An automatic parallelization by the option `-O3` in the single-thread compile command is not performed because the task manager shows the activity of only one thread.) Nonetheless, the results are identical and the user can decide which version they would like to use.

4.3 Relation between pyramid and planet positions

Since my schooldays I have been a physics enthusiast and I have worked in the field of nuclear physics for more than five years. Thus, before we continue to relate pyramids and planets, a remark should be made – see footnote.[4]

How can we compare the positions of the pyramids with those of the planets? As a first attempt, the arrangement of the planetary orbits was drawn to scale on a large sheet of paper, as in Fig. 14. The arrangement of the pyramids was then added in such a way that the Cheops Pyramid was placed on the Earth's orbit, the Chefren Pyramid on Venus' orbit, and – if possible – the Mykerinos Pyramid on Mercury's orbit. The allocation of the pyramids to the planets was performed according to Eqs. (1)–(3). In Fig. 14, five arrangements, A to E, of the pyramid positions are provided, where Earth and Venus (the Cheops and Chefren Pyramids) are always located exactly on their orbits. In most cases, the planet Mercury would not reach its orbit. It is only for the case when Mercury is placed at or near to the aphelion that all three planets are arranged correctly (positions A, Fig. 14).

4 The following mental leap can be seen either as a philosophical comment or as an interesting anecdote. It is obvious that the shapes of planets and pyramids can be symbolized by the two basic figures of circle and triangle. Does the reader know that a circle exists on this planet that is more than 100 m underground with a size much bigger than the pyramids? It has an extent of several km. On the other hand, the gigantic triangle is planned to be realized in the future, and it will probably also be built more than 100 m underground with a dimension of many km. This is a riddle for the reader. If the reader likes, they may try to find out what this is before continuing reading.

We are referring to the Large Hadron Collider (LHC) of the European research organization CERN. It is the largest particle accelerator and the largest machine worldwide and is used for high energy physics. It does not have the exact shape of a circle, due to the presence of some linear sections, but the deviations are small and it is almost a circle. The LHC is positioned between 50 and 175 m underground and has a circumference of 26.659 km. From outside it is invisible, and it is probably impossible to remove it (unless the machines are taken out and the tunnels and giant cavities are filled with rubble). This, thus, is the circle – but what about the large-sized triangle? Does the reader have an idea?

The corresponding project is the Einstein Telescope ET, which will be used in astronomy and astrophysics as a detector of gravitational waves. It will be located deep underground and will have the shape of an equilateral triangle with a side length of 10 km, meaning a circumference of 30 km. It is currently in the planning phase, although where it will be built has not been decided. To the author, this is a funny coincidence. The two most unique and biggest machines in the world – both multi-billion dollar projects – have (or will have) the shapes of a circle and a triangle. (Although mainstream scientists are very successful in physics and astronomy, the scientific understanding concerning archaeological structures can be still improved.)

The aphelion is the position on the orbit of maximum distance to the Sun. Thus, in the following, Mercury has to stand at the aphelion or near to it; otherwise, no solution exists. Interestingly, in Eq. (3), when defining the relation between Mercury and the Mykerinos Pyramid, the aphelion distance of Mercury is used – a remarkable coincidence! In Fig. 14, the north−south alignment of the pyramids also correlates with the main symmetry axis of Mercury's orbit. In principle, this is not necessary in this geometric test, but for the final planetary constellation this is almost the case. The distance from Mercury to the Sun in arrangement A, which is the aphelion distance, can be transferred to the Giza plateau. With this simple geometric (1-dimensional) approach, the distance was determined to be 726 m (compare with Fig. 11).

Later, it became clear that the 3-dimensional approach is the most significant (constellation 12), whereas Fig. 14 takes into account only two dimensions. However, the figure illustrates the main principle very well.

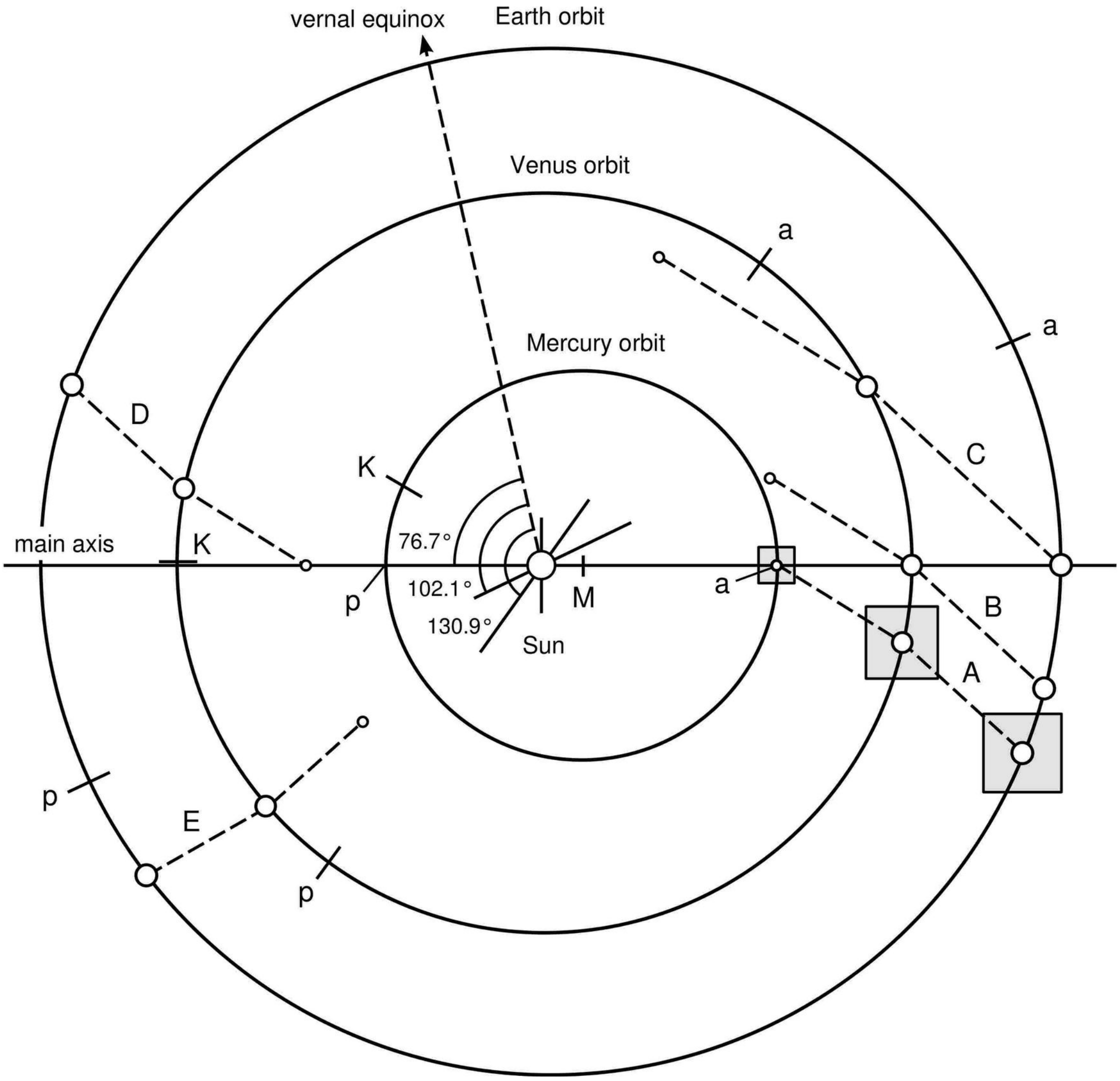


Figure 14: *Approximate to-scale representation of the planetary orbits of Mercury, Venus, and Earth. K represents ascending node, p perihelion, a aphelion, and M the middle of the Mercury orbit. The polygons A to E illustrate each the arrangements of the three great pyramids of Giza. The constellation A fits almost perfectly to the pyramid positions. A solution exists only if Mercury is placed at or near to the aphelion. For better visibility, the planets were magnified by a factor of 500 and the Sun by a factor of 6.*

The next question is: Does this situation ever occur? And, if yes, when? At first, we start with the assumption that Mercury is positioned exactly at its aphelion. What we need are the dates when this actually happens. Fortunately, Jean Meeus derived a formula with the VSOP87 theory for all moments when Mercury stands at the aphelion [18, p. 253]. The Julian date, used here, is:

$$JDE \;=\; 2{,}451{,}590.257 + 87.969\,349\,63\,(k - 0.5)\,, \tag{12}$$

where k is an integer number. For $k = 0$, we obtain the first aphelion passage of Mercury after the beginning of the year 2000. Replacing $(k-0.5)$ with k yields the perihelion passages, respectively. Now, with these aphelion dates we can start to compare pyramid and planetary positions.

4.3.1 1-dimensional comparison

The most simple approach is to compare the angles δ_1 and δ_2 on the Giza plateau (Fig. 11) with the corresponding differences of ecliptic longitudes (L). More precisely, if the index M stands for Mercury, V for Venus, and E for Earth, the following equations must be valid: $L_M - L_V = \delta_1$ and $L_M - L_E = \delta_2$ within a given tolerance. In this case, the "Sun position" is placed exactly south of the Mykerinos Pyramid. Another option is to locate the "Sun position" south of the Chefren Pyramid with the angles δ_1 and δ_2 being adapted accordingly (section 3.3.10). In this "1-dimensional" comparison, only one parameter, the ecliptic longitude L, is considered. For the calculation of the relative error F_{pos}, which is another measure to evaluate a detected constellation, see [5, pp. 133 ff.].

4.3.2 2- and 3-dimensional comparisons

The cases of 2- and 3-dimensional calculations are treated in a similar way. Therefore, we start with the 3-dimensional calculation. Let ***a*** be the vector from the Mykerinos Pyramid to the Chefren Pyramid, ***b*** the vector from the Mykerinos Pyramid to the Cheops Pyramid, and, accordingly, ***a'*** the vector from Mercury to Venus and ***b'*** the vector from Mercury to Earth. The vectors ***a'*** and ***b'*** are derived from the planetary positions, which were calculated before with VSOP87. For example, for the "pyramid date" of constellation 12 [5, 14], JDE = 2,849,079.76330, we find:

$$\boldsymbol{a} \;=\; (385.33,\, -240.04,\, 17.07)^{\mathrm{T}} \tag{13}$$

$$\text{and} \qquad \boldsymbol{a'} \;=\; (0.238520,\, -0.172709,\, 0.035794)^{\mathrm{T}} \tag{14}$$

The numbers in Eq. (13) are given in m and are calculated from Table 3 (compare coordinate system in Fig. 2), and those in Eq. (14) in AU. The astronomical unit (1 AU = 149,597,870.70 km) is the mean distance between the Earth and the Sun. The superscript T means transposed. In this example, the vector ***a*** connects the barycenters of the pyramids. With a and b being the lengths of the vectors ***a*** and ***b***, let $p = b/a$ be the ratio of the distances between the pyramids and $q = b'/a'$, accordingly, the ratio for the corresponding planets. Furthermore, let δ_p be the angle between ***a*** and ***b***, which can be calculated with the inner product $\boldsymbol{a}\cdot\boldsymbol{b}$ by $\delta_p = \arccos(\boldsymbol{a}\cdot\boldsymbol{b}/(a\cdot b))$, and δ_q the corresponding angle between ***a'*** and ***b'***. Now, the conditions $p = q$ and $\delta_p = \delta_q$ imply that the alignments of the pyramids and planets are identical. Because these equations are (probably) never exactly valid, we define the following relative deviation F''_{pos} in percent (δ_p and δ_q in radians) [5, pp. 326, 339]:

$$F''_{pos} \;=\; 100 \cdot \sqrt{\frac{1}{2}\left(\left(\frac{q-p}{p}\right)^2 + (\delta_q - \delta_p)^2\right)} \;\; [\%] \tag{15}$$

For the 2-dimensional calculation, the positions of the pyramids are projected onto the surface of the Earth and the positions of the planets onto the ecliptic plane. Thus, the z-component of each vector is set to zero and Eq. (15) can also be used. The analog name in [5] for the 2-dimensional case is F'_{pos}. Concerning the pyramids and the 3-dimensional approach, the relative deviation of the main constellation 12 is only 0.07 %. In the case of the chambers in the Cheops Pyramid, the calculations are analogous, with the only exception being that the 2-dimensional calculation is not realized. Finally, the 3-dimensional calculation appears to be the most reasonable. The "Sun position" is not predefined and the date is not necessarily restricted to aphelion passages. Note: The factor 1/2 in Eq. (15) causes F'_{pos} and F''_{pos} to be more "relative error per coordinate" than "relative error," which simplifies the comparison of the 1-, 2-, and 3-dimensional calculations (not mentioned in [5].)

4.4 Two fit programs

Different programs for iterative fitting and computing of data are used in P5. Two of them are described in this section in more detail. The first (FITEX) is more complex and was written by G. W. Schweimer. I kindly had access to it from the KfK (Kernforschungszentrum Karlsruhe, today KIT) where I did my PhD. The second (ringfit) was created to improve the processing speed of P4. The improvement is only small, but the equation used seems interesting. Therefore, it is used to calculate the transit phases. Other fit algorithms applied in P4 and P5 are described in their astronomical context in sections 4.2.3, 4.7.1, and 4.7.2.

4.4.1 FITEX

The description here is written on the basis of the program description given by Dr. G. W. Schweimer (KfK, Cyclotron Laboratory, today KIT) [16, 17, and 5]. Originally, the code was written in FORTRAN IV, but has now been adapted to the new compilers GNU Fortran and Intel® Fortran. The program consists of four subroutines (the last four subroutines in P5) and allows us to solve the nonlinear least squares problem. It uses a least squares interpolation between variables and functions or the exact gradient of the functions.

Very often in scientific measurements the problem exists of finding some parameters in a mathematical model, so that the measured data are reproduced by the model in terms of a least squares fit. The mathematical problem is solved if the minimum of the Euclidean norm of the vector $\boldsymbol{F}$ is found by varying the parameter vector $\boldsymbol{X}$:

$$|\boldsymbol{F}(\boldsymbol{X})| = \text{minimum} \tag{16}$$

The components of the vector $\boldsymbol{F}$ are the differences between measured values $\boldsymbol{Y}$ and model values $\boldsymbol{Z}(\boldsymbol{X})$ in terms of the measuring errors $\Delta\boldsymbol{Y}$:

$$F_\mu = \frac{Y_\mu - Z_\mu(X)}{\Delta Y_\mu} \quad , \quad \mu = 1 \ldots m, \tag{17}$$

where μ counts the single data points. For the most part, the solution of Eq. (16) can only be found numerically. Therefore, an optimum procedure does not exist. In the given method the following information about the vector $\boldsymbol{F}$ is used:

1. The vector $\boldsymbol{F}$ has at least as many components as the vector $\boldsymbol{X}$.
2. The solution vector $\boldsymbol{X}$ is known approximately, i.e., the range for each component X_i, $i = 1 \ldots n$, is known.

The functions F_μ are calculated in the main program (P5) by the user. The subroutines of the search program are embedded in the main program and are connected through a question−answer relationship. The search program calculates the expected best vector of parameters and asks the main program for the values of the functions. The main program answers with the function values. The search program stores these values in the memory and asks again, if necessary.

The minimum of the Euclidean norm of the vector ***F*** can be found with an iterative procedure. The estimated best vector of parameters $\boldsymbol{X}_{new}$ is obtained by the linear approximation of the functions ***F***. The linear approximation is

$$\boldsymbol{F}_{lin}(\boldsymbol{X}) \;=\; \boldsymbol{H} + \boldsymbol{G}\cdot(\boldsymbol{X} - \boldsymbol{C}) \qquad \text{with} \qquad \boldsymbol{X} \neq \boldsymbol{C} \tag{18}$$

Here, the vector ***H*** and the matrix ***G*** are the approximations of the function values and of the derivatives at ***X*** = ***C***. The well-known problem of the linear least-squares fit, implemented in the search program as another subroutine, yields a stable procedure to find the vector $\boldsymbol{X}_{new}$, so that the linear approximation $\boldsymbol{F}_{lin}(\boldsymbol{X})$ becomes a minimum. The matrix ***G*** of the derivatives can be calculated analytically for simple functions. For complicated functions, it is more convenient and more effective to determine the derivatives numerically from the function vectors calculated during the earlier iteration process. The latter procedure is used in the P5 program.

Different problems that may show up during the search are fixed by the program. Under certain conditions, it may happen that the new point $\boldsymbol{X}_{new}$ is worse, meaning that it has a larger $\boldsymbol{F}_{lin}$ than the previous best point $\boldsymbol{X}_{old}$. In this case the program would switch to a 1-dimensional search with step-size control along the straight line, connecting $\boldsymbol{X}_{new}$ and $\boldsymbol{X}_{old}$. When calculating the derivatives numerically, another difficulty might be that the rank of one of the used matrices becomes smaller than *n* (number of the components of ***X***), so that the system of supporting points collapses into a subspace. This is fixed by creating a random point in the neighborhood of the previous best point $\boldsymbol{X}_{old}$.

The program terminates the search if $|X_{neu}(i) - X_{min}(i)| \;<\; |E(i)|$ for *i* = 1 ... *n,* where *E*(*i*) are the search accuracies. An estimate of the accuracy of the result follows. If the program does not terminate correctly, an error analysis is carried out. More information is available in [16, 17] and in the source code p5.f95 within the last four subroutines (appendix A1).

4.4.2 Ringfit

A common method to find the roots of a function ($y(x) = 0$) is the secant method. It can be used universally because, in contrast to the Newton-Raphson method, the analytical derivative of the function is not needed. Two points are fitted by a straight line and this line is extrapolated or interpolated to zero. Normally, when calculating the roots of a function, the function is not linear. The idea of the new method is to make the algorithm faster by also taking into account the curvature of the function. Instead of a linear extrapolation, a constant curvature is assumed, which means a circle. Thus, instead of two points, three points are fitted to a circle and the intersections with the x-axis are calculated. Therefore, this algorithm is named ringfit. Iterative application generates the roots of any function if it is continuously differentiable. Because ringfit works well and is slightly faster than the secant method, it is briefly described in a general form.

The equation of a circle is

$$r^2 \;=\; (x - x_0)^2 + (y - y_0)^2 \qquad \text{or} \qquad y(x) \;=\; y_0 \pm \sqrt{r^2 - (x - x_0)^2} \tag{19a, b}$$

with *r* being the radius and (x_0, y_0) being the coordinates of the center of the circle. If three points (x_1, y_1), (x_2, y_2), and (x_3, y_3) of an arbitrary function (near to the x-axis) are placed on the circumference of a circle, we get the x-coordinate, x_0, of the center of this circle by:

$$x_0 \;=\; \frac{(x_1^2 + y_1^2)(y_3 - y_2) \;+\; (x_2^2 + y_2^2)(y_1 - y_3) \;+\; (x_3^2 + y_3^2)(y_2 - y_1)}{2\cdot\left(x_1(y_3 - y_2) \;+\; x_2(y_1 - y_3) \;+\; x_3(y_2 - y_1)\right)} \tag{20}$$

The y-coordinate y_0 is calculated with the same equation by interchanging all x and y and leaving all of the indices unchanged. The reader can verify this result (x_0) straightforwardly by starting with three equations, e.g., Eq. (19a), corresponding to the three initial points, and eliminating r and y_0. To get the radius r, we insert x_0 and y_0 as well as the coordinates of one of the initial points, e.g., x_1 and y_1, into Eq. (19a). The desired intersections of the circle with the x-axis ($y = 0$) are found by setting y in Eq. (19a) to zero and solving the equation for x. This gives

$$x_{1,2}^{(s)} = x_0 \pm \sqrt{r^2 - y_0^2} \tag{21}$$

The superscript (s) represents solution. In most cases, two solutions exist. The nearest one replaces the “worst” of the previous three points, and iterative use of this procedure yields the final solution, which is the root of the original function. However, some aspects have to be considered.

1. We have to find the nearest solution to the previous three points, meaning that we have to decide whether the plus or the minus sign in Eq. (21) applies.
2. In principle, it might happen that the three points are located on a straight line. In this case, the method doesn't work because the denominator in Eq. (20) becomes zero and r becomes infinite. We overcome this situation by checking whether the denominator of Eq. (20) is zero, and if this is true we switch from ringfit to the secant method.
3. The term under the root in Eq. (21) can be negative. This implies that an intersection between circle and x-axis does not exist and, thus, this function does not have any roots. (More precisely, the “circle function” is not a function, but rather a sort of relation because in their range of definition most x-values have two y-values.)
4. Here, ringfit is used to compute the transit phases. Thus, the x-values, representing Julian Ephemeris days, have many digits. If such numbers are squared, in many cases, numerical noise prevents correct results, also called numerical instability. Therefore, in the beginning the three initial x-values are shifted to the origin by a constant time interval to reduce their size. The shift could be, for example, x_2 which even simplifies Eq. (20). At the end of the calculation, the three (new) x-values are shifted back to the old region by the same interval. This should always be done if the differences of the x-values at the beginning are much smaller than the x-values themselves. An example of how the algorithm can be implemented is given in the source code of P5 (subroutine ringfit).

Whereas the secant method extrapolates with straight lines, ringfit extrapolates with circles. The latter routine probably has not much practical relevance because here the speed gain (when using the TYMT test) is about 3 %. In other applications, the improvement can be larger. Nevertheless, it is slightly faster than the secant method and the basic idea and its equations also have an aesthetic aspect. Therefore, the routine is used here.

4.5 Coordinate transformation of planetary orbits

The 2-dimensional comparison of pyramid and planetary positions means that the altitude of the pyramids above the Earth's surface and the planetary positions out of the ecliptic plane are neglected. In other words, the positions are projected perpendicularly to the Earth's surface and to the ecliptic plane, respectively, just by ignoring the z-coordinate. (The astronomical x- and y-axis are placed in the ecliptic plane.) Now, the question is: Why should we use the ecliptic plane for projecting the positions? The ecliptic plane is the plane of the Earth's orbit, the third planet. Would it be better to take Mercury's orbit, since Mercury is the first planet? In principle, it makes sense to take the plane of the orbit of Mercury or Venus as the reference plane – with the new x- and y-axis on it. The question is whether this improves the agreement between pyramids and planets (F_{pos}). In order to check this approach, a coordinate transformation from the heliocentric ecliptic coordinate system to the heliocentric coordinate system of the orbit of Mercury or Venus is necessary.

The main equations of the transformation from the ecliptic to the Mercury orbit coordinate system are given without further explanation. For details and drawings of planetary orbits and their orientation see [5, app. A15]. The x-axis in the ecliptic system is defined in such a way that the Mercury aphelion is placed perpendicularly above the x-axis. In the "Mercury system" the Mercury aphelion is placed directly on the new x-axis. Concerning Mercury (index *M*), let Ω_M be the ecliptic longitude of the ascending node, L_M the ecliptic longitude of the aphelion, and *i* the inclination of the Mercury orbit. We then define $\omega = \Omega_M - L_M$ and

$$\tau = \arcsin\left(\frac{\sin\omega}{\sqrt{1-(\sin i\,\cos\omega)^2}}\right) + \omega - \pi \tag{22}$$

as well as $\xi = \tau - \omega$. Here, π is Ludolph's number and not the longitude of perihelion. For the derivation of Eq. (22) see [5, pp. 331–333]. Now, the transformation can be performed with the rotational matrix $\boldsymbol{R}$:

$$\boldsymbol{R}(\omega, i, \xi) = \begin{pmatrix} \cos\omega\cos\xi - \sin\omega\cos i\sin\xi & \sin\omega\cos\xi + \cos\omega\cos i\sin\xi & \sin i\sin\xi \\ -\cos\omega\sin\xi - \sin\omega\cos i\cos\xi & -\sin\omega\sin\xi + \cos\omega\cos i\cos\xi & \sin i\cos\xi \\ \sin\omega\sin i & -\cos\omega\sin i & \cos i \end{pmatrix} \tag{23}$$

The angles ω, *i*, and ξ are the Eulerian angles. The calculation for the Venus orbit is similar. With this transformation it is possible to conduct a 2-dimensional comparison between pyramids and planets with three different reference planes: the ecliptic plane, the plane of the Mercury orbit, and the plane of the Venus orbit. The main result is that the Mercury and Venus orbits do not yield any significant advantage. Many more details and calculated examples are provided in [5, app. A15].

4.6 "Celestial positions" on the Giza plateau

Let us assume that the three planets Mercury, Venus, and Earth stand in a constellation identical to the arrangement of the pyramids of Giza with the following correlation: Mercury ↔ Mykerinos Pyramid, Venus ↔ Chefren Pyramid, and Earth ↔ Cheops Pyramid. This means that the three planets form a triangle in space and the pyramid positions form a triangle on the Giza plateau. If these triangles are mathematically "similar," meaning they have the same shape (not the same size), then the previous assumption is true. The question is: How can the real Sun position with respect to the planetary positions be transferred to the Giza plateau when taking into account the pyramid positions? Two ways of calculating the "Sun position" on the Giza plateau are explained below by considering 3 dimensions. (For the geometrically predefined "Sun position" at Giza and for the "Sun positions" being free on the Earth's surface in 2 dimensions, see [5]).

4.6.1 "Sun position" by system of linear equations

Here again, the vectors ***a*** and ***b***, pointing from one to another pyramid as explained in section 4.3.2, define the arrangement of the three pyramids of Giza. The corresponding vectors for the planets are ***a'*** and ***b'***. The vectors ***a*** and ***b*** are always constant (because the pyramids do not move), whereas the vectors ***a'*** and ***b'*** change continuously with time. In order to obtain a vector basis for the 3-dimensional space, we create a vector ***d***, perpendicular to ***a*** and ***b***. With the vector product $\boldsymbol{a}\times\boldsymbol{b}$ and $a = |\boldsymbol{a}|$ being the absolute value (as before), we have

$$\boldsymbol{d} = -(\boldsymbol{a}\times\boldsymbol{b})\frac{a+b}{2\cdot|\boldsymbol{a}\times\boldsymbol{b}|} \quad \text{with} \quad \boldsymbol{a}\times\boldsymbol{b} = (a_y b_z - a_z b_y,\; a_z b_x - a_x b_z,\; a_x b_y - a_y b_x) \tag{24}$$

Analogously, we get a vector ***d'*** for the planets. (The letter ***d*** instead of ***c*** is used for consistency with [5] because in [5] ***c*** was already defined as a vector from the Chefren Pyramid to the Cheops Pyramid.) Note that the bases ***a***, ***b***, and ***d*** as well as ***a'***, ***b'***, and ***d'*** are not orthogonal, which is not necessary here. Now, we obtain the solution – the (transferred) “Sun position” – in two steps.

The three vectors ***a'***, ***b'***, and ***d'*** represent a basis of the 3-dimensional space. Thus, first we expand the vector ***s'***, which is the vector from Mercury to the real Sun, with respect to the basis ***a'***, ***b'***, and ***d'***. This means that the following system of inhomogeneous linear equations (SLE) must be solved:

$$\boldsymbol{a'}\, x_1 + \boldsymbol{b'}\, x_2 + \boldsymbol{d'}\, x_3 = \boldsymbol{s'} \tag{25}$$

After solving the SLE (25) [5, p. 341], we build a linear combination of the basis ***a***, ***b***, and ***d*** in the pyramid area with the solution x_1, x_2, and x_3 and obtain the “Sun position” ***s*** on the Giza plateau by

$$\boldsymbol{s} = \boldsymbol{a}\, x_1 + \boldsymbol{b}\, x_2 + \boldsymbol{d}\, x_3 \tag{26}$$

One more aspect must be considered. The coordinate system in Fig. 2 is relevant. If we use the center of mass as the pyramid positions, the position of the Mykerinos Pyramid is not at the origin of our coordinate system, but rather a quarter of the pyramid height above that. Therefore, we have to add a quarter of the height, which is 16.285 m, to the z-component of the result ***s***. Thus, the coordinates of the “Sun position” (constellation 12) in the pyramid area (details in [5, app. A16]) are:

$$s_x = -665.1 \text{ m}, \quad s_y = 22.8 \text{ m}, \quad \text{and} \quad s_z = 273.1 \text{ m} \tag{27}$$

4.6.2 “Sun position” by coordinate transformation and FITEX

Another possible way to obtain the “Sun position” is to transform the planetary positions (coordinates) to the pyramid positions by translation, rotation, and change in the size by a “scale factor.” In this case, the position of the Sun can also be transferred to the Giza plateau. At first, the problem of calculating the corresponding parameters and especially the rotation angles seems difficult, but it becomes easy if we also include FITEX. Thus, the solution is found by the search program. All components needed are still present in P4 and P5. For the rotation in space, we take the rotational matrix ***R*** of Eq. (23).

At first, a point in time is calculated by P5 (VSOP87) when the planetary constellation and the arrangement of the pyramids match each other (F''_{pos} being minimized). This means that the arrangements – both forming a triangle – are mathematically “similar.” Then the positions of the planets are adapted to those of the pyramids by translation, rotation, and “downsizing” in the 3-dimensional space. For the translation, three parameters, X_1, X_2, and X_3, are needed; the rotation in space means another three parameters, X_4, X_5, and X_6, and the change in size is given by one parameter, X_7. The calculation is an iterative process. At the beginning, X_1 to X_7 are chosen more or less arbitrarily. Then the program FITEX optimizes these seven parameters by iteratively minimizing the Euclidean distances between the transformed positions of the planets and the corresponding pyramid positions. If x, y, z and x', y', z' are the coordinates of a planet before and after the transformation, the full transformation is given by

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = X_7 \cdot \boldsymbol{R}(X_4, X_5, X_6) \cdot \begin{pmatrix} x + X_1 \\ y + X_2 \\ z + X_3 \end{pmatrix} \tag{28}$$

The search program FITEX works efficiently. The number of iterations necessary to find the solution X_1 to X_7 is approximately 50 to 150 for each constellation within ±15,000 years from present time, although seven parameters have to be optimized simultaneously. But how do we get the “Sun position"? In the heliocentric coordinate system, the Sun is placed at the origin. Thus, we apply the transformation of Eq. (28) to the zero vector $(x, y, z)^T = (0, 0, 0)^T$. As in section 4.6.1, for the date of constellation 12 we get the following coordinates of the "Sun":

$$s_x = -667.5 \text{ m}, \quad s_y = 21.3 \text{ m}, \quad \text{and} \quad s_z = 272.4 \text{ m} \tag{29}$$

The differences in the results between Eqs. (27) and (29) are about 1 and 2 m, which seems reasonable. The transformation Eq. (28) is also used for the chamber positions in the Cheops Pyramid and – once the parameters have been found – for transforming the positions of the outer planets Mars to Neptune to the pyramid area. This second procedure is preferred because the positions do not match exactly 100 % and the small deviations are balanced by minimizing the distances with FITEX. Some examples of other "Sun positions" and "planetary positions" in the Giza area, calculated using this second method, are listed in sections 3.4.2–3.4.4, 3.4.6, and 3.4.11–3.4.16.

4.6.3 Additional "planetary positions"

The previous section describes two methods of calculating the "Sun position" inside the Cheops Pyramid (see Fig. 6). On the left half of Fig. 6 there are some additional positions inside the pyramid. These belong to the transformed planets at the "pyramid date" and at the "conjunction (syzygy) date." These positions do not seem as important as those defined by the "chamber date." However, for the sake of completeness, we describe how they can be computed. To make it clearer, these positions do not refer to the date of the chambers, but rather to the dates of syzygy and the pyramids, although these positions belong to the coordinate system of the Cheops Pyramid.

The calculation is straightforward. For the date of the chambers, the positions of the planets are adapted to the chamber positions by coordinate transformation and the fit program FITEX. The corresponding seven parameters, X_1 to X_7, are kept for later use. Next, the planetary positions are calculated for the associated pyramid date, being 44 days later with Mercury at aphelion. Finally, we repeat the coordinate transformation with these new data by using the previous seven parameters, X_1 to X_7, and get the "pyramid positions" inside the Cheops Pyramid (Fig. 6). Because we need a fixed coordinate system, we use VSOP87A (J2000.0), although the results, when calculated with VSOP87C, are nearly identical. The tools in the program already exist. The trick is that we have two different points of time and must know how to use them correctly. We can do the same for the date of the planetary conjunction (syzygy) as well as for the middle of the Mercury transit. Concerning the latter date, the planetary positions are not shown in Fig. 6 because they are not much different than those of the conjunction.

What about the coordinate system of the pyramids? Here, the origin is placed at the center of the Mykerinos Pyramid. Similarly, we can also calculate the transformed planetary positions for all dates – provided previously – on the Giza plateau and in the urban area of Giza, respectively. The region is shown in Fig. 15, with the planetary orbits plotted accordingly. In this case, the procedure of applying the dates is reversed. From the transformation of the planetary to the pyramid positions, we obtain seven new parameters, X_1 to X_7, for the "pyramid date." The same coordinate transformation is then done for the other points of time. Note that the "planetary orbits" in Fig. 15 are tilted against the Earth's surface by about 24.5° (see Fig. 2) so that the visible shape of the orbits becomes slightly elliptical. Some numbers are provided further down in Table 6. The tilted small rectangle at the "Sun position" (Fig. 15) is a concrete platform 25 m wide by 50 m long aligned to the center of the Chefren Pyramid, which still existed in 2003. Today, the shape of the platform has been changed.

If the reader is interested in where these positions can be found at Giza, it would be more convenient to have the exact geographical latitude and longitude rather than the Cartesian coordinates in m. In this case, it is easy to find the locations with a GPS receiver (GPS coordinates in section 3.4.13). The corresponding calculation is described in the following section.

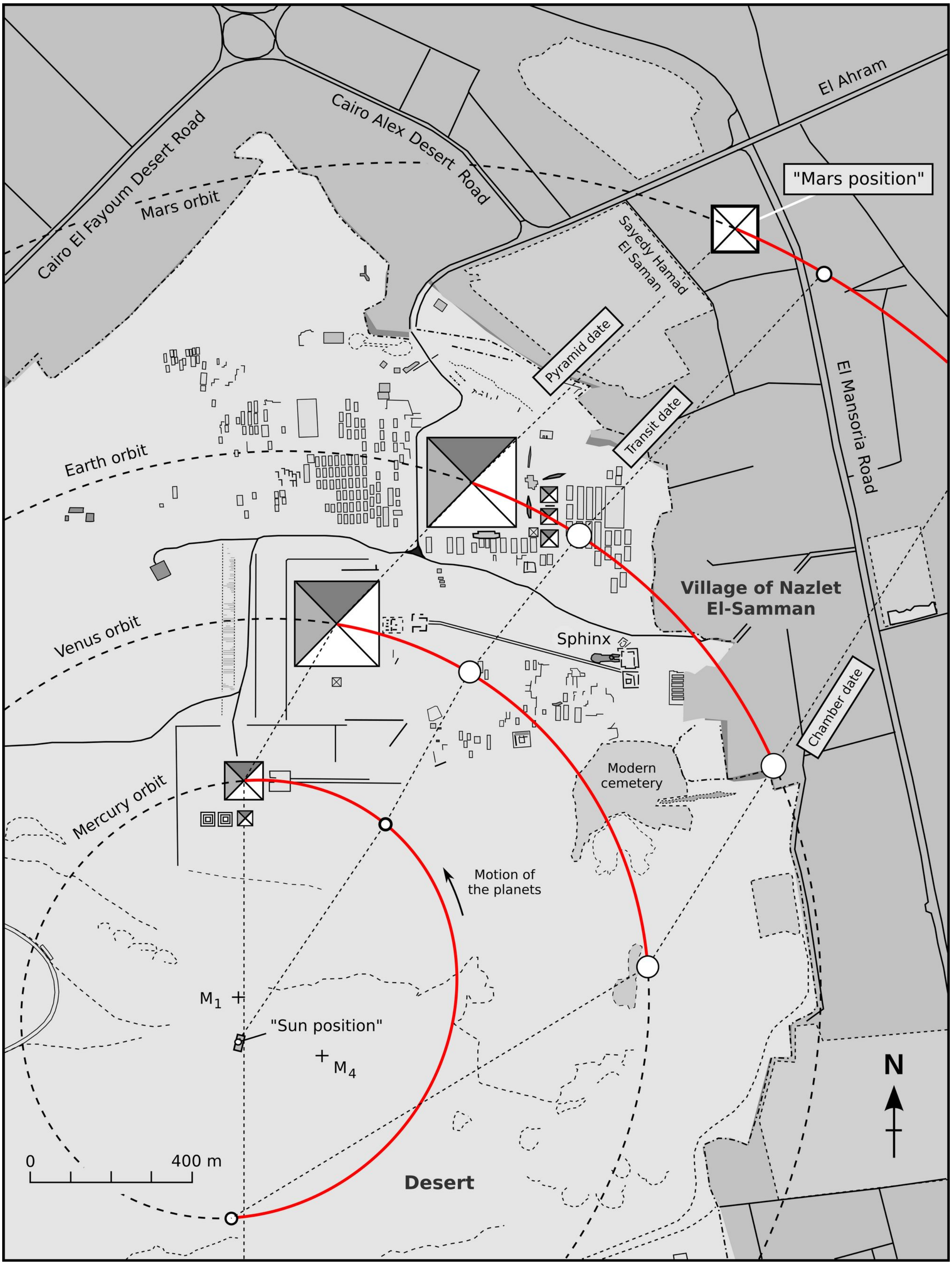


Figure 15: *Pyramid plateau of Giza and neighboring village. The transferred planetary orbits are projected vertically to the Earth's surface. The sizes of the planets are magnified with respect to the orbits. The Mars position, which belongs to the pyramids, is represented by a white pyramid (29° 59.095' N, 31° 8.461' E). Its size is adapted roughly in proportion to Mars. The points M_1 and M_4 are the orbital centers for Mercury and Mars. Other planetary positions belong to the date of the Mercury transit and to the "chamber date" (AD 3088). The GPS coordinates can be calculated with option 334 and are provided in section 3.4.13. Background created on the basis of Google Maps; © 2015 Google, ORION-ME.*

4.6.4 Geographical coordinates

The conversion to latitude and longitude is not trivial if done properly. One reason for this is that we have to match a flat area, given in Cartesian (rectangular) coordinates, to the surface of a sphere; another reason is that the Earth is not an exact sphere, but rather an ellipsoid or spheroid. To get accurate results, we have to consider the mathematical definition of the geographical latitude. The cross section of the Earth along the rotational axis is an ellipse. Thus, we begin with some basic equations.

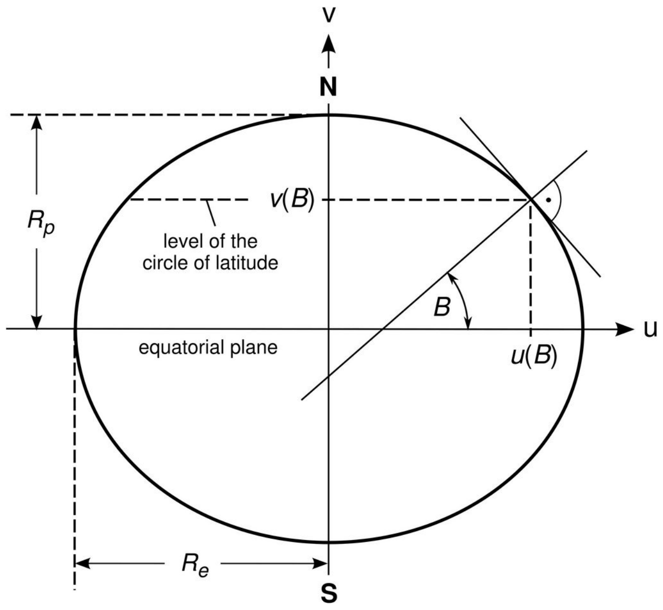


Figure 16: *Schematic elliptical-shaped cross section of the Earth. The geographical latitude B is the cutting angle between the tangent normal and the equatorial plane.*

If the Earth was a sphere with radius *R,* then the coordinates in Fig. 16 would be $u = R \cdot \cos(B)$ and $v = R \cdot \sin(B)$. (Here, the common identifiers *x* and *y* are replaced by *u* and *v* in order to avoid confusion with the rectangular coordinates *x, y,* and *z* on the Giza plateau.) When considering the elliptic shape of the Earth's cross section, the calculation becomes a bit more complicated. With R_e and R_p being the Earth's equatorial and polar radii (see Tab. 7), the equation for the Earth ellipse in Fig. 16 is

$$\left(\frac{u}{R_e}\right)^2 + \left(\frac{v}{R_p}\right)^2 = 1 \qquad (30)$$

It follows

$$v(u) = \pm R_p \sqrt{1 - \left(\frac{u}{R_e}\right)^2} \qquad (31)$$

As shown in Fig. 16, the geographical latitude is the angle *B* of the intersection between the tangent normal and the equatorial plane. The tangent normal per definition is aligned perpendicularly to the tangent, whose slope is the derivative of the elliptical-shaped function with respect to *u.* More precisely, the derivative is the tangent of $B - \pi/2$. From Eq. (31) we obtain

$$\frac{dv}{du} = \mp \frac{R_p u}{R_e \sqrt{R_e^2 - u^2}} = \pm \tan\left(B - \frac{\pi}{2}\right) \qquad (32)$$

In order to get *u* as a function of *B,* we solve Eq. (32) for *u*. The positive solution is

$$u(B) = R_e \left[1 + \left(\frac{R_p}{R_e} \tan B\right)^2\right]^{-\frac{1}{2}} \qquad (33)$$

Using the procedure described in section 4.6.3, the P5 program calculates the rectangular coordinates *x, y,* and *z* of any planetary position that is transferred from space to the pyramid area. As mentioned previously, the x-axis points to the north, the y-axis points to the west, and the z-axis points upward. The origin of the coordinate system is placed in the center of the base area of the Mykerinos Pyramid. In this case, the z-coordinate, being more or less the position above or under the ground, is not relevant. Only *x* and *y* are converted to the geographical coordinates *B* and *L,* enabling the use of GPS. (In the calculation below, the height of this coordinate system of ca. 72 m above sea level is neglected. The Cartesian coordinate system of the chambers is different.)

The center of the Mykerinos Pyramid is located at a latitude of B_0 = 29° 58.35175' N and a longitude of L_0 = 31° 7.69455' E (or B_0 = 29.972529° N, L_0 = 31.128243° E, accuracy ≈ ± 0.000010°), measured in Giza by averaging the GPS coordinates of the four pyramid corners. This defines the origin of the coordinate system. If *x* and *y* are the rectangular coordinates of a calculated "planetary position" measured from the Mykerinos Pyramid, then we can calculate the corresponding geographical coordinates *B* and *L*. The differences $\Delta B = B - B_0$ and $\Delta L = L - L_0$ are related to *x* and *–y* in the "pyramid system" and to *x* and *z* in the "chamber system." In the following, the subscript 0 always refers to the point of origin.

The latitude is calculated in two steps. First, we determine an approximate value of the difference in latitude by $\Delta B_a = x \cdot 360°/U$, with *U* = 40,008 km as the circumference of the Earth, measured across the poles. (Here, the subscript *a* always means approximate.) Thus, an approximate value of the desired latitude is given by $B_a = B_0 + \Delta B_a$. Next, we determine the exact latitude *B.* For the Mykerinos Pyramid, we get the geocentric coordinates u_0 and v_0 by inserting B_0 into Eq. (33) and then applying Eq. (31). Similarly, we obtain approximate values u_a and v_a with Eqs. (33) and (31) by inserting B_a. The Euclidean distance x_a between two points having the latitudes B_0 and B_a and the same longitude, is

$$x_a = \sqrt{(u_a - u_0)^2 + (v_a - v_0)^2} \tag{34}$$

Next, we correct the value ΔB_a by calculating the difference $\Delta B = \Delta B_a \cdot |x/x_a|$. On the one hand, distances like *x* and *y* mean straight lines, and on the other hand, the surface of the Earth is not flat but slightly curved. Concerning the pyramids, the distances are in the range of 1 or 2 km, meaning that the differences in latitude and longitude are less than 0.02°. For such small angles α, we get a very good approximation: $\sin\alpha \approx 2\sin(\alpha/2) \approx \alpha \approx \tan\alpha$, where α is given in radians. The reader might verify that the term $2\sin(\alpha/2)$ is the Euclidean distance between two points on a sphere with radius 1. (The analogous distance would be measured in a straight line through the Earth.) Thus, we neglect the curved nature of the Earth's surface and get the final latitude $B = B_0 + \Delta B$.

When calculating the longitude, in principle there is another problem. If we have the exact coordinates *x* and *y,* then it makes little difference whether we first go *x* meters to the north and then *y* meters to the east (at latitude *B*), or if we go *y* meters to the east (at latitude B_0) and then *x* meters to the north. The reason for this is simple. If a constant west−east distance $|y|$ is shifted from the equator "upward" toward the North Pole, the corresponding difference in geographic longitude, ΔL, becomes continuously larger. Although this effect is quite small for shifts of a few km, we balance the result by using the arithmetic mean of the latitudes: $B_m = (B_0 + B)/2$. This means that we first go *x*/2 meters to the north, then *y* meters to the east, and again *x*/2 meters to the north.

Once more, we use Eq. (33) and calculate $u(B_m)$. With $2\pi u(B_m)$ being the circumference of the circle of latitude B_m, we obtain

$$\frac{\Delta L}{360^o} = \frac{y}{2\pi u(B_m)} \tag{35}$$

which yields ΔL. Note that *y* means –y for the pyramids and *z* for the chambers. Finally, we get the geographical longitude by $L = L_0 + \Delta L$. (If this procedure is applied to Teotihuacán, the distances in the area should be modified first by a factor $R/(R+h)$ with *h* being the height above sea level.)

Note: The perimeter *U* of a circle with radius *R* is given by $U = 2\pi R$. Surprisingly, the perimeter of an ellipse U_{ell} can be calculated only numerically. However, more than 100 years ago, the Indian mathematician Srinivasa Ramanujan found the following analytical approximation for the circumference of an ellipse (using the Earth's radii):

$$U_{ell} \approx \pi(R_e + R_p)\left(1 + \frac{3\lambda^2}{10 + \sqrt{4 - 3\lambda^2}}\right) \quad \text{with} \quad \lambda = \frac{R_e - R_p}{R_e + R_p} \tag{36}$$

This formula is very interesting because, for low and medium eccentricities, it is extremely precise and probably nobody (on Earth) knows whether it can be deduced mathematically or how Ramanujan found it. An example of "planetary positions" in the Giza area is provided in Table 6 by using the date of the Mercury transit (calculation with option 0 or 334, section 3.4.13, see Fig. 15).

Table 6: *Geographical positions at the date of the Mercury transit (May 18, 3088, 19:20:59, TT).*

corresponding planet	Mercury	Venus	Earth	Mars
Latitude (North)	29° 58.29609'	29° 58.49812'	29° 58.68005'	29° 59.03875'
Longitude (East)	31° 7.91754'	31° 8.04605'	31° 8.21763'	31° 8.58980'

For the chamber system (Fig. 6), such calculations do not make much sense because the chambers are separated by only a few meters and there is no GPS reception inside the pyramid. Anyway, in P5 the geographical coordinates are also calculated for the chambers (slightly corrected).

4.7 Syzygy

4.7.1 Planetary conjunctions

The condition for planetary conjunctions is that the ecliptic longitudes L of all participating planets are similar within a given angle dL_0, e.g., dL_0 = 5°. The ecliptic latitudes, which describe the positions out of the ecliptic plane, are neglected. The two main options are "3 planets in conjunction" (Mercury, Venus, and Earth) and "4 planets in conjunction" (Mercury, Venus, Earth, and Mars). In order to save computation time, the chronological search happens mostly in "large steps" with a special search after each step. These "large steps" are (mostly) the synodical period of Venus and Earth of approximately 584 days. We start with a conjunction and after each step, when Venus and Earth stand again in conjunction, the overall range dL, including all participating planets, is minimized as a function of time. If the minimized angle dL_{min} is smaller than the limit dL_0, a new syzygy is found.

For the minimization of dL, being an iterative process, the difference in L for all planets has to be checked pairwise. Now, three planets mean three differences and four planets mean six differences. So, after the minimization procedure, the condition is that the maximum of all differences must be lower than the given limit dL_0. The minimization algorithm uses three points of time with equal time intervals. Let the angular ranges dL_1, dL_2, and dL_3 be the associated function values. If the corresponding three points of time are in ascending order, then the algorithm to minimize dL is as follows: At the beginning, both time differences are 5 days. For d$L_1 \leq$ d$L_2 \leq$ dL_3, the three points of time are shifted to the left (to earlier times) by one interval; for d$L_1 >$ d$L_2 >$ dL_3, they are shifted to the right; and for d$L_1 >$ d$L_2 \leq$ dL_3, they move closer together by the (optimized) factor of 5. If the difference between two times is lower than the search minimum ε or if d$L_1 \leq$ d$L_2 >$ dL_3, meaning "numerical noise," the procedure is terminated and the solution is found (subroutine fitmin, first method). In P5, a check follows to determine whether a simultaneous Mercury or Venus transit happens – see, e.g., section 3.4.8. Therefore, we continue with the transits in front of the Sun.

4.7.2 Transit phases

Three different ways to determine these Mercury and Venus transits are provided in P5. The first two options are quite simple. In the case where, e.g., Mercury has the same ecliptic longitude as the Earth, a check is performed using plane geometry for whether the ecliptic latitude of Mercury B_m is small enough for the planet to stand in front of the solar disk. In the second option, the condition of "identical ecliptic longitudes" is replaced by "minimum separation" between the planet and the Sun. These two options are not very precise because the finite speed of light is neglected. Therefore, only the third option is explained in more detail.

This option includes the calculation of geocentric phases and minimum separation of a transit (see Figs. 8 and 17). Here, the term geocentric means as seen from the center of the Earth. For the calculation of the geocentric phases, we need the radii of Mercury, Venus, and the Sun, which are summarized in the following table. The radius of Mercury is taken from Seidelmann et al. [44, p. 173], that of Venus from *Transits* [25, p. 16], and the solar radius is taken from a recent measurement of Brown & Christensen-Dalsgaard [29]. The given radius of Venus includes the opaque atmosphere of a height of nearly 50 km.

Table 7: *Optical size of the celestial bodies and the Earth's radii: IERS (2003).*

	radius [arc sec]	radius [km]
Sun (equatorial radius)	958.966	695,508
Mercury	3.3638	2439.7
Venus	8.4100	6099.5
Earth, equatorial radius	8.7941	6378.1366
Earth, polar radius	8.7647	6356.7519

When taking the solar radius of 695,990 km, used by Meeus [25], the results are identical to those of Meeus in almost all cases, apart from some rounding effects. (If desired, the solar radius can be adapted easily in the source code p5.f95.) As an example, we now take the data of Mercury, but the arguments are analogous for Venus. The letters L, B, and r characterize the heliocentric spherical coordinates of a planet, and the subscripts E and M refer to Earth and Mercury. Let α be the separation between Mercury and the Sun as seen from the center of the Earth, then we find

$$\alpha = \arctan\left(\frac{r_M \sqrt{1 - \cos^2\beta}}{r_E - r_M \cos\beta}\right) \tag{37}$$

with
$$\cos\beta = \sin B_E \sin B_M + \cos B_E \cos B_M \cos(L_E - L_M) \tag{38}$$

The angle β is the separation of Mercury and Earth as seen from the center of the Sun. Eq. (37) is deduced by plane geometry from the astronomical triangle Earth-Mercury-Sun, and Eq. (38) is, in principle, the spherical law of cosines (trigonometry in 3 dimensions). For $B_E = 0$, which is the case when using the ecliptic of date (VSOP87C), Eq. (38) reduces to

$$\cos\beta = \cos B_M \cos(L_E - L_M) \tag{39}$$

Eq. (37) was used in the beginning in P4 because it is universally valid. In contrast to this approach of spherical trigonometry, another possibility is provided on the basis of vector analysis. If $\boldsymbol{r}$ is a vector from the Sun to a planet in rectangular coordinates, and by applying the inner product of two vectors and the absolute value (length) of a vector $|\boldsymbol{r}|$, we get the separation by

$$\alpha = \arccos\left(\frac{-\boldsymbol{r}_E \cdot (\boldsymbol{r}_M - \boldsymbol{r}_E)}{|\boldsymbol{r}_E| \cdot |\boldsymbol{r}_M - \boldsymbol{r}_E|}\right) \tag{40}$$

Both Eqs. (37) and (40) yield the same results, but, ultimately, Eq. (40) is used (in P4 and P5) because the calculation is slightly faster. Considering the transit of Venus, Eqs. (37) to (40) can be used by replacing the indices M with V. But how do we get the exact geocentric transit phases? If s and s' are the angular radii (semidiameters) of the Sun and Mercury or Venus as seen from the

Earth, then we obtain the outer contact points 1 and 4 with

$$\alpha = s + s' \tag{41}$$

and the inner contact points 2 and 3 with

$$\alpha = s - s' \tag{42}$$

(Compare Figs. 8 and 17.) But, we must be careful. If, for example, Eq. (41) is fulfilled and we have calculated the planetary positions for one point of time, it does not mean that we see the planet in contact with the Sun. If the light is coming from the “contact point” on Mercury's surface to the Earth, it needs approximately five or six minutes. During this time, the Earth has moved away from the point where we wanted to make the observation. In short, we have to consider the finite speed of light.

Let us assume that the light from the Sun's circumference passes Mercury at time t_M and reaches Earth at time t_E. The difference $\Delta t = t_E - t_M$ is the travel time of the light. If t_M and the position of Mercury are given, we need the position of the Earth to calculate Δt; on the other hand, we need Δt to calculate the position of the Earth. Thus, it seems that we have a problem. Fortunately, this can be solved iteratively. In the following, c is the speed of light and ε is the search accuracy, e.g., ε = 0.1 s. The time t_M is given and the problem now is to determine the exact time t_E, when the light, starting from Mercury at t_M, reaches the Earth. This problem can be solved with the following fixed point algorithm:

Step 1: Calculate the position of Mercury $\boldsymbol{r}_M$ with VSOP87 at time t_M and set initial travel time of light (arbitrarily) to Δt = 320 s.

Step 2: Calculate the position of Earth $\boldsymbol{r}_E$ with VSOP87 at time $t_E = t_M + \Delta t$.

Step 3: Calculate the optical path length between Mercury and Earth by $\Delta r = |\boldsymbol{r}_E(t_E) - \boldsymbol{r}_M(t_M)|$ and the travel time of light by $\Delta t_{new} = \Delta r / c$.

Step 4: As long as $|\Delta t_{new} - \Delta t| \geq \varepsilon$, replace Δt with Δt_{new} and continue with Step 2; otherwise, stop this routine and the solution is $t_E = t_M + \Delta t_{new}$.

Furthermore, the minimum separation is found using a procedure with three points (separations α) as a function of time t, which are α_1, α_2, and α_3 at times t_1, t_2, and t_3. These points define a type of hyperbolic function with the following form:

$$\alpha(t) = a \cdot \sqrt{(t-b)^2 + c^2} \quad \text{with} \quad b = t_2 + \frac{1}{2} \cdot \frac{(\alpha_2^2 - \alpha_1^2)(t_3 - t_2)^2 + (\alpha_3^2 - \alpha_2^2)(t_1 - t_2)^2}{(\alpha_2^2 - \alpha_1^2)(t_3 - t_2) + (\alpha_3^2 - \alpha_2^2)(t_1 - t_2)} \tag{43a, b}$$

The parameters a and c need not be further specified. Because t is given as a large number *JDE*, the addition and subtraction of t_2 in Eq. (43b) is a trick to avoid numerical instability (as for ringfit, section 4.4.2). Next, the minimum at $t = b$ replaces the worst of the three previous points, which iteratively yields the moment of nearest approach (subroutine fitmin, second method). Note that the transit calculations are partly performed in a different way to J. Meeus [25]. Nevertheless, if the solar radius of 695,990 km, applied by Meeus, is used in P5, the results are identical in almost all cases.

4.7.3 Position angles of transit

The position angles refer to the transiting planet when it is in contact with the Sun's limb. The angles are measured from the y-axis (Fig. 17), which points to the celestial North Pole. They also correlate with the apparent motion of the Sun due to the Earth's rotation. Jean Meeus provides a procedure for calculating the apparent positions of Mercury and Venus on the solar disk during transit [25, pp. 14 ff.]. Unfortunately, for Mercury, the method is available only for the years between AD 1600 and 2300. Because we are interested in the year AD 3088, another way must be found.

Let us assume an Earth reference system that is not rotating and independent from the orientation of the Earth’s axis (CRS, celestial reference system), and another system that is fixed to the Earth (TRS, terrestrial reference system). If $\boldsymbol{x}_{CRS}$ and $\boldsymbol{x}_{TRS}$ are two position vectors belonging to the same local point as well as to the two different systems, the transformation between both vectors at a time t is given by [45, 46]

$$\boldsymbol{x}_{CRS} = \boldsymbol{P}(t)\cdot\boldsymbol{N}(t)\cdot\boldsymbol{U}(t)\cdot\boldsymbol{X}(t)\cdot\boldsymbol{Y}(t)\cdot\boldsymbol{x}_{TRS} \tag{44}$$

The matrices $\boldsymbol{P}$ and $\boldsymbol{N}$ take into account precession and nutation, $\boldsymbol{U}$ the rotation, and $\boldsymbol{X}$ and $\boldsymbol{Y}$ the polar motion of the Earth. Although in our case the matrices $\boldsymbol{N}$, $\boldsymbol{U}$, $\boldsymbol{X}$, and $\boldsymbol{Y}$ can be neglected, $\boldsymbol{P}$ would still be needed. However, instead of explicitly using the time-dependent precession matrix $\boldsymbol{P}(t)$, the position angles are calculated in the following four steps:

1. The positions of the planets Mercury (or Venus) and the Earth in Cartesian coordinates – calculated with VSOP87C (ecliptic of epoch, including the matrix $\boldsymbol{P}$) – are rotated around the x-axis by an angle that is the obliquity of the ecliptic of that epoch. We have to take the x-axis because it connects the solar center with the Earth's position at the beginning of spring (vernal equinox). By this rotation, the x-y plane becomes parallel to the plane of the Earth's equator. The obliquity of the ecliptic ε_e, which varies slightly in time, is taken from Axel D. Wittmann [47, p. 203]:

$$\varepsilon_e = 23.4458042^\circ - 0.856033^\circ \cdot \sin\left(0.015306 \cdot (T + 0.50747)\right) \tag{45}$$

   The time T is measured in Julian centuries as in Eq. (7) and the argument of the sine function is given in radians. Other equations for ε_e with polynomials exist, but Eq. (45) has the advantage of having no “runaway properties” for large T [47]. Mathematically, the transformation is performed by using the rotational matrix of Eq. (73) in section 4.9.3.

2. Now, the new positions of Mercury or Venus and of the Sun are translated by the (negative) coordinates of the new Earth position. This means that the origin of the heliocentric coordinate system is shifted to the Earth's center and so becomes geocentric.

3. The new rectangular coordinates of Mercury (Venus) and the Sun are transformed into spherical coordinates.

4. From these geocentric coordinates, the position angle of Mercury – or, accordingly, Venus – with respect to the solar center is calculated by equations taken from André Danjon [48, p. 36] and Jean Meeus [25, p. 15], respectively. In the following, α_S and δ_S are the apparent right ascension and declination of the center of the Sun, and α_P and δ_P are the corresponding angles for the planet Mercury or Venus. With $\Delta\alpha = \alpha_P - \alpha_S$, $\Delta\delta = \delta_P - \delta_S$, and K being an auxiliary quantity, we get

$$K = \frac{206264.8062}{1 + \sin^2\delta_S \cdot \tan\Delta\alpha \cdot \tan(\Delta\alpha/2)} \tag{46}$$

$$x = -K\cdot\left(1 - \tan\delta_S \cdot \sin\Delta\delta\right)\cdot\cos\delta_S\cdot\tan\Delta\alpha \tag{47}$$

$$y = K\cdot\left(\sin\Delta\delta + \sin\delta_S\cdot\cos\delta_S\cdot\tan\Delta\alpha\cdot\tan(\Delta\alpha/2)\right) \tag{48}$$

   The constant 206,264.8062 = 360·60·60/(2π) is the number of arc seconds in one radian. The zero position of right ascension is not relevant because declination and differences of right ascension are unaffected. The quantities x and y are the rectangular coordinates of the planet given in arc seconds. Finally, the position angle P, measured from the y-axis (Fig. 17), is

$$P = \arctan\left(\frac{-x}{y}\right) \tag{49}$$

With $\cos P$ having the same sign as y, we get P in the correct quadrant [25]. In the P5 program, this is realized as follows: If we have $y \cdot \cos P < 0$, then P is replaced by $P + 180°$. The transit in AD 3088 (Fig. 17) is not central, but it is the first in a new transit series. Due to the convention, taken from the NASA Eclipse Web Site, this series has the number 20. It comprises nine transits and will last from 3088 to the year 3456.

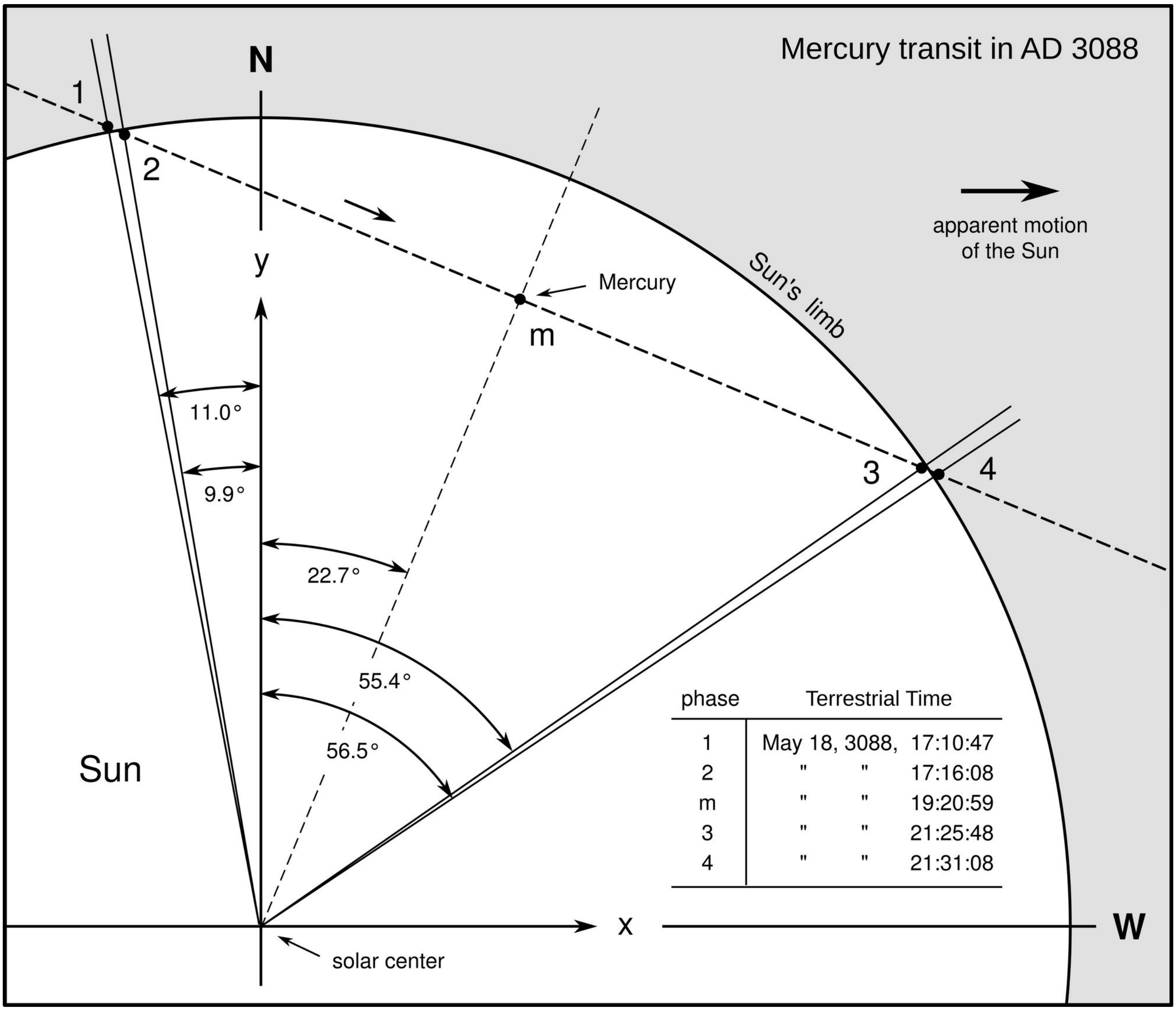


Figure 17: *True-to-scale representation of the Mercury transit in AD 3088 with position angles. If calculating the data with the previously used solar radius 695,990 km [25] and not with the current value of (695,508 ± 26) km [29], the deviations are a maximum of 0.06° for the position angles and 18 s for the times of day. These differences are rather small.*

Several iterative search algorithms are used in the P5 program. The contact points 1 to 4 are determined with the subroutines ringfit and secant in combination with the fixed point algorithm (section 4.7.2, subroutines vsop1tr and vsop2tr), whereas the nearest approach is calculated by a special minimum search (subroutine fitmin, second method). Other methods are those of Newton and Raphson, used to solve Kepler's equation (subroutine vsop3), the procedure to minimize the angular range of a planetary conjunction (subroutine fitmin, first method), and FITEX, the multi-parameter fit program used, e.g., to determine the “Sun position” (last four subroutines in P5).

A few remarks should be made about the characteristics of grazing transits. In principle, there are three different kinds of grazing transits. All calculations in the P5 program are performed with the assumption that the observer is positioned at the center of the Earth. This yields the geocentric transit phases (times in the tables). In the case of a geocentric grazing transit, only three transit phases are provided: the two outer contact points and the minimum separation because the planet never moves completely on the solar disk. If this transit can be seen as a full transit from other parts of the Earth, then it is also named a partial transit. The second possibility is that from the geocentric position, the planet does not touch the Sun's limb, but, there is a grazing transit from other points on the Earth. In this case, only one transit phase can be calculated – this is the nearest separation between planet and Sun. The third possibility means that it looks like a full transit from the geocentric position but from other places on the Earth it is a grazing transit. In this case, we have five transit phases, as for a full transit, but from parts of the Earth it is a grazing (partial) transit. In the computed tables, these three cases – marked with m for Mercury and v for Venus – can be distinguished easily. They have three, one, or five transit phases, respectively.

4.7.4 Transit series

The transits of Mercury or Venus in front of the Sun can be combined in the so-called transit series. Different ways of combining the transits are possible and are described in detail in [25, pp. 7–13]. The main patterns are successive transits of Mercury every 46 years and of Venus every 243 years. Each series has a number and different series are serially numbered according to their first appearance. The series of Mercury, starting on May 18, 3088, has the number 20 (see last column in the table in section 3.4.7). The corresponding paths of Mercury along the Sun's disk are provided in Fig. 18. The figure is drawn on the basis of the position angles, calculated with P5 (VSOP87).

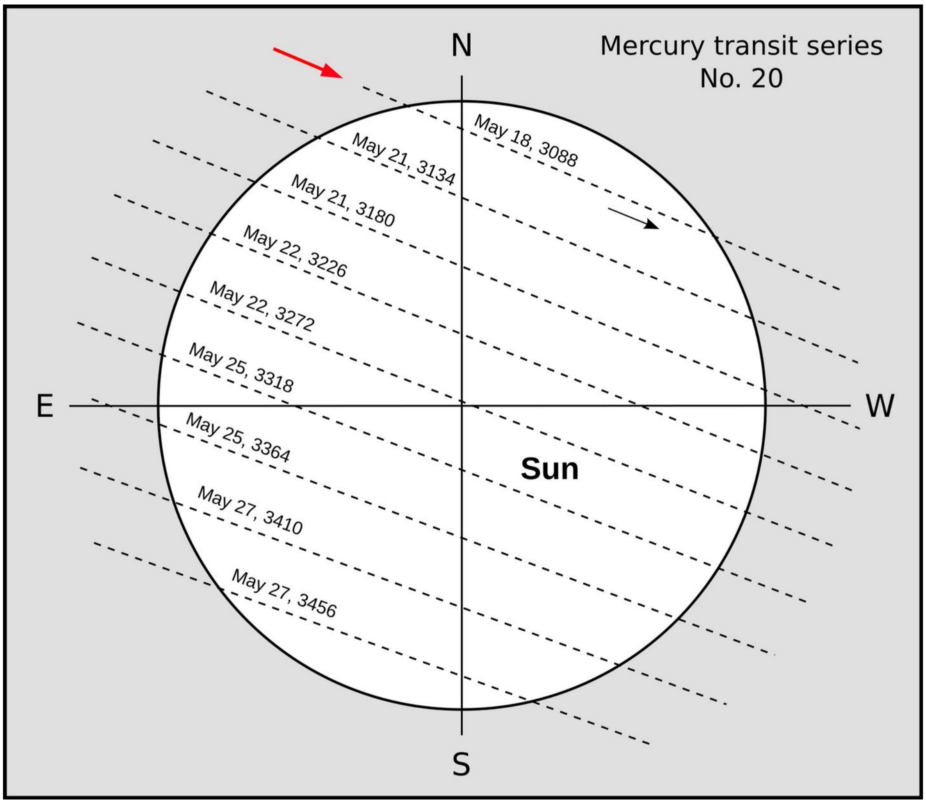


Figure 18: *The complete transit series of Mercury, having the number 20 (according to the serial numbers on the website of the NASA/Goddard Space Flight Center (URL 9) – see also section 3.4.7). The transit series begins on May 18, 3088.*

Now, if a new transit is found by the P5 program, the question is: How do we get the corresponding serial number? For Mercury, the time differences between transits in the same series are multiples of 46 years. In the P5 program, the date (JDE) of each first transit of a new series is stored. Therefore, the date of the new transit is compared with these transit dates, already stored. If the time difference between the new and one of the stored transit dates is a multiple of 46 years, the new transit has the same serial number as the stored transit date. If there is no connection to a preceding transit, the new one gets a new serial number. To check for the multiple of 46 years, the modulo operation (mod) can be used. Let J_{prev} and J_{new} be the decimal years of one of the stored older transits and the new transit. Then the corresponding relation for Mercury is:

$$(J_{new} - J_{prev}) \bmod 46 \ < \ \varepsilon \tag{50}$$

Here, ε is a short time period, e.g., 0.03 years. This description is a little bit simplified. Instead of using years in the P5 program, the periods of 46 and 243 years are provided in Julian days. For Mercury, the averaged time interval is Δt_M = 16,802.200 Julian days and for Venus it is Δt_V = 88,756.137 Julian days. In reality, some of the already finished transit series can show up again after a longer time period. Due to the convention, these transits receive a new serial number.

Another example is given here independently of the pyramids. The recent Venus transit in the year 2012 has the serial number 5. If we follow this series No. 5 into the future, we find that in the years between approximately 5000 and 7200 a fantastic sequence of about eight successive central Venus transits will occur within this series. For the data and details see [49]. The corresponding Venus passages are represented in Fig. 19. The transits are shown from the beginning only up to the central transits. During later transits of this series, Venus will also pass in front of the northern half of the Sun so that Fig. 19 would become confusing. Additionally, the precision of VSOP87 decreases in this remote future.

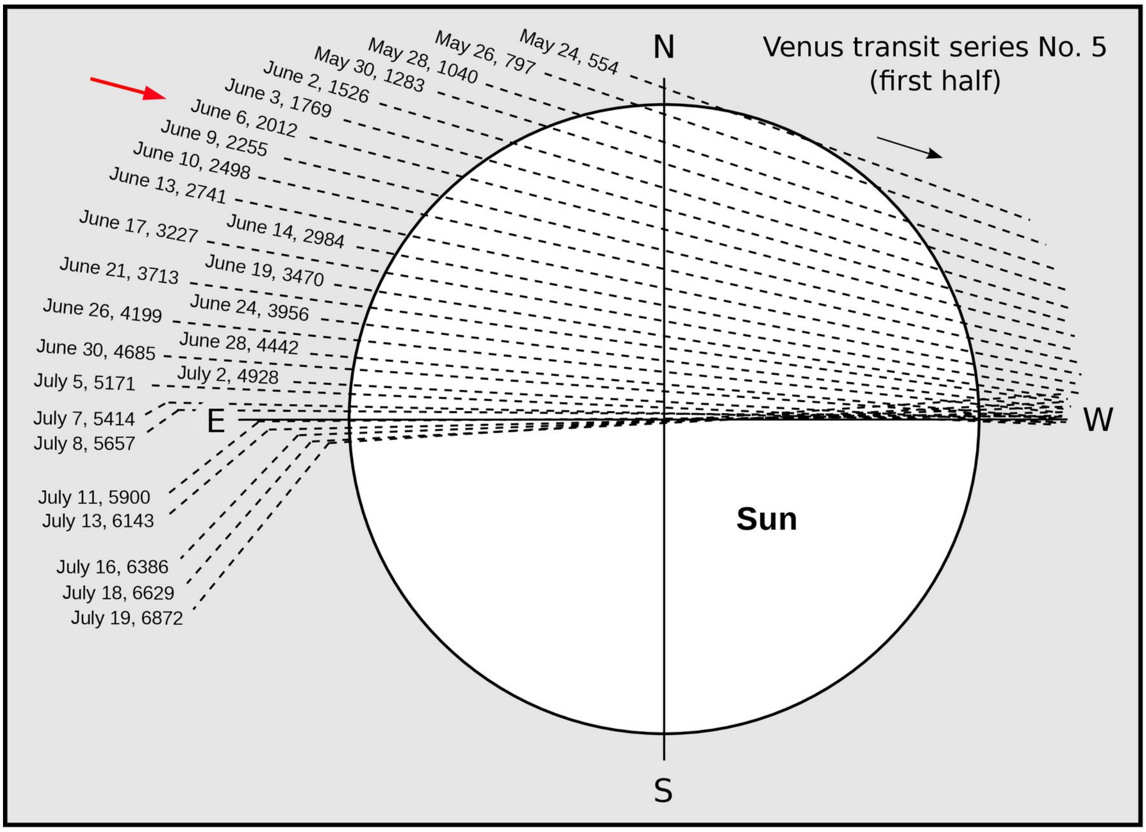


Figure 19: *Venus transits, first half of series No. 5 (number convention as in Fig. 18, only Gregorian calendar, TT), from the beginning in the year 554 up to the central transits between the years 5000 and 7200.*

More details could be discussed, e.g., special circumstances that arise when computing conjunctions or methods to improve the processing speed, but here only the main points are given. In principle, all the details can be found in the source code (appendix A1).

4.8 Universal Time

Terrestrial (dynamical) Time (TT) is a linear time scale with constant day lengths. It is appropriate for astronomical purposes in order to handle large time spans accurately. Universal Time (UT) is continuously adjusted to the Earth's rotation, which decelerates due to tidal friction. This means that the length of a day very slowly increases. Because the second is the basic time constant, a leap second is occasionally introduced in order to keep the time of day in phase with the Earth's rotation. These leap seconds are applied in UTC (Coordinated Universal Time), which means that the difference between UTC and TT always changes discontinuously by one second. The difference between UT and TT changes continuously but not uniformly because the deceleration of the Earth varies slightly from time to time.

Because the Earth's deceleration for times far in the past or in the future is not precisely known, UT (and also UTC) includes some uncertainties during these times. For such time periods, UT can only be extrapolated. TT is, by definition, a precise measure of time and is used in astronomy for long time spans if the Earth's rotation is not relevant. TT is equivalent to JDE and is used in the VSOP87 theory. If the time of day is important, e.g., for historical events on Earth, UT or UTC should be applied. The P5 program allows for a conversion from TT to UT. The equations, provided further down, are used to calculate the time difference:

$$\Delta T = TT - UT \tag{51}$$

These equations are taken from the NASA Eclipse Web Site: Polynomial Expressions for Delta-T (URL 6), and are reproduced here because they seem to be available only on the Internet. The polynomials up to 7th degree were created by Fred Espenak and Jean Meeus, based on the works of Morrison & Stephenson [50] and Stephenson & Houlden [51].

To apply the equations, a decimal year *J* is required. Espenak and Meeus provide the following equation: *J* = year + (month – 0.5)/12. For consistency with [5] and [14], *J* (Jahr) instead of *y* (year) is used. The maximum error would be 0.5 months and the average error about 8 days, which is sufficiently small. However, we use the decimal year given by Eqs. (68) and (69) in section 4.9.1. The average error of 0.5 days is even smaller and the application is easier because in these equations the year is given directly as a function of *JDE.*

Before the year -500 (astronomical counting), i.e., 501 BC, and from the present into the future, ΔT has to be extrapolated based on the reasonable assumption that the Earth's rotation decelerates more or less constantly. The polynomials are valid only within the corresponding time periods. The result ΔT is given in seconds (Fred Espenak, Jean Meeus [30]):

$J \le -500$:

$$\Delta T = -20 + 32\,u^2 \quad \text{and} \quad u = \frac{J-1820}{100} \tag{52}$$

$-500 < J \le 500$: $\quad u = \dfrac{J}{100}$

$$\begin{aligned}\Delta T = {} & 10583.6 - 1014.41\,u + 33.78311\,u^2 \\ & - 5.952053\,u^3 - 0.1798452\,u^4 \\ & + 0.022174192\,u^5 + 0.0090316521\,u^6\end{aligned} \tag{53}$$

$500 < J \le 1600$: $\quad u = \dfrac{J-1000}{100}$

$$\begin{aligned}\Delta T = {} & 1574.2 - 556.01\,u + 71.23472\,u^2 \\ & + 0.319781\,u^3 - 0.8503463\,u^4 \\ & - 0.005050998\,u^5 + 0.0083572073\,u^6\end{aligned} \tag{54}$$

$1600 < J \le 1700$: $\quad t = J - 1600$

$$\Delta T = 120 - 0.9808\,t - 0.01532\,t^2 + \frac{t^3}{7129} \tag{55}$$

$$1700 < J \le 1800: \quad t = J - 1700$$
$$\Delta T = 8.83 + 0.1603\,t - 0.0059285\,t^2 + 0.00013336\,t^3 - \frac{t^4}{1174000} \qquad (56)$$

$$1800 < J \le 1860: \quad t = J - 1800$$
$$\Delta T = 13.72 - 0.332447\,t + 0.0068612\,t^2 + 0.0041116\,t^3 - 0.00037436\,t^4 + 0.0000121272\,t^5 - 0.0000001699\,t^6 + 0.000000000875\,t^7 \qquad (57)$$

$$1860 < J \le 1900: \quad t = J - 1860$$
$$\Delta T = 7.62 + 0.5737\,t - 0.251754\,t^2 + 0.01680668\,t^3 - 0.0004473624\,t^4 + \frac{t^5}{233174} \qquad (58)$$

$$1900 < J \le 1920: \quad t = J - 1900$$
$$\Delta T = -2.79 + 1.494119\,t - 0.0598939\,t^2 + 0.0061966\,t^3 - 0.000197\,t^4 \qquad (59)$$

$$1920 < J \le 1941: \quad t = J - 1920$$
$$\Delta T = 21.20 + 0.84493\,t - 0.076100\,t^2 + 0.0020936\,t^3 \qquad (60)$$

$$1941 < J \le 1961: \quad t = J - 1950$$
$$\Delta T = 29.07 + 0.407\,t - \frac{t^2}{233} + \frac{t^3}{2547} \qquad (61)$$

$$1961 < J \le 1986: \quad t = J - 1975$$
$$\Delta T = 45.45 + 1.067\,t - \frac{t^2}{260} - \frac{t^3}{718} \qquad (62)$$

$$1986 < J \le 2005: \quad t = J - 2000$$
$$\Delta T = 63.86 + 0.3345\,t - 0.060374\,t^2 + 0.0017275\,t^3 + 0.000651814\,t^4 + 0.00002373599\,t^5 \qquad (63)$$

$$2005 < J \le 2050: \quad t = J - 2000$$
$$\Delta T = 62.92 + 0.32217\,t + 0.005589\,t^2 \qquad (64)$$

$$2050 < J \le 2150: \quad \Delta T = -20 + 32 \cdot \left(\frac{J-1820}{100}\right)^2 - 0.5628 \cdot (2150 - J) \qquad (65)$$

$$J > 2150: \quad \Delta T = -20 + 32\,u^2 \quad \text{and} \quad u = \frac{J-1820}{100} \qquad (66)$$

As recently stated on the NASA Eclipse Web Site, a small correction of ΔT is necessary, meaning that for the years $J < 1955$ and $J > 2005$ the time interval $c = -0.000012932 \cdot (J - 1955)^2$ s must be added to the calculated ΔT. (This small correction is new in P5.) By using Eqs. (52)–(66) (except for the years $1955 \le J \le 2005$), UT is now obtained as follows: UT = TT – (ΔT + c). Note that Eqs. (52) and (66) are identical. All equations are also implemented in the calendar program DATUM-2. The uncertainty in ΔT (URL 12) is also taken from the NASA website [31] and fitted by polynomials for use in DATUM-2 (for details see [14, app. A5]). To get an idea, some results of ΔT with uncertainties (±) are provided below:

J = – 2000 :	ΔT = (775 ± 62) minutes
J = 2000 :	ΔT = (63.9 ± 0.1) seconds
J = 3000 :	ΔT = (74 ± 31) minutes
J = 20,000 :	ΔT = (293 ± 97) hours ≈ (12 ± 4) days

4.9 Computational changes from P3 to P4/P5

When reproducing the results in the tables of book 1 [5] with the P5 program, in some cases slight numerical changes can be found. The astronomical calculations based on the VSOP87 theory are unchanged. This includes the dates, based on JDE, almost all positions like the “Sun position” at the Giza plateau, and other astronomical quantities. However, some other calculations have been improved and the changes − compared to the previous version P3 − are provided below. (Only the small correction of ΔT and UT, which is described at the end of section 4.8, is not included here.) New additional options and all new features of P4/P5 compared to P3 are listed in the program header of the P5 source code (appendix A1) and are also specified in sections 3.3 and 3.4.17.

4.9.1 Decimal year

In some tables the date is not given as JDE but as a decimal year number. This is merely intended to assist the reader in knowing, for example, what JDE = 2,456,282.5 means. If the corresponding decimal year J = 2012.97 is given, it becomes clear that the given date is somewhere at the end of the year AD 2012. In the first book [5, p. 315], the decimal year J was approximated by the following linear function of JDE:

$$J = \frac{JDE}{365.248} - 4711.9986 \qquad (67)$$

When comparing this with the calendar date, this equation has an error of less than 12 days for the time interval 11,000 BC to AD 4000. Before and after this period, when going further into the past or into the future, the error increases linearly with respect to ΔT. For the year AD 10,000, the deviation from the calendar date is about 34 days, and for the year AD 100,000, the error is approximately 1.5 years, which can be checked easily with the DATUM-2 program. The reason for these discrepancies is the existence of two different calendars, the Julian and the Gregorian calendars, with a calendar reform in the year AD 1582. This means that two different linear functions, which are linear on a large scale, are approximated by one linear function in Eq. (67).

The simple solution to this problem is to use *two* linear functions instead of one. Thus, for the Julian calendar and the Gregorian calendar, respectively, we have

$$(0 \le JDE < 2299160.5) \qquad J = \frac{JDE}{365.25} - 4712.0 \qquad (68)$$

$$(JDE < 0 \text{ or } 2299160.5 \le JDE) \qquad J = \frac{JDE - 2451545.0}{365.2425} + 2000.0 \qquad (69)$$

Here, the decimal year, based on the Julian calendar, is only used for the time period $0 \le JDE <$ 2,299,160.5, which are the years between 4712 BC and AD 1582. The upper limit is evident because of the calendar reform. One reason for the lower limit is that the Julian calendar becomes completely out of phase with the seasons before 4712 BC. Furthermore, in those years no historical events are known, thus meaning it would make no sense to use the Julian calendar. The Gregorian calendar is a substantial improvement. However, with 365.2425 days per year and the current value of 365.24219 days [52], an additional correction of one day is needed in 3226 years.

Now, the average deviation between the decimal year and the calendar date is around ± 0.5 days for all times, regardless of how far we go into the past or future. Nevertheless, for our purposes, the difference between Eq. (67) and the system of Eqs. (68) and (69) is relatively small, e.g., for the moment of minimum separation of the Mercury transit during the planetary constellation 12. Eq. (67), applied in the first book, yields J = 3088.365, whereas with Eq. (69) we get J = 3088.379. Computations with the VSOP87 theory are unaffected since they are based on JDE and not on J.

The period of 3,800 years

In this context, the period of about 3,800 years, described in [5, pp. 132, 136], needs an additional comment. At the end of a period of 3,800 years and 1 month, the planets Mercury, Venus, and Earth have nearly the same positions as at the beginning, because for all three planets this period is almost equal to an integer number of orbital periods. (This helped estimate the range of validity of vsop3 in Fig. 13. See also section 4.2.5.) The time interval is about 1,387,980.4 Julian days. If we divide this number by the 365.25 days of a Julian year, we get 3,800.0832 years, which is almost equal to 3,800 years and 1 month. Division by the 365.2425 days of one Gregorian year yields 3,800.1613 years, equal to 3,800 years and 2 months. Thus, the period of 3,800 years and 1 month, discussed in [5], is primarily valid for the Julian calendar. On the basis of the Gregorian calendar, after the year AD 1582, the period is 3,800 years and 2 months. Although the two periods differ by 1 month because of the different length of the years, the physical time period is exactly the same.

4.9.2 Position tolerance

When the planetary constellation of Mercury, Venus, and Earth is fitted to the pyramid positions (or chamber positions), an accuracy of this fit in percent is given by the relative error F_{pos}, F'_{pos}, or F''_{pos}, respectively. (F refers to the German word *Fehler*, meaning error, fault, or mistake.) In the following we use only F_{pos}, although the equations are also valid for F'_{pos} and F''_{pos}, which are each based on a different geometrical approach. In order to get a position error dr of the calculated "Sun position" in m (in [5] also called Δs), the length of the position vector of the "Sun position" $r_S = |\boldsymbol{r}_S|$ is multiplied by F_{pos}, giving $\mathrm{d}r = r_S \cdot F_{pos}$ [5, e.g., Tab. 17, p. 149]. The origin of the corresponding coordinate system is placed in the center of the Mykerinos Pyramid (pyramid positions) or at the base of the Cheops Pyramid (chamber positions), which is an arbitrary choice in both cases. Although the resulting position errors are quite reasonable, it is more convenient to measure the position vectors from the common (bary)center of the three pyramids and from the common center of the three chambers, respectively. The coordinates of these two centers (position vector $\boldsymbol{r}_{CM}$) are merely the arithmetic average of the corresponding rectangular position coordinates of the three pyramids or three chambers. Let r_a be the average distance of the three pyramids (chambers) from this center and r_{Sun} the distance of the "Sun position" from this center, given by

$$r_{Sun} = |\boldsymbol{r}_S - \boldsymbol{r}_{CM}| \tag{70}$$

Then, the position error of the "Sun position" (dr) in m is calculated by the following equations:

For $r_{Sun} > r_a$:
$$\mathrm{d}r = r_{Sun} \cdot F_{pos} \tag{71}$$

For $r_{Sun} \leq r_a$:
$$\mathrm{d}r = \frac{r_a}{2}\left(\left(\frac{r_{Sun}}{r_a}\right)^2 + 1\right) \cdot F_{pos} \tag{72}$$

Because F_{pos} is given in percent, F_{pos} must be divided by 100 before being inserted into Eqs. (71) and (72). If the "Sun position" is located near to the common center of the three chambers, meaning that r_{Sun} is almost zero, then the relative position error dr, calculated with Eq. (71), is also nearly zero. This would not be reasonable and is avoided by using the parabolic function in Eq. (72).

Eqs. (70) to (72) are used for the positions of the "Sun" and all the "planets" – by replacing r_{Sun} accordingly – except for Mercury, Venus, and Earth. For these three planets, the deviations between transformed planetary positions and pyramid (chamber) positions can be determined exactly by calculating the corresponding Euclidean distances. These values are marked with < (see options 3, 230, and 231 in sections 3.4.3, 3.4.11, and 3.4.12, respectively). Finally, the previous equations are analogously used to determine the uncertainty of the Mercury aphelion position for the constellations 13 and 14.

4.9.3 Algebraic sign of X_5

One method for calculating the "Sun position" in 3 dimensions is to use coordinate transformations and the least squares fit program FITEX, in which the planetary positions are fitted to the pyramid or chamber positions by adjusting the seven parameters X_1 to X_7. The orientation is adapted by using the rotational matrix $\boldsymbol{R}(X_4, X_5, X_6)$ of Eq. (23). With X_4, X_5, and X_6 being the Eulerian angles, $\boldsymbol{R}$ is the product of three matrices $\boldsymbol{D}_z(X_6) \cdot \boldsymbol{D}_x(X_5) \cdot \boldsymbol{D}_z(X_4)$; see also [5, pp. 335 ff.]. In the following, the algebraic sign of X_5 is discussed. Let α be the rotational angle; then, for example, a rotation around the x-axis is given by the matrix

$$\boldsymbol{D}_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \tag{73}$$

(Note: It seems that, accidentally, all rotations in [5] are given by the transposed (inverse) matrices compared to the normal convention. Because this is only a question of agreement and because it does not change the results – except for the sign of the rotational angles – we kept this allocation here.) The angle X_5 is the tilt angle between the planes of the Earth's surface and of the transformed Earth's orbit [5, p. 345]. Both planes are shown schematically in Fig. 2. When geometrically considered, it can be established that, if X_5 is a solution matching the planetary positions with the pyramid (chamber) positions, $-X_5$ is also a solution. In the latter case, the angles X_4 and X_6 have to be replaced by $X_4 \pm \pi$ and $X_6 \pm \pi$, respectively, where the plus and minus signs can be chosen arbitrarily and independently for both quantities. Thus, for rotations in 3 dimensions we get the following matrix identity:

$$\boldsymbol{D}_z(X_6) \cdot \boldsymbol{D}_x(X_5) \cdot \boldsymbol{D}_z(X_4) = \boldsymbol{D}_z(X_6 \pm \pi) \cdot \boldsymbol{D}_x(-X_5) \cdot \boldsymbol{D}_z(X_4 \pm \pi) \tag{74}$$

This can be shown easily with the matrix in Eq. (23). By modifying the angles correspondingly, all changes of algebraic signs of the trigonometric functions cancel each other, implying that the matrix remains the same. It follows that the sign of X_5 has no meaning if X_5 is given without X_4 and X_6. Therefore, most tables in the second book [14] list only the absolute values of X_5. On the other hand, the P5 program always yields the actually found algebraic sign of X_5. Eq. (74) can be demonstrated, for example, with a postcard. After defining the x-, y-, and z-axes, as well as X_4, X_5, and X_6, the reader gets the same result by rotating the postcard manually by using either the given angles on the left or the right side of Eq. (74).

4.9.4 Date of constellations 13 and 14

For the constellations 13 and 14, the date is not fixed to the planetary passage through aphelion or perihelion. Instead, the exact point of time was found by manually minimizing the relative error F''_{pos}. For the results in the first book [5], when using the P3 program, the Julian days (*JDE*) were rounded to three digits after the decimal point. In the P4 and P5 programs, the minimization of F''_{pos} is done automatically and the *JDE* results are accurate for about five digits after the decimal point. In order to achieve consistency between different calculations within the second book, the dates are not rounded as before. Thus, when reproducing the results in the first book with the P5 program, there are slight differences in the planetary coordinates concerning constellations 13 and 14. Nevertheless, these tiny differences are not important. They are mentioned here so that the user of the P5 program knows (if comparing the results with [5]) where these deviations come from.

4.10 Further specific features concerning Giza

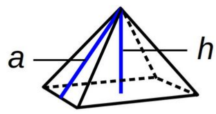
Two additional discoveries with respect to constellation 12 (AD 3088) are presented below in sections 4.10.2 and 4.10.3. These are not directly related to the use of the P5 program, but they do support the general findings. They are also mentioned to show how the overall picture of the planetary correlation is even more complex than originally expected. In sections 4.10.4 and 4.10.5, we learn more about "Sun positions" and "secret chambers." It follows a detailed analysis concerning the accuracy of Eq. (1) in sections 4.10.6 and 4.10.7, but before these aspects are explained, some information is given in the next section concerning mathematical speculations.

4.10.1 Matching coefficients

If archaeological facts, like measured data, are mixed with mathematical speculations to show certain relations, criticism sometimes arises, stating that with such "mathematical playing around" anything can be proven. Truthfully, under special circumstances, this criticism is justified. However, it is not always true. To clarify, we will specify what "special circumstances" means in this context by using two examples.

In addition to classical archaeology, two relations that define the size of the Cheops Pyramid have been found in the literature. First, the distance from Earth to the Sun (1 AU = 149.6 million km) is said to be 1 billion times greater than the height of the Cheops Pyramid (146.59 m [10, vol. IV, p. 1228]). Second, the height of the triangular side of the Cheops Pyramid, being the distance from the base line to the top of the pyramid (186.43 m), multiplied by 600, should be the distance corresponding to 1° difference in longitude at the equator (111.32 km). The relative error of the first equation varies up to 3.6 % because the distance between Earth and the Sun is not constant due to the elliptical shape of the Earth's orbit. The accuracy of the second relation is about 0.5 %, which is somewhat better. The reader can verify this easily. Nevertheless, both relations have a serious disadvantage: They both contain an arbitrary factor, namely 1 billion (1,000,000,000) and 600. The problem is that with such factors, or better "matching coefficients," just about anything can be proven. The corresponding equations are given below together with a small sketch. The crossing out of both equations stresses that both of them are meaningless. The reason is given below.

1. Distance Earth – Sun: $r = 1{,}000{,}000{,}000 \cdot h_{Cheops}$
2. Distance d for 1° difference in longitude: $d_{(\Delta L = 1°)} = 600 \cdot a$



Let us take two arbitrary quantities (e.g., the height of the Eiffel Tower and the distance between London and New York) and let us allow matching coefficients, consisting of one digit (1 to 9) and an arbitrary number of zeros (e.g., 100, 4,000, 70, 300,000, etc.). In this case, it can be easily shown that with the corresponding matching factor, an average accuracy of about 10 % can be achieved. We can illustrate this with an example. Let us assume that the ratio between two quantities is exactly 550. Thus, we need a factor of 550 to get an equation that is perfectly valid. According to the previous assumption concerning matching coefficients, the nearest available factors are 500 and 600. The relative error of an equation, using 500 or 600, would be approximately 10 %. A mathematically more detailed discussion is given in [5, pp. 64 ff.]. Now, a deviation of 10 % is more than the errors in the above two equations, so we could misleadingly assume that both equations are still significant. However, this is not the case.

The given pyramid has five characteristic lengths: the height h, the height of the side face a, the base length, the diagonal in the base area, and the distance from a corner to the top of the pyramid. If we take, for example, five astronomical lengths for comparison, e.g., the distance from Earth to the Moon, the circumference of the Earth, etc., then we have 25 different combinations for the five lengths in the pyramid. The 25 corresponding equations do not have an accuracy of 10 % each – some of them have a lower and some a higher accuracy, meaning statistical scattering. Now, it can be shown mathematically that, on average, at least one of these combinations has an error of less than 1 %! This means that we have to look only for the smallest error of all 25 relations to obtain an accuracy of better than 1 %. It follows that these equations containing a matching coefficient (like 600 or 1 billion) have no significance! Using these factors we can prove anything. We could even easily bridge several orders of magnitude. In short, equations with these matching coefficients are irrelevant! The same can be achieved if more complicated equations are used. This includes squares (e.g., x^2, where x might be any quantity), other mathematical powers (x^3, x^4, …), square roots, constants like π, etc., or even two or more matching coefficients instead of one.

Another important criterion is that the physical units, e.g., m, kg, etc., must match. For example, after combining some archaeological quantities in an equation, someone might get the number 2.99 and could claim: “Here, we have the speed of light.” In actuality, the speed of light is $2.9979 \cdot 10^8$ m/s. The relative error of the pure digits is about 0.3 %, which is not bad, but the physical unit of velocity, in this case m/s, is missing. Furthermore, if using the unit m/s, the order of magnitude is wrong by a factor of 100,000,000. The last factor is a matching coefficient, and is not even mentioned. Thus, the value of 2.99 has absolutely no meaning with respect to the velocity of light.

We will keep in mind that matching coefficients are not allowed. However, without factors like this it is almost impossible to find an equation that relates an arbitrary quantity with fundamental physical or astronomical constants. So, what about the three equations (1) to (3) of the planetary correlation? All of them are simple and of the same kind: the rule of proportion. No matching coefficients are used and the physical units are correct. Moreover, the physical quantities create an overall picture, which makes sense. Thus, they are not of the category “matching coefficients,” as explained previously. Especially, Eq. (1) is analyzed in detail in sections 4.10.6 and 4.10.7 and also in [14]. Two other astonishing aspects that support the planetary correlation are described below.

4.10.2 Obliquity of the ecliptic

Figure 2 in the introduction shows how the transformed planetary orbits are tilted against the Earth's surface. The tilting angle between the transformed ecliptic plane (plane of the Earth's orbit) and the Earth's surface is 24.47° (= X_5; see X_5 in sections 3.4.3 and 3.4.4). This is arrived at by using the VSOP87 theory in the P5 program, in which X_5 is one of the parameters X_1 to X_7 characterizing the coordinate transformation from the positions of the planets to those of the pyramids.

Why didn't the master builders construct these two planes to be coplanar, but instead tilted them against each other? The answer is simple: the obliquity of the ecliptic, ε, is about 23.45°, which is the angle between the plane of the equator and the plane of the Earth's orbit (ecliptic plane). By the way, this obliquity is the reason why we have four seasons per year.

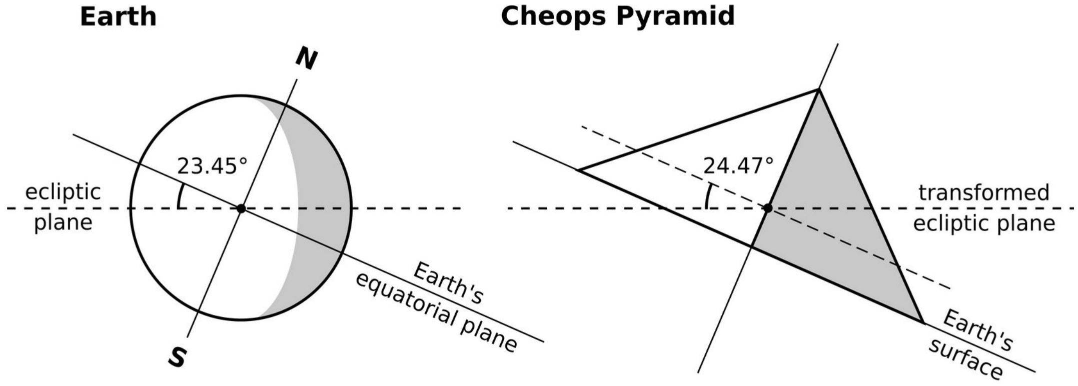


Figure 20: *Correlation between the Earth and the Cheops Pyramid – as seen from a southwest direction – with respect to the ecliptic plane (see Fig. 2 and for 24.47° see X_5 in the table of section 3.4.3).*

The given correlation means that the angle between the plane of the Earth's equator and the ecliptic plane correlates with the angle between the Earth's surface and the ecliptic plane after coordinate transformation. The difference of 1° is small. If we take the year 2968 BC, being roughly the average of 3030 BC and 2905 BC (age of the Cheops Pyramid determined by accelerator mass spectrometry, AMS, see page 2 [11, 12]), we obtain $\varepsilon = 24.03°$ on the basis of Eq. (45). This yields an even better agreement.[5] An illustration is given in Fig. 20 (see also Fig. 2). This is perfectly in keeping with the correlation between pyramids and planets and can explain why the transformed planetary orbits in Fig. 2 are considerably tilted against the Earth's surface in Giza. (Remark: The best agreement, given by $\varepsilon = 24.30°$, would be obtained around 8300 BC.)

4.10.3 The riddle of midwinter

Figure 21 shows that inside the Cheops Pyramid the "Sun position" is located south of the subterranean chamber. If we look from the chamber to the "Sun position 2" (Fig. 23), the direction is about 33.6° upward from the horizontal direction. (With $\Delta x = 16.29$ m, $\Delta y = 10.97$ m and $\Delta z = 2.68$ m being the differences of the respective coordinates of "Mercury" and "Sun position" – see table in

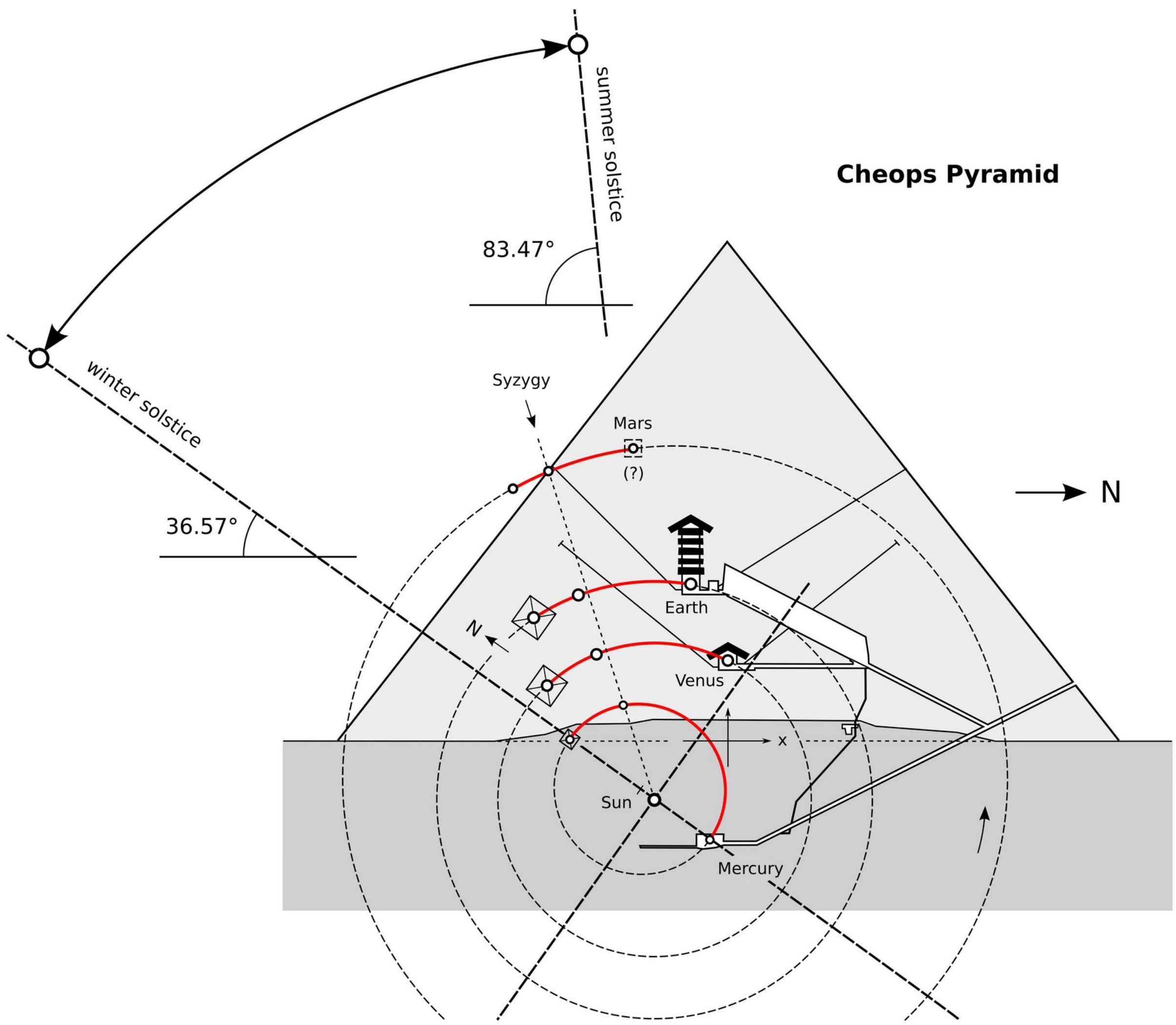


Figure 21: *Cross section of the Cheops Pyramid (AD 3088) with highest Sun position in midwinter and midsummer.*

5 I got this idea of considering the variability of the tilted ecliptic for the first time from Lisa Anders, who sent me a mail in 2015. Later, two other independent mails from Franz Grabendorfer and Christoph Opalka presented the same idea, among other aspects. Thus, I am indebted to these persons, who attentively watched my talk about the pyramids on the Internet. Of course, the time-dependent effect is included in the astronomical calculations (VSOP). However, at the beginning, when trying to explain the angle of 24.47° and the "midwinter problem," this was not considered.

section 3.4.11 – the angle is calculated by arctan($\Delta y/((\Delta x)^2 + (\Delta z)^2)$) = 33.60°.) The real Sun has its highest position above the horizon in the south at noon. The question is: Is it possible that the real Sun stands in the same direction of approximately 34° above the horizon?

The highest daily position of the Sun is dependent on the time of year. In summer the maximum angle above the horizon at Giza is 83.47°. This date is called midsummer or the summer solstice. In winter, the lowest angle of the Sun in Giza at noon is 36.57°. Figure 21 provides the geometrical arrangement. It shows that when looking from the "Mercury position" in the subterranean chamber to the "Sun position" in midwinter, the real Sun stands almost in the same direction. The angular difference of ca. 3° is small but not very small. For the astronomical comparison with the planetary positions, the positions are assumed to be in the middle of the east walls of each chamber.

In order to explain the angular discrepancy of 3°, two aspects can be considered. As mentioned before, the obliquity of the ecliptic varies with the date. Let us again take the potential date of construction of the Cheops Pyramid (2968 BC) based on AMS [11, 12]. The height of the Sun, h, above the horizon is calculated by $h = 90° - \varepsilon - B$, where B is the geographical latitude of the Cheops Pyramid. With ε = 24.03° and B = 29° 58.7524' [14], the height of the Sun is h = 36.0°. For the year 3088, we obtain ε = 23.30° and h = 36.7°. Both values of h do not considerably change the result.

The second point is that the "Sun position 2" in the pyramid is not exactly south of the "Mercury position," as can be seen in Fig. 23. The deviation from the south direction to the east, if looking from "Mercury position 2 (Fig. 25)," is arctan($\Delta z/\Delta x$) = 9.34°. In this direction, the real Sun in the sky is not yet at its highest point, but approximately 0.6° lower. (This angle was determined with the program *Stellarium* [23].) Thus, if considering this aspect and taking the year 2968 BC, the discrepancy of 3.0° is reduced to 36.0°– 33.6°– 0.6° = 1.8°, which is slightly better, but not good enough. Therefore, the midwinter problem still remains an open question.

With respect to the corresponding date 2968 BC, it is interesting that almost at the same time (2970 BC, astronomical year numbering), the very rare event of a central Venus transit occurs, which can be found with the quick start option 22.

4.10.4 "Sun position" and concrete platform

In Fig. 15 we see the "Sun position" located approximately 670 m south of the center of the Mykerinos Pyramid. This position is defined precisely by the planetary positions in the year 3088 with an uncertainty of about 1 m if the VSOP87 theory with 3-dimensional coordinate transformation is applied. The question is: Do we find anything special at this location? The answer is yes.

In 2003 I visited this spot and found a platform of concrete. This platform was definitely not from ancient times, but rather from modern times. The center of the platform was located at latitude 29° 57.9898' north and longitude 31° 7.6842' east. The numbers are an average of a few measurements, performed on different days with a GPS receiver (Garmin eTrex Summit). The theoretical "Sun position," calculated with VSOP87, is latitude 29° 57.9905' north and longitude 31° 7.6813' east (section 3.4.13). At that time, the platform had dimensions of about 25 · 50 m^2 and was oriented with its long middle axis towards the center of the Chefren Pyramid. The distance between the measured and the theoretical position is only 5 m. One year later, the whole platform was covered with a half-meter deposit of sand, which had been put there artificially and not by wind transportation. Today, in 2025, the platform is again mostly free of sand. Meanwhile, its shape has been changed and can easily be seen on Google Earth or Google Maps (satellite view).

What is the purpose of this platform? It is located in the desert, surrounded by sand and hilly ground. There is no paved vehicle access, and therefore it cannot be used as a parking area, although this is what it looks like. The question is: Is the remarkable coincidence of the platform position and the "Sun position" accidental or not? Of course, a coincidence is possible, but the probability of this seems low. The calculation of the "Sun position" yields a vertical coordinate of

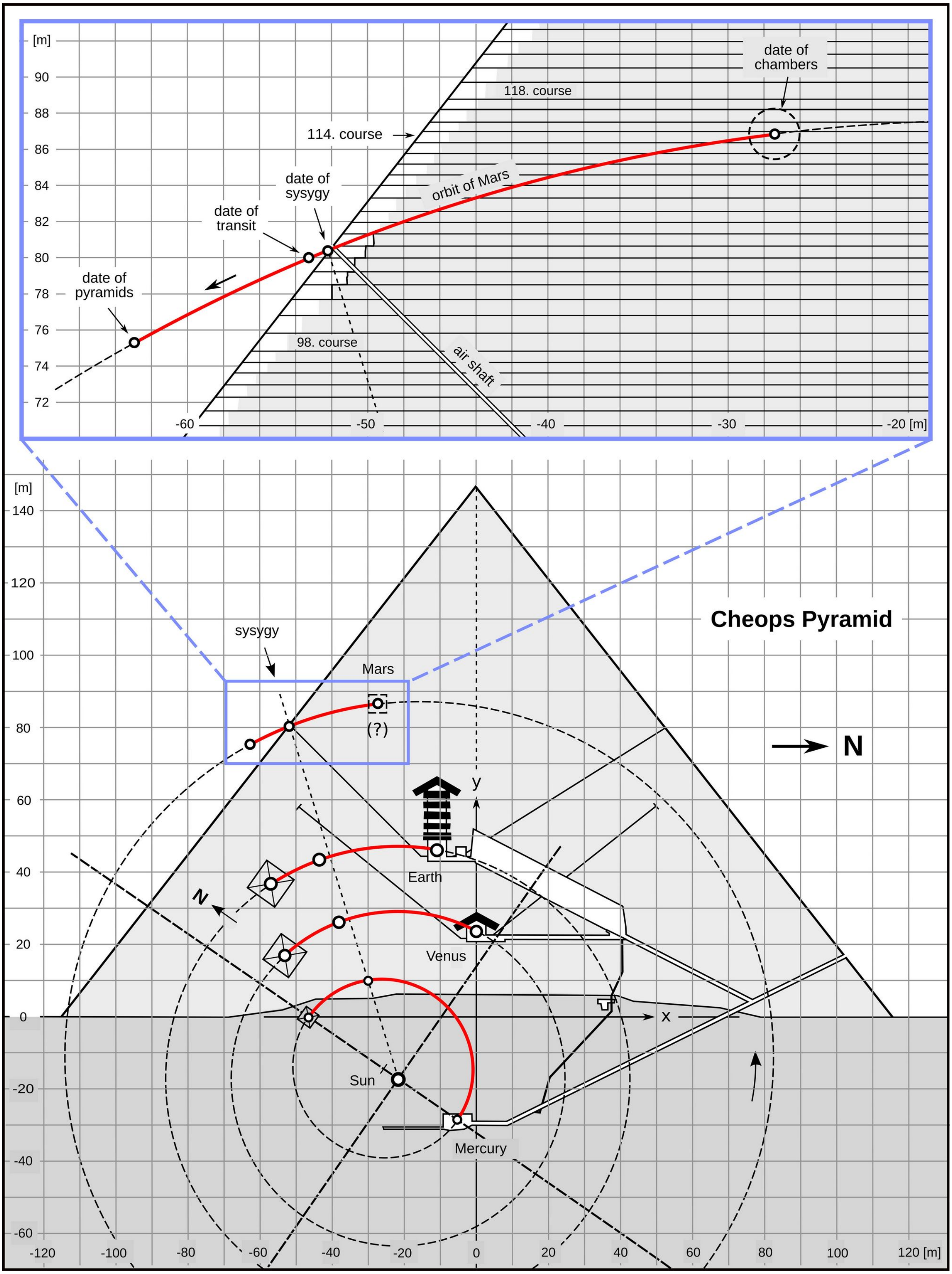


Figure 22: *Cross section of the original state of the Great Pyramid with details of the "Mars position" and its environment (upper inset) during the astronomical events in the year AD 3088. The levels of the courses were measured by Sir W. M. F. Petrie [6, Map VIII] – see also [5, pp. 392–393]. The shape of the blocks of stone around the opening of the southern air shaft is taken from a drawing by Maragioglio and Rinaldi [9, part IV, map 2, Fig. 2]. This drawing, where we find the numbers of the courses, was published in 1965. The exact "planet positions" in the pyramid concerning the dates of sysygy, transit, and pyramids were calculated with the quick start option 331. The alternative is 330 (section 4.10.5).*

272.36 m above the base level of the Mykerinos Pyramid. Thus, the “Sun position” is placed more than 250 m above the ground. So, if someone digs beneath this platform, they will probably not find any treasure, other than sand.

4.10.5 “Secret chambers”

A better chance for a successful search is given in the Cheops Pyramid. Beside the “Sun position” beneath the pyramid, there is a “Mars position” about 40 m above the King's chamber. The massive volume of the Pyramid consists of more than 200 courses of stone blocks. Fortunately, Sir W. M. F. Petrie accurately measured the level of each course [6, map. VIII; 14, Tab. 15] so that it is possible to locate the course of the “Mars position.” A true-to-scale drawing of the pyramid's cross section with a grid for higher graphical precision and better visibility of the distances is provided in Fig. 22.

“Mars position”

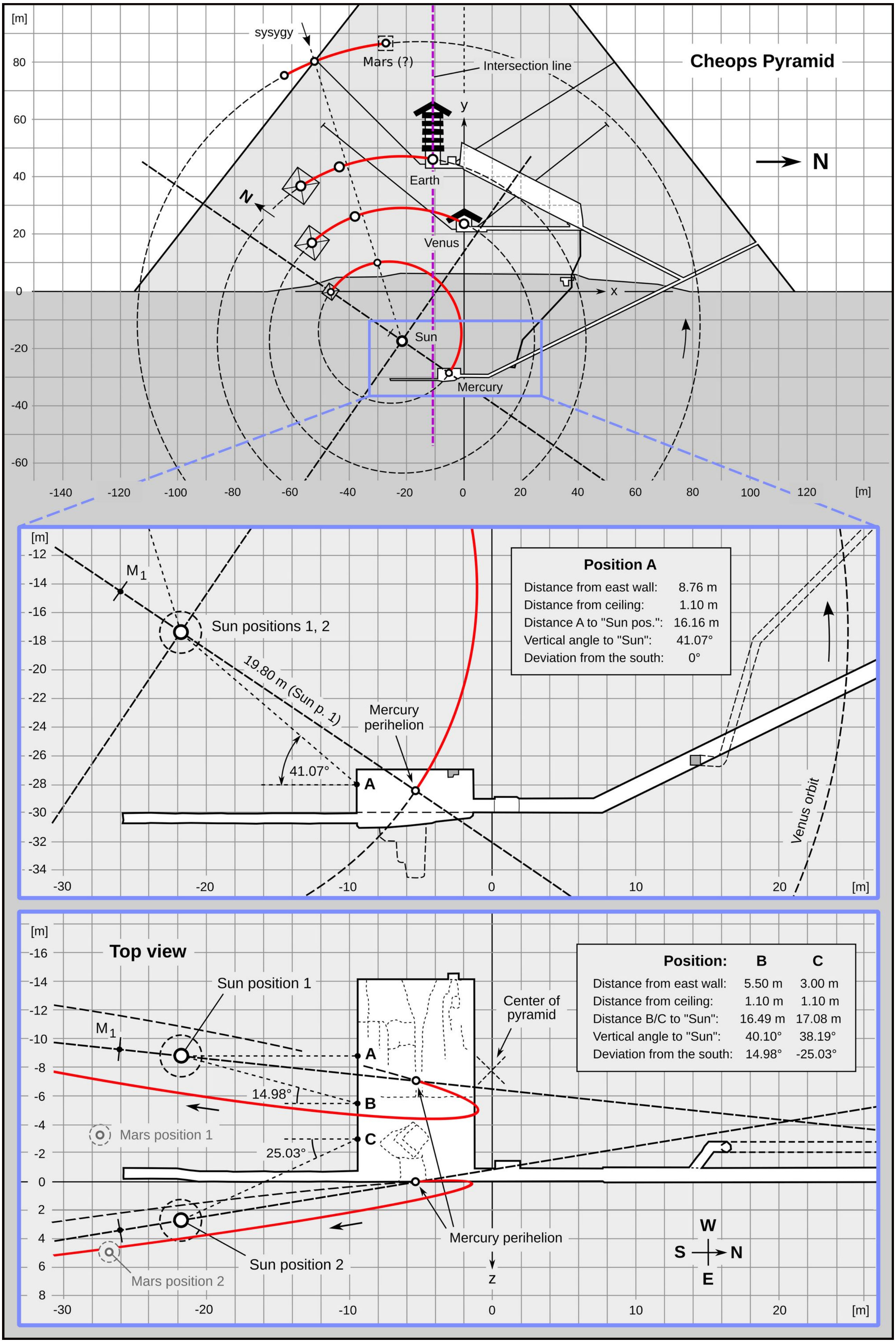
According to the data of Petrie, the 114. course covers the height from 86.385 m to 86.957 m, measured from the pyramid base. As the vertical coordinate of the “Mars position 2” is 86.77 m, the position is located within the 114. course. The horizontal distance from the original southern pyramid surface is about 19.5 m. But how is the positioning in the east−west direction? The position of the central pyramid axis to the middle plane of the corridors (Fig. 3) is 7.20 m to the west [9, part IV, map 3, Fig. 2].[6] Adding half of the corridors’ width (0.53 m) [9, part IV, map 6, Fig. 4] gives 7.73 m to the east walls. The “Mars position 2” is located 4.89 m to the east from the common plane of the east walls. It follows that we find the “Mars position 2” about 12.6 m to the east of the vertical middle plane of the pyramid (see top view in Fig. 23). The coordinates of all of the transformed planetary positions inside the pyramid can be computed with the options 330 and 331.

Southern air shaft

An interesting fact is that in Figs. 22 and 23, the “Mars position” at the date of the planetary conjunction is placed almost exactly at the original opening of the southern air shaft of the King's chamber. The orbit of Mars in the year 3088 can be calculated very precisely with the VSOP87 theory. The astronomical precision is better than 1 arc second for the heliocentric coordinates. If we transfer this precision to the “planetary positions” in the Great Pyramid, we obtain an accuracy of better than 1 mm for the transformed Mars positions. The chamber positions in the Great Pyramid have an uncertainty of approximately 10 cm, which is still very good. The largest error comes from combining chamber and planetary positions and is given by the relative error in percent (see, e.g., the program output in section 3.4.11). The computed error of 0.25 % for comparison with the chambers means a spatial uncertainty of 19 cm for the “Mars position.” But what about the z-coordinate, fixing the position in the east−west direction? The distance of the middle of the southern air shaft (entrance) from the east wall of the King's chamber is 2.46 m [9, part IV, map 7, Fig. 10].

Figure 23 (right): *Cross section of the Great Pyramid with details of the two alternative “Sun positions” and “planetary positions” in AD 3088, to the south of and above the subterranean chamber. The dimensions of the chamber and corridors were taken from drawings by Maragioglio and Rinaldi [9, part IV, maps 3 and 4]. The points A, B, and C were chosen arbitrarily as possible access points to start a borehole in order to examine the transformed Sun position. The position uncertainties of the “Sun position” and “Mercury position” are around 20 cm. From point A, the drilling should be oriented exactly to the south with an angle of ca. 41.1° above the horizontal plane. (This is only an example for how to proceed if an inspection were be done.) If the residual rock structures in the subterranean chamber are an obstacle, the drilling can also be started closer to the east wall, for example at point B. In this case, the drilling direction is not exactly to the south but has an angular deviation of ca. 15.0° to the west and a vertical angle of 40.1°. Notice that the distance between Mercury at perihelion and the Sun position 1 of 19.80 m is not calculated using the “Mercury position” (section 3.4.12) but based on the coordinates of the spatial middle of the chamber (Tab. 4). On the basis of the Mercury position, the distance would be 19.85 m. The drawn orbits are orthogonal projections. However, the alternative “Sun position 2” with the starting point C for drilling and the “Mars position 2” are the best solution.*

6 In the given reference, the central axis of the pyramid is drawn to the east of the corridors, which is not correct. Actually, the central axis is located to the west of the corridors, which can be seen, for example, in the top view of the Great Pyramid [5, Fig. 163], published originally by Piazzi Smyth [33, p. 1235].



Cheops Pyramid
N
sysygy
Mars (?)
Intersection line
y
Earth
Venus
Sun
Mercury
x
[m]
80
60
40
20
0
-20
-40
-60
-140
-120
-100
-80
-60
-40
-20
0
20
40
60
80
100
120
Position A
Distance from east wall: 8.76 m
Distance from ceiling: 1.10 m
Distance A to "Sun pos.": 16.16 m
Vertical angle to "Sun": 41.07°
Deviation from the south: 0°
M1
Sun positions 1, 2
19.80 m (Sun p. 1)
Mercury perihelion
41.07°
A
Venus orbit
Top view
Sun position 1
Center of pyramid
Position: B C
Distance from east wall: 5.50 m 3.00 m
Distance from ceiling: 1.10 m 1.10 m
Distance B/C to "Sun": 16.49 m 17.08 m
Vertical angle to "Sun": 40.10° 38.19°
Deviation from the south: 14.98° -25.03°
14.98°
B
C
25.03°
Mars position 1
Mars position 2
Sun position 2
Mercury perihelion
z
W
S
N
E

At the chambers date the corresponding distance of the “Mars position 2” is 4.89 m in the other direction (to the east, book option 230). If we consider the southern air shaft being exactly oriented in a north−south direction, we get an east−west distance of the “Mars position” of 7.35 m to the east of the opening of the air shaft.

If a horizontal borehole were drilled from the south side of the pyramid to examine the location of the “Mars position,” the east−west positioning should be determined with both options “middle axis of the south side” and “position of the air shaft” because the exact north−south alignment of the air shaft between the King's chamber and the pyramid's surface is unsure. The middle axis of the pyramid's south side can be fixed by triangulation from the two south corners of the pyramid. As mentioned before, the “Mars position 2” is located ca. 12.6 m eastward from the central axis of the pyramid and 7.35 m eastward from the air shaft. We have an interesting analogy: At the end of the transit, Mercury leaves the Sun's disk, and, shortly before, “Mars” leaves the Great Pyramid.

“Sun position”

The “Sun position” is located to the south and above the subterranean chamber. The coordinates are given with an accuracy of about 10 cm. In order to examine how this position can be reached, a detailed view of the area around the subterranean chamber is given in Fig. 23.

The main difference of Fig. 23 to the corresponding figure in the previous version (P4) is that instead of one “Sun position,” we have two. Position 1 was calculated with the coordinates as the spatial middle of the chambers, which we call assumption 1. Position 2 is obtained by using the middle of the east walls of the chambers (assumption 2). The main reason for using position 1 was the assumed correlation between the vertical plane of the east walls of chambers and corridors (purple plane in Fig. 24) with the plane of the Earth's orbit. The angle between both planes is 4.183° and the relative position error is 0.57 % for assumption 1, whereas for assumption 2 the angle is 13.957° and the relative error 0.25 %. The idea was that the vertical plane in the pyramid has a meaning and the Earth's orbit fits best to this plane with assumption 1, although the error of 0.57 % was slightly larger in this case.

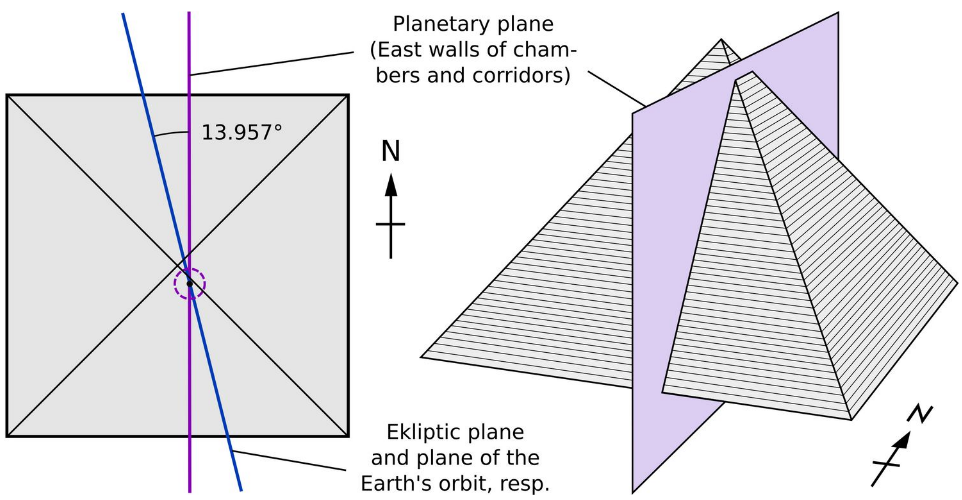


Figure 24: *Sketch of the Cheops Pyramid with planetary plane (purple), being defined by the east walls of chambers and corridors in the pyramid, and the transformed ecliptic plane (blue line). Both planes intersect at a vertical line (small circle), in a collinear position with the middle axis of the east wall of the King's chamber – see dashed purple line in Fig. 23 and purple circle in Fig. 25 (Earth 2).*

Nevertheless, a much better idea was found. After again scrutinizing assumption 2 (east walls), this alternative seems to make much more sense because of the following four reasons:

1. In assumption 1, the plane of the Earth's orbit with its deviation of 4.183° does not fit well to the vertical pyramid plane. In assumption 2, however, the transformed positions of the three planets Mercury, Venus, and Earth are located exactly in this plane. This means that these three

planets precisely define this plane and we have a perfect correlation (0° deviation). Therefore, we designate this the "planetary plane," which now has an exact meaning.

2. With the "Sun position 2" (east walls), the correlation fits best with an error of 0.25 %. The spatial middles of the chambers mean a 0.57 % error and the west walls a 2.19 % error.

3. After checking the orientation of the transformed Earth's orbit (ecliptic plane) in the pyramid, it was found that it has a *vertical* orientation with a deviation of 13.957° from the north direction. The intersection line of the plane of the Earth's orbit with the planetary plane runs vertically through the middle of the east wall of the King's chamber (Earth) – see the purple line in Fig. 23 and the purple circle in Figs. 24 and 25. This has to do with the fact that the absolute value of the rotation angle X_6 = –90.11° is almost 90° (section 3.4.11).

4. If the point of time is not fixed to the perihelion passage of Mercury, the relative error can be minimized by searching for the optimum date before or after the perihelion passage. The optimum date (minimum error) for the spatial middle of the chambers is 14.7 hours and for the west walls 56 hours prior to the perihelion passage. Surprisingly, the optimum date for the east walls is only 5 to 6 seconds (!) after the perihelion passage, which is virtually a perfect coincidence. In the corresponding program run (option 10, section 3.4.6), the time difference is zero (dt = 0.000 days) because of only three digits after the decimal point.

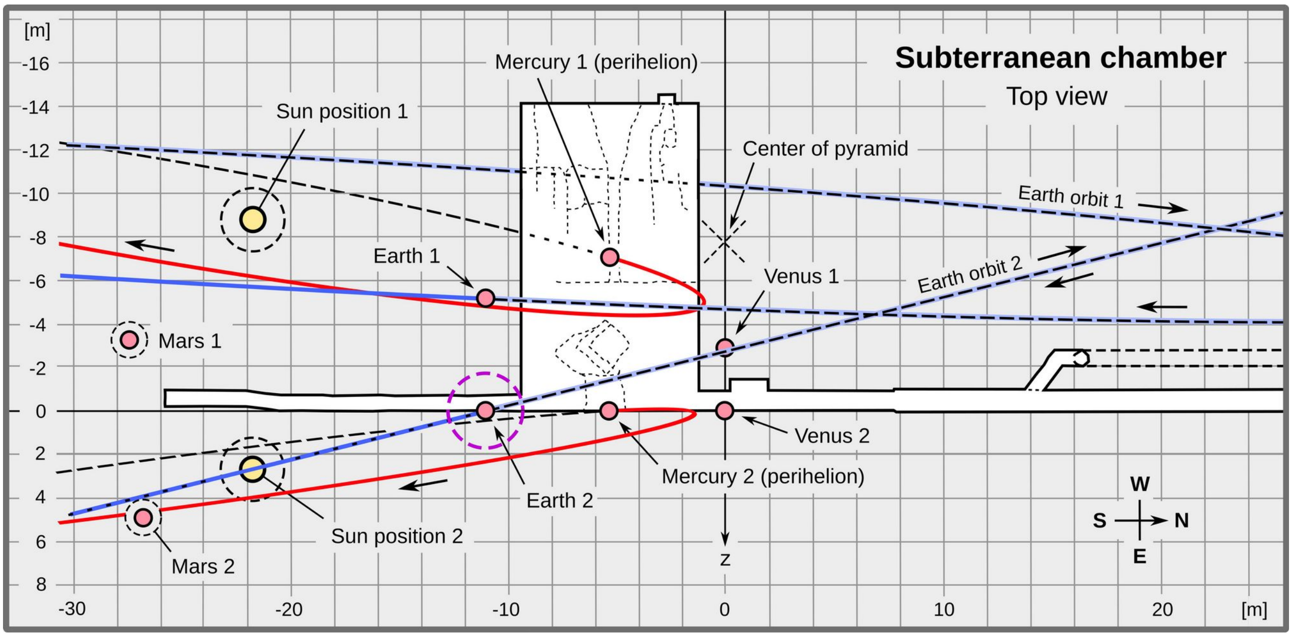


Figure 25: *Top view of the subterranean chamber in the Cheops Pyramid (planetary constellation on April 17, 3088, 6:41:13 am, TT). Again, the alternative arrangements of the planets and the Sun (assumptions 1 and 2), based on the spatial centers of the chambers (1) and on their east walls (2) are provided. Additionally, the Earth's orbit is depicted in blue for both assumptions. In the second case, the Earth's orbit is a straight line, because it is vertically oriented.*

In Fig. 25, the top view of the subterranean chamber from Fig. 23 is provided once again, now with the orthogonal projections of the Earth's orbit (blue lines). These two orbits are constructed accurately by calculating the position of Earth for several successive points in time. It can be seen that for assumption 2 the Earth's orbit is given by a straight line because of its vertical orientation. Additionally, the positions of Venus and Earth are depicted at the time of the perihelion passage of Mercury. For assumption 2, the (transformed) three planets are all placed in the vertical planetary plane, defined by the coordinate z = 0. It follows that the most probable candidates for positions of undetected chambers in the pyramid are the "Sun position 2" and the "Mars position 2." For the exact coordinates, see the program output in section 3.4.11. Assumption 1 is less probable but not impossible (coordinates in section 3.4.12).

4.10.6 Analysis of equation (1)

The main Eq. (1) on page 12 defines the base length and size of the Cheops Pyramid. For convenience, Eq. (1) is provided again here:

$$\frac{S_{Cheops}}{c \cdot 1\,\mathrm{s}} = \frac{V_{Earth}}{V_{Sun}} \tag{1}$$

Originally, we used the following quantities in Ref. [5]: the average of the four base lengths of the pyramid is S_{Cheops} = 230.364 m [53, 54], one light second is 299,792,458 m/s · 1 s = 299,792,458 m, and the volume ratio of Sun to Earth is V_{Sun}/V_{Earth} = 1,301,000 [55].

This yields a difference of 0.03 % between the two sides of Eq. (1), which is a very good agreement. The reader should note that the equation does not contain any arbitrary additional number in order to improve the identity. (Concerning the unit of time, 1 second, see [5, app. A9].) However, we will now check the accuracy of Eq. (1) in much greater detail. For this, we do not rely on the Sun-Earth volume ratio but instead calculate the volumes separately. The most comprehensive calculation was the determination of the Earth's volume because we need to use more than just the Earth's given radii for the calculation: $V_{ell} = (4\pi/3)\, R_e^2 \cdot R_p$. This only yields the volume of the reference ellipsoid on the basis of the sea level, and it follows that all land and ice masses are ignored. Thus, we start with the calculation of the Earth's real volume.

The Earth's volume

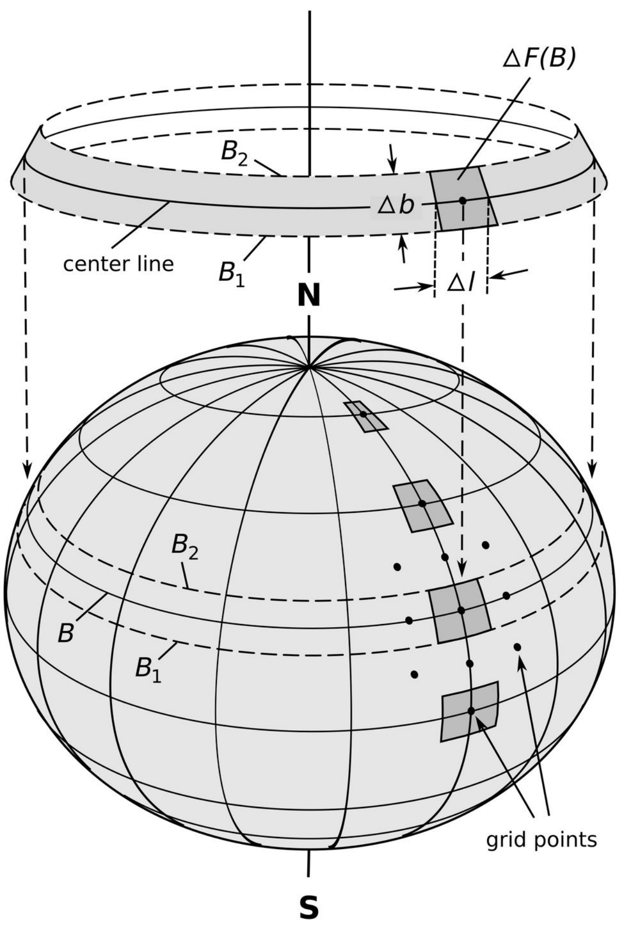


***Figure 26:** Spheroid of the Earth with some points of the topographic grid. B, B_1, and B_2 are geographical latitudes.*

The topographic data are taken from the Worldbath 5 arc-minute grid [56] (URL 13). If we had the average height of the land and ice masses, we could add this average to the equatorial and polar radii and thus obtain the real volume of the Earth. However, the calculation of the average height is not simple. We cannot just take the arithmetic mean of all the topographic data because the grid points cover areas of different size. This can be seen in Fig. 26, where the areas become smaller if the grid points approach the poles.

Along one circle of latitude the corresponding area of the grid points does not change; along one circle of longitude the size of the areas always changes. Another problem is that the main shape of the Earth is not a sphere but a spheroid (ellipsoid). Thus, we approximate the areas of all points of a certain latitude *B* using a strip, which is actually a conical frustum (see upper ring in Fig. 26). The lateral surface area of this frustum is the length of the circle of latitude *B* times the width, Δ*b*, of the ring.

(In the computer program TOPO, the area of each grid point is divided into several parallel strips in order to improve the accuracy.)

The Worldbath 5 arc-minute grid consists of 4320 × 2161 = 9,335,520 points (longitude × latitude). The area belonging to one point on the ring (frustum) exactly is $\Delta F(B) = \Delta b \cdot \Delta l$ (see Fig. 26), in which the lower and upper boundaries of the area are given by B_1 and B_2. With the distances $u(B)$ and $v(B)$ calculated according to Eqs. (33) and (31) in section 4.6.4 (Fig. 16), we obtain

$$\Delta l = \frac{2\pi\, u(B)}{4320} \quad \text{and} \quad \Delta b = \sqrt{(u(B_2)-u(B_1))^2 + (v(B_2)-v(B_1))^2} \qquad (75a, b)$$

The program TOPO not only allows for calculations concerning the whole Earth's surface, but also with respect to certain areas, defined by the borders given by the geographical latitude and longitude. In order to calculated the average height $\bar{h}$ for a certain (flat) area F, the main equation is

$$\bar{h} = \frac{\sum_i h_i \cdot \Delta F_i(B)}{\sum_i \Delta F_i(B)} \qquad (76)$$

The subscript i counts the grid points within the given area, and h_i and $\Delta F_i(B)$ are the height and area belonging to a single grid point. Concerning the calculation of $\bar{h}$ and the choice of ΔF_i, there are different possibilities. Although these alternatives do not influence the final result by much, we will clarify this point, which also makes sense with respect to the volume calculation.

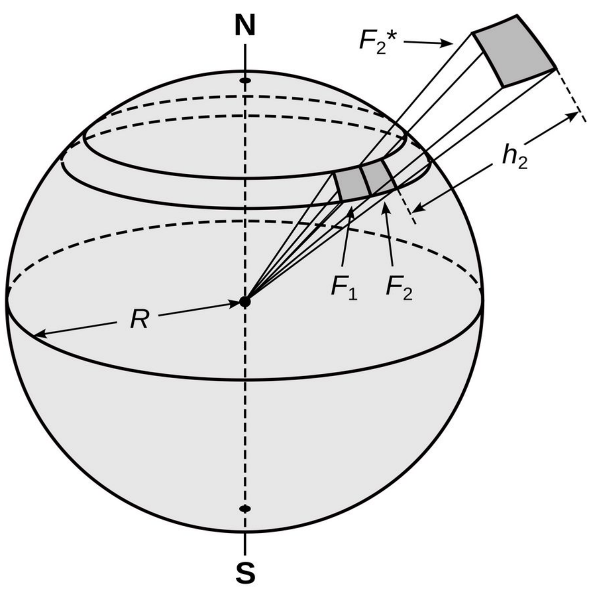


Figure 27: *Schematic drawing of areas and volumes, used for the determination of the average height.*

A simple approach could be to multiply the area belonging to a grid point with its height in order to obtain the volume of the corresponding landmass. However, this approach is not very precise. In Fig. 27, we have two areas F_1 and F_2 of the same size, but with different heights. In order to make the effect clear, the height of area F_1 is $h_1 = 0$ (sea level) and that of area F_2 is equal to the Earth's radius, meaning $h_2 = R$ (R at this location). Because of the extreme height, the area of the "real" Earth's surface at this height is $F_2^* = 4\,F_2$.

If we insert the areas F_1 and F_2 into Eq. (76) as follows:

$$\bar{h} = \frac{h_1 F_1 + h_2 F_2}{F_1 + F_2}\,, \qquad (77)$$

we get an average height of $\bar{h} = 0.5\,h_2$. By using the solid angles instead of the areas in Eq. (76), the result is identical. But, if we apply the "real" areas F_1 and F_2^*, implying that we replace F_2 in Eq. (77) by F_2^*, the result is $\bar{h} = 0.8\,h_2 = 0.8\,R$. The reason for this is that vertical lines on Earth are not parallel, because they point (roughly) toward the Earth's center.

Nevertheless, these two approaches are both approximations. We get a better result if we calculate the volume between the areas F_2 and F_2^*, bordered by the four straight lines. Concerning the whole volume of the Earth, we would ask: When all landmasses and ice masses sink completely into the ocean, how much would the sea level rise globally? With F_s being an area at sea level and h the corresponding (constant) height, the volume, V, of the landmass of this area is not just $F_s \cdot h$. Instead, we find:

$$V = \int_0^h F(h')\,dh' = \int_0^h F_s \cdot \left(\frac{R+h'}{R}\right)^2 dh' = F_s \cdot h \cdot \left[1 + \frac{h}{R} + \frac{1}{3}\left(\frac{h}{R}\right)^2\right] \qquad (78)$$

This not only provides the correct Earth volume, it also yields the exact average $\overline{h}$. It can be shown that for $h << R$, the latter average $\overline{h}$ is very close to the arithmetic mean of the two averages, calculated on the basis of F_2 and F_2*. The three different options for calculating $\overline{h}$ are summarized below. The calculations are based on

a) the areas on the surface of the reference ellipsoid (approximation),
b) the areas on the real Earth's surface (approximation),
c) the volume of the landmass according to Eq. (78) (most precise).

Whereas cases a) and b) would be correct for a flat surface, the product "surface × height" is only an approximation for the curved surface of the Earth. By including volume effects of higher order, option c) yields the best result for the average height and also the correct Earth volume.

The program TOPO calculates the desired quantities and consists of the following six files:

- topo — executable program file
- topo.f95 — Fortran source code (appendix A2)
- [X]data.tsv — topographical data (5 arc-minute grid, Worldbath) [56] (URL 13)
- zlakes.txt — correction data for upper and lower lakes (appendix A2)
- readme.pdf — brief information about the program
- out.txt — output file with the results (which is overwritten every program run)

After unpacking the file topo-program-08-2025.zip, the program can be easily started by typing `./topo` ↵. Since the program not only calculates in relation to the whole Earth, but also for a limited area, the borders, given by the geographical latitude and longitude, have to be defined at program start. The input for the whole Earth in degree is: latitude –90 to 90 and *extended* longitude 0 to 360. (Thus, if the longitude is negative, meaning west, please add 360°. Due to a technical reason, the area must not pass the Greenwich meridian, 0°.) The original program output for the whole planet is:

```
                    EARTH`S VOLUME INCLUDING LANDMASS
                   (Worldbath, 5 arc-minute resolution)

==============================================================================
      Earth`s equator. radius  :    6378136.6 m   (IERS conventions, 2003)
      Earth`s polar radius     :    6356751.9 m   (IERS conventions, 2003)
      Earth`s mean radius      :    6371000.4 m
      Geograph. latitude [deg] :     -90.00  to   90.00    used grid points
      Extended longitude [deg] :       0.00  to  360.00             9335520

      Kind of average              ell.-based    vol.-based   (arithm. mean)
      ----------------------------------------------------------------------
(A)   Average height (sea bed) :  -2386.073 m   -2385.081 m   ( -1893.804 m)
(B)   Average h.    (sea level) :    235.555 m     235.619 m   (   383.251 m)
(B*)  Average h. (upper lakes) :    235.560 m     235.625 m   (   383.260 m)
      ----------------------------------------------------------------------

      Covered area ..... (sea level, integrated) :    5.100655569E+08 km^2
      Ellipsoid surface ........... (analytical) :    5.100655569E+08 km^2
      Surface of sphere ......... (equal volume) :    5.100645336E+08 km^2

      Volume correction ............. (as per A) :   -1.216092375E+09 km^3
(C)   Volume of landmass ............ (as per B) :    1.201857753E+08 km^3
(C*)  Volume of landmass + lakes ... (as per B*) :    1.201884022E+08 km^3
      Volume of upper lakes ........... (C* - C) :    2.6269E+03      km^3

(D)   Earth`s volume .... (ellipsoid, sea level) :    1.083207113E+12 km^3
(E)   Earth`s volume + landmass ........ (D + C) :    1.083327299E+12 km^3
(F)   Earth`s vol. + landm. + lakes ... (D + C*) :    1.083327302E+12 km^3
==============================================================================
```

In the output, only the options a) `ell.-based` and c) `vol.-based` are provided. Additionally, the covered area is calculated and, in the case of the whole Earth, the integrated area can be compared with the analytically calculated surface of the reference ellipsoid. Again, R_e and R_p are the equatorial and polar radii of the Earth. Then, the equation for the surface F_E of this (oblate) ellipsoid is

$$F_E \;=\; 2\pi R_e^2 \left[1 + \left(\frac{R_p}{R_e} \right)^2 \frac{\operatorname{artanh} \epsilon}{\epsilon} \right] \;=\; 2\pi \left[R_e^2 + \frac{R_p^2}{2\epsilon} \ln \left(\frac{1+\epsilon}{1-\epsilon} \right) \right] \qquad (79)$$

with $$\operatorname{artanh} \epsilon \;=\; \frac{1}{2} \ln \left(\frac{1+\epsilon}{1-\epsilon} \right) \quad \text{and} \quad \epsilon \;=\; \sqrt{1 - \left(\frac{R_p}{R_e} \right)^2}$$

Equation (1)

In [5], it was shown that the base lines of the Cheops Pyramid are probably intentionally of slightly different lengths, based on different mathematical constants! The average is a virtual length, but the base line of the south side really exists. Therefore, it seems reasonable that the south side, attributed to the Sun (!), is used in Eq. (1). The improvement of the accuracy of Eq. (1) is briefly listed below. The right column shows the successively decreasing deviation between the right and left side of Eq. (1). Note: The volume of the Sun is $V_{Sun} = (4\pi/3)\, R^3_{Sun} \cdot (1-f)$, with f being the oblateness (flattening).

physical quantities		**deviation**
Original accuracy, based on the given physical values on page 98:	→	0.0297 %
The theoretical average base length of the Cheops Pyramid of 230.364 m is replaced by the length of the southern base line of 230.454 m [53, 54].	→	0.0094 %
The real Earth volume including ice and landmass, $1.0833273 \cdot 10^{12}$ km^3, and the oblateness of the Sun of 0.0008 % [57, 58] again improve the result.	→	0.0011 %
The Earth's rotation is slowed down by the tide friction and, therefore, 5000 years ago (AMS dating [11, 12]), the second was shorter by ca. 0.0001 % [5].	→	0.0010 %
Due to a least squares adjustment concerning the asymmetric shape of the Cheops Pyramid, Model B [5], the length of the south side is 230.4553 m.	→	0.0004 %

Overall, this means that the original accuracy of 0.03 % is improved to a value less than or equal to <u>0.001 %</u>, which is an improvement by a factor 30. The uncertainty of the solar equatorial radius R_{Sun} = (695,508 ± 26) km [29] corresponds to an error of 0.0037 %. In the formula for the solar volume needed in Eq. (1), the radius is included as R^3_{Sun}. Thus, the relative uncertainty due to the radius is multiplied by 3, which yields approximately 0.01 %. This means that the uncertainty introduced by the solar radius alone, is roughly ten times larger than the deviation in Eq. (1), implying that Eq. (1) does not contain any significant error! <u>The equation fits perfectly!</u> Meanwhile, topographic data with higher spatial resolution are available, including: ETOPO: 1 arc-minute grid up to 15 arc-second grid [59] (URL 14). However, the result would probably be more or less the same.

Equations (2) and (3)

Concerning Eq. (2) on page 12, the volume of Venus is not precisely known, because corresponding topological data of the Venus surface has not been available. Eq. (3) has the problem that the base lines of the Mykerinos Pyramid were not finally finished and are thus not well defined. The surfaces of the granite stones at the pyramid base are in a rough and unfinished state.

4.10.7 Determination of the solar radius

The radius of the Sun has been measured several times with different methods during the last centuries and more often during the last decades. When comparing the methods, it turns out that the different techniques yield systematic differences concerning the solar radius. Although these differences are relatively small, they influence the accuracy of Eq. (1) and we therefore briefly present these measurement techniques.

1. The measurement of the photospheric radius with a fixed telescope and by using the Earth's rotation is a very direct method. For example, a telescope with an appropriate Sun filter and cross lines is directed to the south, where the Sun has its highest position. The time is then measured from the moment the solar limb contacts the cross line until the other side of the Sun leaves the cross line. This time span allows for the calculation of the solar radius.
2. When observing a Mercury transit, the time also is measured from the beginning until the end of the transit in order to obtain the solar radius. In principle, a Venus transit is also useful. However, Venus transits are rare and the next Venus transit will take place in the year AD 2117, whereas the next Mercury transit will occur in AD 2032.
3. The third method determines the so-called helioseismic radius. For example, a drop of water in the air can vibrate slightly and in a similar way the Sun performs slight spatial vibrations. A special kind of vibration (the so-called f modes) yields the solar radius.
4. The last method is a direct measurement of the angle that we obtain from one side of the solar limb to the other side. Together with the exact distance of the Sun from the observer on Earth, this yields the radius of the Sun.

If we observe the Sun with a telescope equipped with a solar filter, the solar limb looks relatively sharp. Nevertheless, by using more sophisticated instruments with a higher optical resolution the solar limb shows a considerable diffuseness. Furthermore, the diffuseness varies slightly within the solar cycle of about 11 years and also the distance to Earth changes during a year. Therefore, the described methods are more complicated (which is not important at this point).

Altogether, the results for the solar radius, measured with the different methods, show a variation of up to ± 500 km, corresponding to ca. ± 0.07 %. But be aware of the fact that the publications were based on experimental work of very different extent. Thus, it seems that the most recent works are not generally the most precise ones.

When quoting stellar radii in astronomy, the solar radius is sometimes used as a length unit (as also in Teotihuacán, section 5.2.1) and so it would be reasonable for the scientific community to agree upon one value. Therefore, in 2015 the IAU (International Astronomical Union) defined the nominal solar radius (as a length unit) to be exactly 695,700 km, taking into account all the measurements done so far. It was further stated that this value would be kept constant as a length unit, although future measurements could yield a better result for the solar radius. In the case of new measurements, the expected difference is estimated to be within approximately ± 200 km.

If we use the value from the IAU in Eq. (1), the uncertainty increases from 0.001 %, based on [29], to ca. 0.08 %, which is considerably larger but still very good. So, in principle, the astronomical measurements can be discussed, but the effect on Eq. (1) is not very serious. Furthermore, we could ask the question whether the master builders relied on the photospheric radius, the helioseismic radius, or on a different standard. We guess that they used the photospheric radius, because that is what we <u>see</u> from Earth. Therefore, the measurement in [29], being made very accurately and continuously over six years using method 1, seems to be a good choice. The result is also less than 200 km from the IAU value. (If we use the final values on page 101 and calculate the solar radius on the basis of Eq. (1), we obtain 695,508.9 (± 1.0) km – in perfect agreement with [29].)

5. The pyramids of Teotihuacán

From our point of view, the second most famous pyramid area on Earth after Giza is Teotihuacán (see Fig. 28). In 2005, I gave a seminar talk about our work in materials science at a conference in Cancún, Mexico (XIV. International Materials Research Congress, IMRC). While there, I used the opportunity to visit some of the archaeological sites in Mexico. In Teotihuacán, an interesting pattern was found, which can be explained by astronomical aspects. This finally led to a new planetary correlation described in this chapter.

5.1 Planetary correlation and data

A satellite image of Teotihuacán and a drawing with the main constructional features are given in Fig. 28. A very strange phenomenon is the six massive barriers on the Avenue of the Dead, indicated in red (see also Fig. 9). People have to climb over them in order to walk along the avenue. From a top view, the broad avenue looks like a scale with the barriers being markers on the scale.

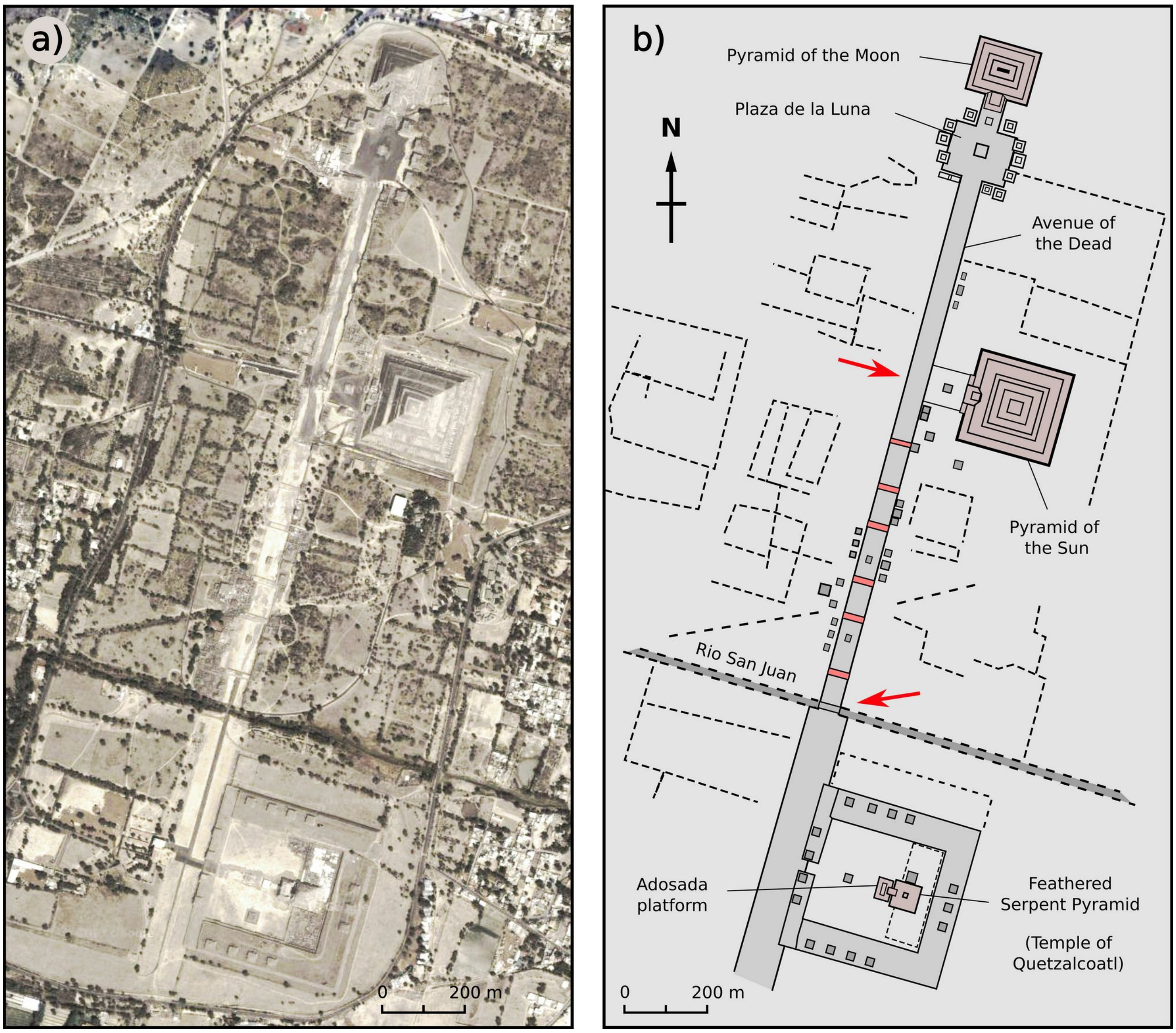


__Figure 28:__ Archaeological site of Teotihuacán. __a)__ Satellite image, Google Maps, © 2014 Cnes/Spot Image, DigitalGlobe. __b)__ Schematic drawing with the main archaeological buildings and terrain edges. Eight significant positions are marked on the Avenue of the Dead. These are provided by the six strange barriers (red), the Pyramid of the Sun, and the Rio San Juan (red arrows). For a better visibility of the barriers in a), the reader can magnify this picture on the computer monitor.

The American civil engineer Hugh Harleston Jr. (1925–2013) propounded that the pyramid area of Teotihuacán represents Earth and our solar system. Although we will not use his calculations and interpretation concerning the planets, we take his main idea. Tentatively, we correlate the planetary positions with the markers on the main avenue. According to Kepler's first law, a planet moves on an elliptical orbit, with the Sun being positioned in one of the two foci of the ellipse. The main features of such an orbit are shown in Fig. 29. With the semi-major axis *a* and the eccentricity *e*, the largest and the shortest distance of the planet to the Sun can be calculated. The shortest distance *q* (planet at perihelion) and the largest distance *Q* (planet at aphelion) are determined by

$$q = a \cdot (1 - e) \quad \text{and} \quad Q = a \cdot (1 + e) \qquad (80a, b)$$

According to Fig. 29, we have $a^2 = b^2 + (a \cdot e)^2$ implying

$$a = \frac{b}{\sqrt{1 - e^2}} \quad \text{and} \quad b = a \cdot \sqrt{1 - e^2} \qquad (81a, b)$$

where *b* is the semi-minor axis. Thus, in astronomical tables, only *a* and *e* are listed because the other data can be easily calculated by means of *a* and *e*.

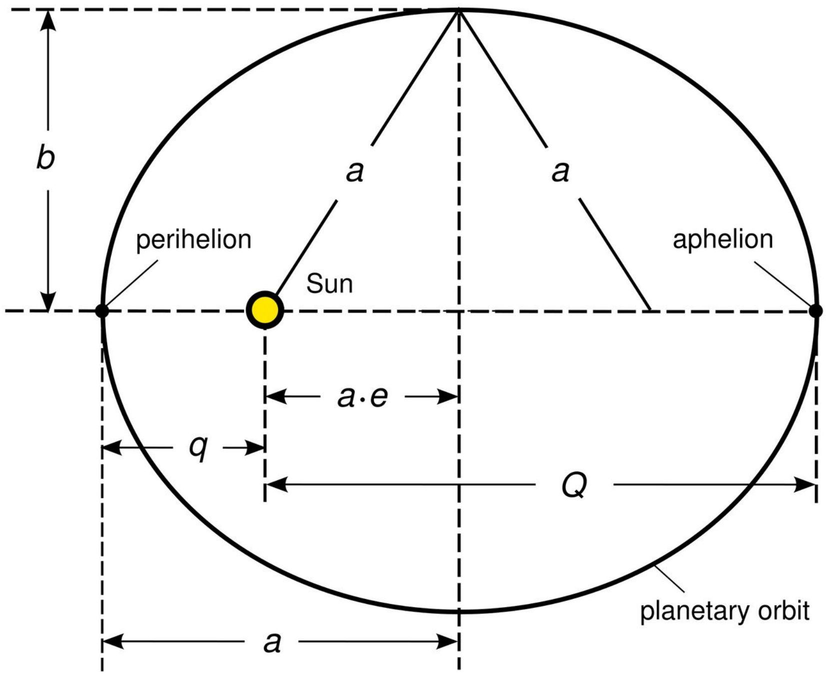


Figure 29: *Shape and main features of the elliptic orbit of a celestial body moving around the Sun.*

In addition to the six barriers, two additional positions are given by the Pyramid of the Sun and the Rio San Juan (see red arrows in Fig. 28 b). If the distances between these eight positions and the planetary distances are presented in one diagram, a linear correlation cannot be found. The reason is that the astronomical distances of the planets increase exponentially if we go from the inner to the outer planets. In order to display such an exponential behavior in a diagram, it is common practice in science to use the logarithms instead of the numbers themselves. Remark: The exponential increase of the planetary distances is approximated by the Titius-Bode law.

The next three sections describe the assignment of the planets, the methods for determining exact positions at the pyramid site, and the geographical and astronomical data used.

5.1.1 Assignment of the planets

The solution that works almost perfectly using logarithms is given in Fig. 30. Pivoting the position of the Pyramid of the Sun along the given circular arc by 90° yields another position attributed to Earth and one of the six barriers is assigned to the asteroid belt.

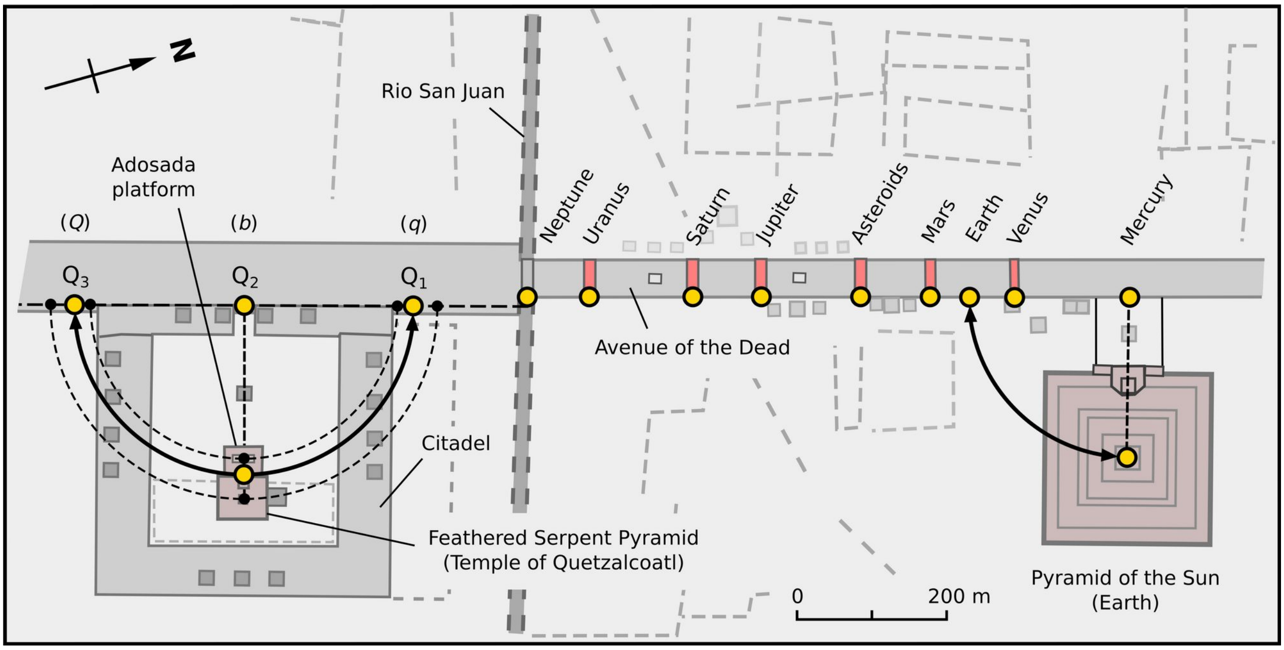


Figure 30: *Mapping of the main positions on the Avenue of the Dead with the planets of the Solar system. The area of the Feathered Serpent Pyramid on the left will be discussed in sections 5.2.2 and 5.2.3.*

5.1.2 Different measurement methods

In order to perform the calculations of the planetary correlation in Teotihuacán, we need the *relative* distances along the Avenue of the Dead. In other words, we need the relations (quotients) between different distances. This means that these distances can be determined using any unit of length. They can be measured with a ruler on a map, they can be measured with a measuring tape at the pyramid site, or they can be calculated from the GPS coordinates. Actually, there are four methods, as follows:

1. Measuring distances with a ruler on a map or on the computer monitor, e.g., in Google Maps, HERE WeGo, or other satellite-based maps. After magnification, even Fig. 28 a) can be used.

2. Determining the GPS-coordinates in Google Maps, HERE WeGo, etc., by pinpointing the positions with the mouse cursor and calculating the real distances from the geographical coordinates according to the method described in section 4.6.4 (applied in reverse).

3. Using a measuring tape at the pyramid site in Teotihuacán. This also requires angular measurements if the positions of the two great pyramids are measured from the top. These pyramid positions can also be measured by the average values of the four corner positions. Alternatively, a laser range finder can be used to determine distances.

4. Determining the geographical coordinates directly in Teotihuacán with GPS and subsequent calculation of the distances.

The first two methods can be applied easily by everyone. The third and fourth methods require a visit to Teotihuacán. Although the third method (measuring tape) yields the most precise results, the other three methods are completely adequate. So, it does not matter, whether the distances are measured in mm (on a map), in, m, km, mi, or another unit of length. Therefore, it is easy for the reader to check all of the following numbers and results of the planetary correlation.

When determining the pyramid positions on satellite images, the optical distortion of the pictures has to be taken into account (see Fig. 31). When a satellite takes a photograph, only the part vertically below the satellite is without distortion. Other parts of the image have a slight "side view." The distortion at ground level is almost zero, but the top parts of high buildings are shifted more or less to the side. The available geographical coordinates are valid on the ground level. Thus, in order to obtain the correct pyramid position, the top of the pyramid is not a good choice. Instead, one should take the four corners of the pyramid at ground level and draw the diagonals between the corners. The intersection of the diagonals is then the correct position.

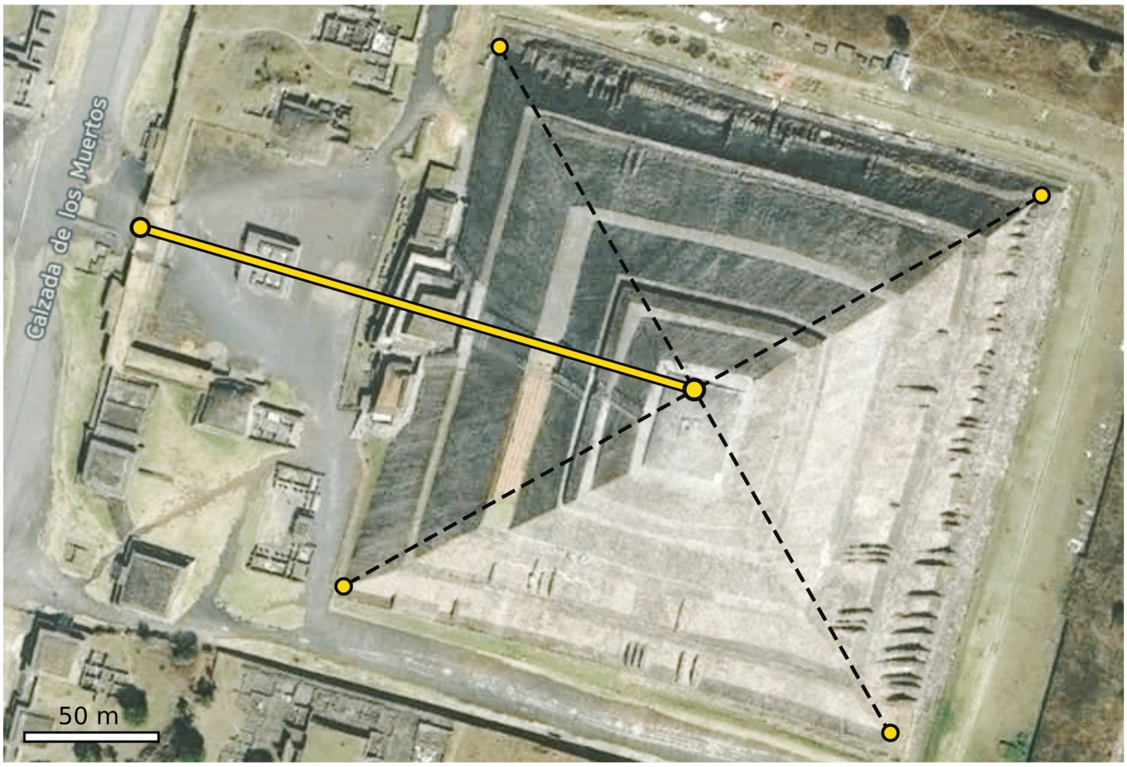


Figure 31: *Pyramid of the Sun. The positions of the pyramids are mostly not their tops because of the photographic, perspective distortion. Because the GPS coordinates are valid for the ground level, the central pyramid position is given either by the intersections of the diagonals of the pyramid base or by the arithmetic mean of the coordinates at the four corners. The horizontal distance, given by the yellow line, is about 214.8 m according to the GPS data in Table 8. The altitude of this area is ca. 2304 m above sea level (MoonCalc). Satellite image: © 2017 HERE, 2014 DigitalGlobe, INEGI.*

5.1.3 Geographical and astronomical data

The main positions, according to the pyramids, the temple, and the barriers on the Avenue of the Dead, are numbered from 1 to 18. The GPS coordinates were determined with the second method by using Google Maps and are listed in Table 8. As the reader can see in Fig. 30, the positions along the Avenue of the Dead are not in the middle of the road or on its west side, but on its east side. The reason is that the position of Earth would be correct only at the east side. Note that the last two columns in Table 8 are only examples that represent the distances. These distances can also be measured in any other unit of length.

Table 8: *Geographical positions and distances according to Fig. 30. For the position of the Pyramid of the Moon, see Fig. 34. The left side of the table contains the locations in Teotihuacán and the corresponding celestial bodies. The geographical coordinates (GPS) were obtained by pinpointing the positions in Google Maps. The distances in m, measured from the Pyramid of the Moon, are calculated from the GPS data and the alternative numbers in the last column are measured accurately with a ruler on a large computer monitor (Google Maps). The numbers with the stars (*) are positions beside the avenue, the numbers with a cross (†) are the sum or difference of two distances. The positions Q1a–Q3a at the Feathered Serpent Pyramid are represented in Fig. 36.*

No.	**Locations** (Fig. 30 [26, 27])	**cel. body**	**geogr. lat.**	**geogr. long.**	**dist. [m]**	**d [mm]**
1	Pyramid of the Moon	Sun	19.699662	−98.843713	0.00	0.0
2	Plaza de la Luna	“Sun unit”	19.697947	−98.844212	197.00	51.9
3	“Avenue pos.”, Sun Pyr.	Mercury	19.692982	−98.845651	767.16	200.0
4	barrier 1	Venus	19.691620	−98.846028	923.08	240.0
5	Pyramid of the Sun	Earth	19.692415 *	−98.843693 *	981.92 †	254.5 †
6	barrier 2	Mars	19.690632	−98.846302	1036.20	270.2
7	barrier 3	Asteroids	19.689801	−98.846546	1131.72	295.4
8	barrier 4	Jupiter	19.688594	−98.846890	1270.16	331.0
9	barrier 5	Saturn	19.687797	−98.847053	1359.83	355.5
10	barrier 6	Uranus	19.686594	−98.847465	1499.71	391.4
11	Rio San Juan	Neptune	19.685788	−98.847712	1592.64	415.5
12	Q1a (Feath. Serpent Pyr.)	Sedna (*q*)	19.681881 *	−98.846180 *	1712.25 †	446.7 †
13	Q1	“ (*q*)	19.681952 *	−98.846438 *	1740.44 †	453.8 †
14	Q1b (Adosada platform)	“ (*q*)	19.682001 *	−98.846622 *	1760.48 †	458.4 †
15	Q2	“ (*b*)	19.682515	−98.848481	1963.62	511.5
16	Q3b (Adosada platform)	“ (*Q*)	—	—	2166.75 †	564.6 †
17	Q3	“ (*Q*)	—	—	2186.80 †	569.2 †
18	Q3a (Feath. Serpent Pyr.)	“ (*Q*)	—	—	2214.98 †	576.3 †

Table 9: *Semi-major axes a and eccentricities e of the planetary orbits of different sources and points of time. The first two columns of a and e are taken from [28]. The four columns on the right are calculated on the basis of analytical formulas derived by J. Meeus [18] with VSOP82 [1]. The calculated data are rounded to seven digits. The semi-major axes a are almost constant over time, whereas e changes significantly. The equatorial (photospheric) solar radius is taken from [29].*

celestial body	***a* [km]** [28]	***e*** [28]	***a* [km]** AD 2000	***e*** AD 2000	***a* [km]** AD 200	***e*** AD 200
Sun (radius)	695508	—	—	—	—	—
Mercury	57900000	0.2056	57909080	0.2056318	57909080	0.2052562
Venus	108200000	0.0068	108208600	0.0067719	108208600	0.0076607
Earth	149600000	0.0167	149598000	0.0167086	149598000	0.0174250
Mars	227900000	0.0933	227939200	0.0934006	227939200	0.0917479
Jupiter	778300000	0.048	778298400	0.0484949	778297800	0.0454151
Saturn	1427000000	0.056	1429394000	0.0555086	1429400000	0.0615223
Uranus	2869600000	0.046	2875039000	0.0462959	2875039000	0.0468121
Neptune	4496600000	0.010	4504450000	0.0089881	4504450000	0.0088728

Table 10: *Astrophysical data (osculating orbital elements, retrieved March 2024) of the known trans-Neptunian objects (TNOs) with diameters ≥ 800 km. The semi-major axes a are given in astronomical units (1 AU = 149,597,870.700 km [60]). U is the orbital period and D the diameter of the celestial body. The orbital characteristics a, e, and U of Pluto are taken from [61], and those of the other objects from [62]. The uncertainties are 1-sigma. The different sources of the diameters are provided in the last column. If the shape deviates significantly from a sphere, only the mean diameter is given (volume equivalent).*

TNO	***a*** **[AU]**	***e***	***U*** **[years]**	***D*** **[km]**	**Ref.**
Sedna	541.63 ± 0.16	0.85900 ± 0.00004	12605.6 ± 5.7	995 ± 80	[63]
Eris	68.1532 ± 0.0003	0.432320 ± 0.000004	562.649 ± 0.004	2326 ± 12	[64]
Gonggong	67.194 ± 0.003	0.49853 ± 0.00003	550.81 ± 0.03	1230 ± 50	[65]
Makemake	45.28556 ± 0.00005	0.165770 ± 0.000001	304.7528 ± 0.0005	1430 (mean)	[66]
Quaoar	43.31263 ± 0.00003	0.039511 ± 0.000002	285.0559 ± 0.0003	1110 (mean)	[67]
Haumea	42.89167 ± 0.00004	0.199928 ± 0.000001	280.9103 ± 0.0004	1595 (mean)	[68,69]
Salacia	42.33422 ± 0.00004	0.100628 ± 0.000002	275.4518 ± 0.0004	846 ± 21	[70,71]
2002 MS_4	41.77603 ± 0.00003	0.146773 ± 0.000002	270.0219 ± 0.0003	800 ± 24	[72]
Pluto	39.481687	0.248808	247.945	2376.6 ± 3.2	[73]
Orcus	39.14621 ± 0.00002	0.2276546 ± 0.0000005	244.9305 ± 0.0002	917 ± 25	[74,75]

Three alternative values of the semi-major axis *a* and the eccentricity *e* are listed in Table 9 for the eight planets of the solar system. The first alternative is taken from [28] and the second and third are calculated with the P5 program, based on the algorithm created by Jean Meeus (VSOP82). The time-dependent data can be calculated with P5 for any date between 30,000 BC and AD 30,000. However, for dates in the distant past or future, the increasing uncertainty of the results must be taken into account. For Pluto (J2000) and the other trans-Neptunian objects (TNOs), the quantities *a*, *e*, and the orbital period *U* in Table 10 are taken from [61, 62]. In addition, the diameters of the TNOs are provided in the last column, taken from [63–75].

5.2 Comparison of pyramid area and solar system

The analysis of the planetary correlation is provided in the next four sections. The first section covers the eight planets and the Sun and in the second section we deal with the trans-Neptunian objects that lead us to the TNO Sedna. Finally, the third and fourth sections provide an interpretation of the Citadel and the possibilities of a scientific investigation of Sedna.

5.2.1 Quantitative analysis of the correlation

In order to mathematically check the planetary correlation, the geographical and astronomical data are presented in Fig. 32. The coefficient of determination R^2, a measure of the significance of the correlation, is the square of the correlation coefficient R, given (for example) by:

$$R = \frac{n\sum d_i p_i - \sum d_i \cdot \sum p_i}{\sqrt{n\sum d_i^2 - \left(\sum d_i\right)^2} \cdot \sqrt{n\sum p_i^2 - \left(\sum p_i\right)^2}}, \tag{82}$$

where n is the number of data points, d_i are the positions on the avenue, and p_i the logarithms of the perihelion distances and of the solar radius, respectively. The summation index i runs from 1 to n. (For convenience, these bounds of summation are omitted.) The adjusted coefficient of determination, $\overline{R}^2$, taking into account the number s of free model parameters, is given by [76]

$$\bar{R}^2 = 1 - (1 - R^2) \cdot \frac{n-1}{n-s} \tag{83}$$

Linear regression means $s = 2$. In our case, R^2 and $\bar{R}^2$ are almost identical. Therefore, we use R^2 (Eq. (82) squared). After performing the linear fit, the parameters u and v of the linear fit function, $f(x) = ux + v$, are listed in the program output. They are calculated using the following formulas:

$$u = \frac{n \sum d_i p_i - \sum d_i \cdot \sum p_i}{n \sum d_i^2 - \left(\sum d_i\right)^2} \tag{84}$$

$$v = \frac{\sum d_i^2 \cdot \sum p_i - \sum d_i \cdot \sum d_i p_i}{n \sum d_i^2 - \left(\sum d_i\right)^2} \tag{85}$$

In the corresponding equation, presented in Fig. 32, the parameters u and v are given directly as decimal numbers. Since q indicates the perihelion distance, we have $p_i = \log(q_i/\text{km})$, if q_i is measured in km. The logarithmized perihelion distances of the eight planets as a function of the distances on the Avenue of the Dead exhibit an almost perfect correlation with $R^2 = 0.99962$. If the Pyramid of the Moon is taken as a marker on the scale and is attributed to the logarithm of the solar radius $R_{Sun} = 695{,}508$ km [29], the corresponding point fits perfectly to the planetary correlation in the diagram and the coefficient of determination increases to 0.99980 (Fig. 32).

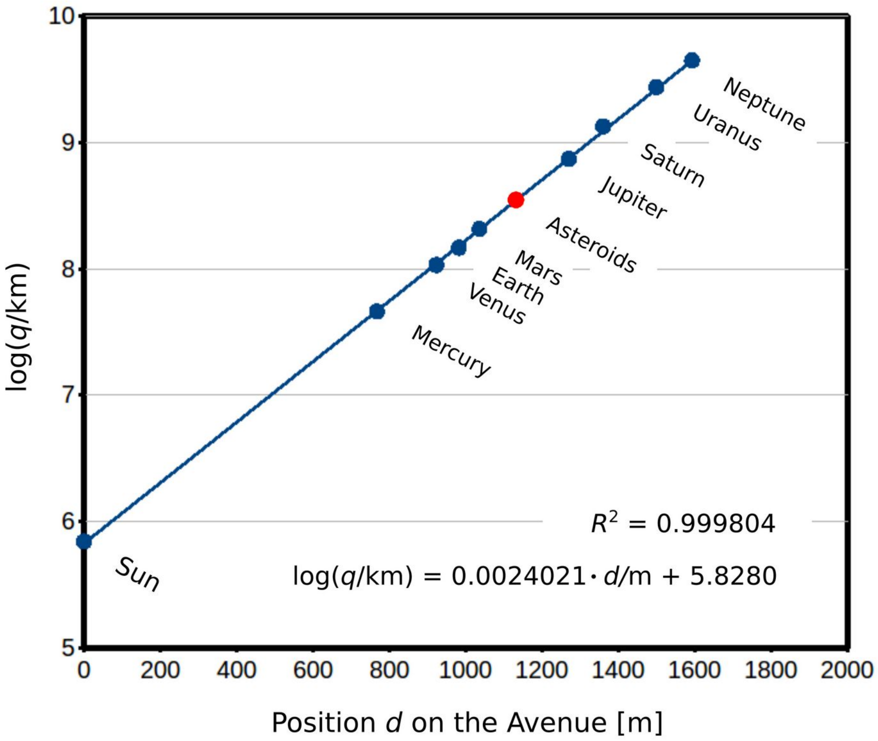


Figure 32: *Logarithmized perihelion distances as a function of the positions on the Avenue of the Dead (GPS data). The position of the Sun is given by the logarithm of the solar radius. Astronomical distances are provided in km. The geographical distances are measured from the position of the Pyramid of the Moon and given in m. The red point indicates a position within the asteroid belt.*

Now, the parameters $u = 0.0024021$ and $v = 5.8280$ in the equation of the linear fit function (Fig. 32) seem arbitrary, which has to do with the choice of the man-made units of length (m and km). After analyzing this equation in detail, a very interesting modification was found by applying the following “natural” units of length: As an astronomical length unit, we use the solar radius, which is measured from the solar center, as are the perihelion distances. In Teotihuacán, the unit of length is defined by the distance between the center of the Pyramid of the Moon and the central platform of the Plaza de la Luna. This distance is named the “Sun unit,” with a length of about u_{Sun} = 197 m and is depicted in Figs. 10 and 34. If the logarithmic base 3 is used instead of 10, the correlation is defined almost perfectly by the coefficients u = 1 and v = 0. (Note: The coefficient of determination, R^2, is used as a simple criterion to evaluate the correlation. If another mathematical approach is considered more appropriate, it would probably yield a similar result.)

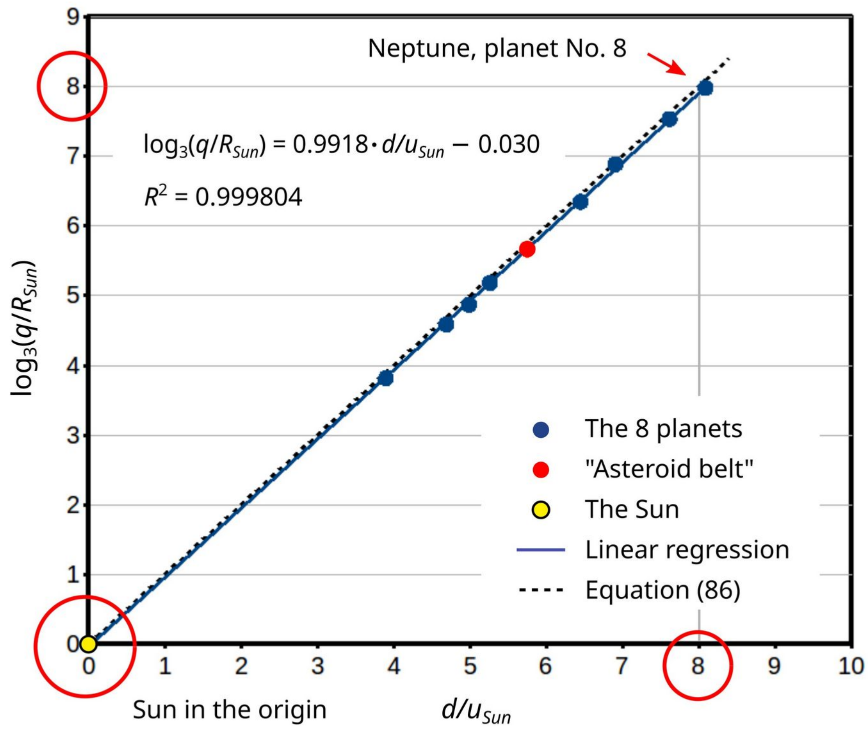


Figure 33: *The same correlation as in Fig. 32 with “normalized” scales, logarithmic base 3, and identical R^2 (GPS data).*

It follows that the resulting equation, Eq. (86), has a very basic form with some extraordinary properties. It does not contain any additional arbitrary number and the data point of the Sun is placed in the origin, as the Sun represents the center of the solar system. Furthermore, on both axes of the diagram, the eighth planet Neptune is related to the number 8 (see Fig. 33). Detailed information is provided in [26, 27]. We obtain

$$\log_3\left(\frac{q_i}{R_{Sun}}\right) = \frac{d_i}{u_{Sun}} \tag{86}$$

with i = 0, ..., 9. The index i = 0 is related to the Sun (solar radius), 1 means Mercury, 2 Venus, 3 Earth, 4 Mars, 5 the belt of asteroids, 6 Jupiter, 7 Saturn, 8 Uranus, and 9 Neptune. This equation means that the logarithm of the perihelion distances is identical to the distances in Teotihuacán if they are measured in the appropriate units of length. In addition, the Sun also fits perfectly into this picture. Note that the coefficient of determination does not depend on the length units used, on the logarithmic base, or on the zero position on the scale. The next figure, an extension of Fig. 10, shows a comparison of the scale in Teotihuacán and the corresponding astronomical scale. The positions on the blue scale are accurately calculated using the perihelion distances of the planets.

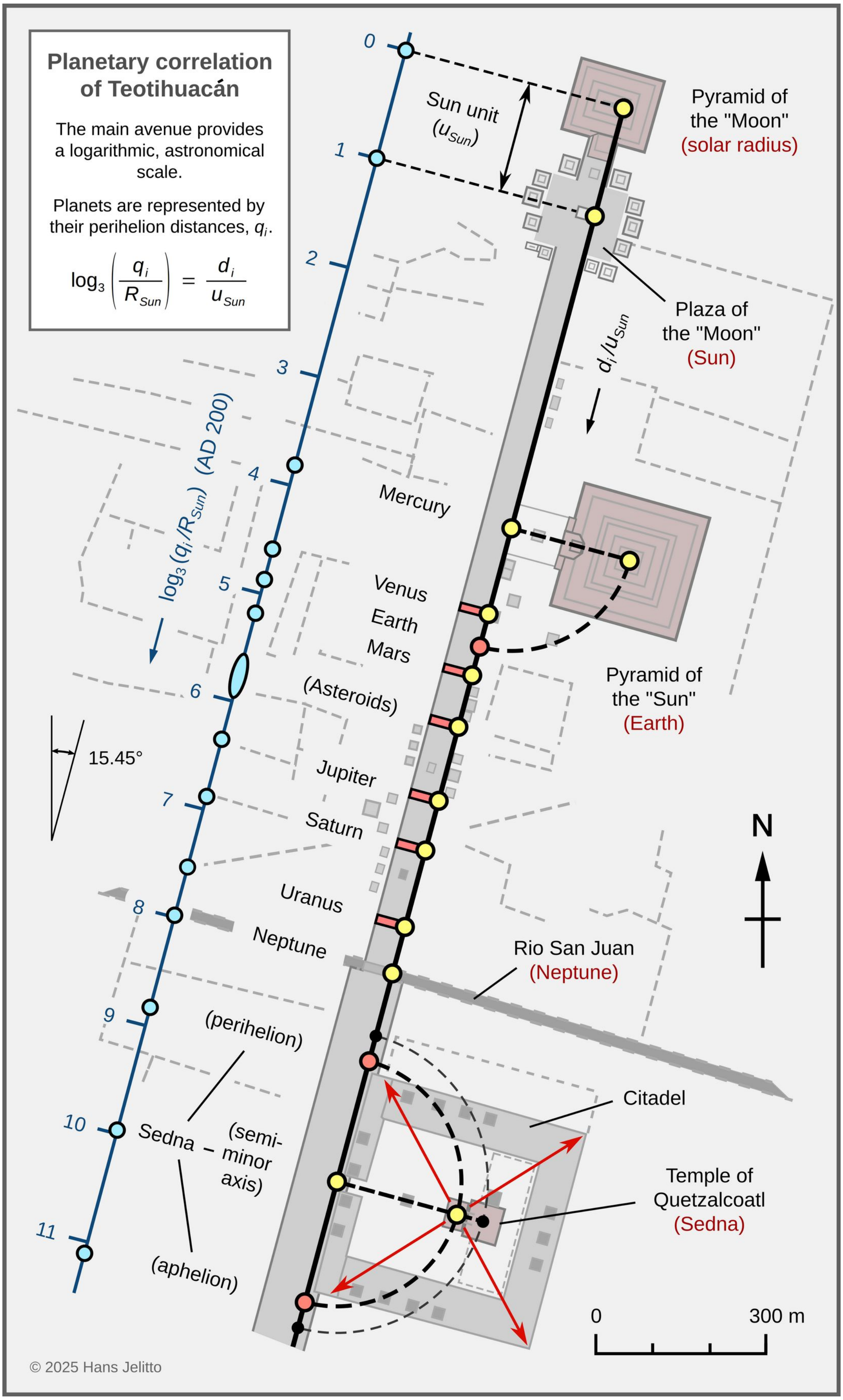


Figure 34: *True-to-scale overview of the pyramid area of Teotihuacán with the corresponding logarithmic astronomical scale. The range of the asteroid belt of 2.10 to 3.27 AU is taken from the NASA JPL website (URL 15). Note that the numbers on the blue scale apply to the astronomical and local distances (Tab. 9, AD 200, and Tab. 8). The drawing of the archaeological site is based on a satellite image (Google Maps); see Fig. 28 a). For the mathematical inclusion of Sedna, see [26].*

If this correlation was intended by the master builders, it follows that the Pyramid of the Sun does not represent the Sun but instead the planet Earth, and the Pyramid of the Moon represents the Sun. If comparing the astronomical and the “archaeological” axes accurately, small differences concerning the positions can be seen. The reason for this might be that the astronomical data are calculated for the year AD 200 (Table 9), which is roughly the date of construction of this area according to the current archaeological state of knowledge. Therefore, the next question is: If we leave the year AD 200 and proceed further into the past or future, how does the coefficient of determination change? For this, a time scan was computed from the year 18,000 BC until AD 4000 and the result is shown in Fig. 35 – see GPS data in section 3.4.10.

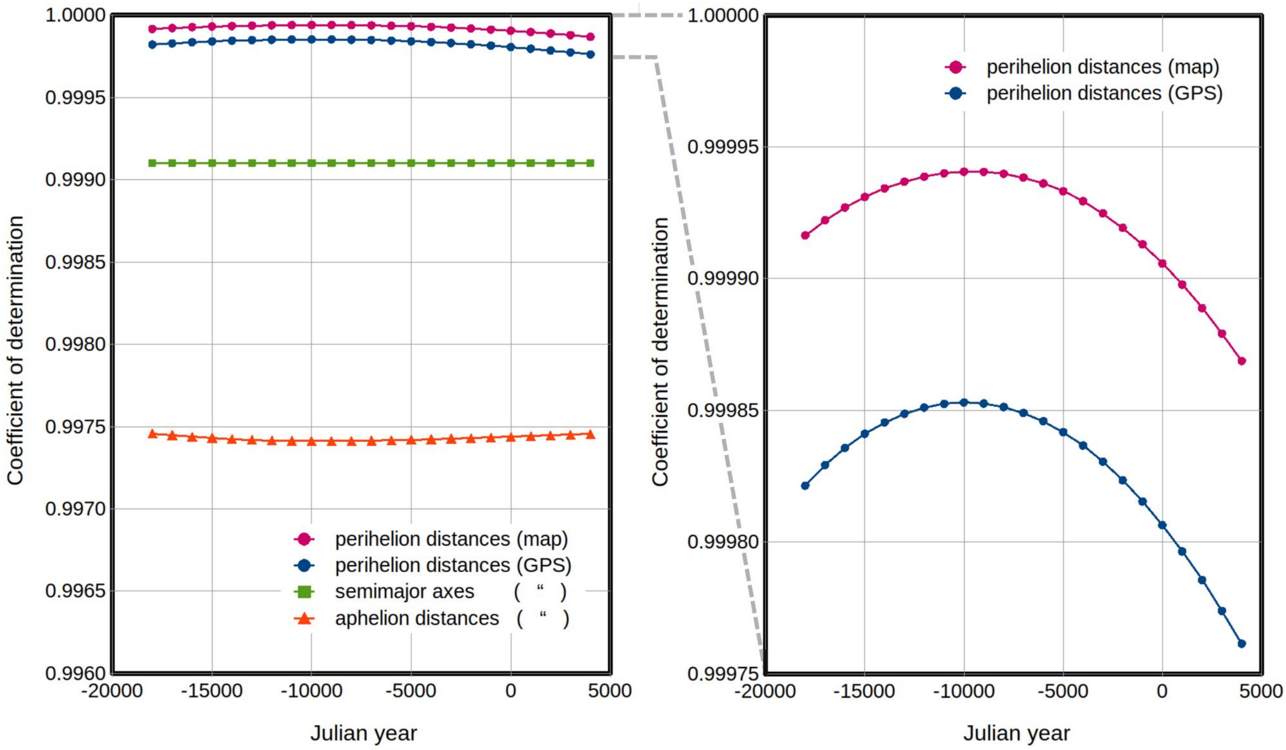


Figure 35: *Coefficient of determination, R^2, of the planetary correlation as a function of time from the year 18,000 BC until AD 4000, including the solar radius in each case. The astronomical distances are calculated on the basis of polynomials of third degree [18, pp. 200 ff.], created by Jean Meeus and based on VSOP82 [1]. The right diagram is a stretched presentation of the upper two curves of the left diagram. The maxima of R^2 are 0.99985 in 9930 BC (GPS data) and 0.99994 in 9570 BC (satellite map).*

The point in time of the maximum R^2 has an uncertainty of about a few hundred years because the distance of the points in Fig. 35 are 1000 years each and, thus, the maximum is relatively flat. Another question concerns the accuracy of the calculations in the remote past. For the years between 2000 BC and AD 6000, the VSOP theory has a very high precision of around 1 arc second (depending on the planet). Nevertheless, the equations from Meeus can be used for times much further in the past. For 2000 BC, this means 4000 years in the past. With some reservations, we extend this time interval by a factor of five, arriving at 18,000 BC. For polynomials of the third degree, a rough extrapolation of the error to 18,000 BC is given by the factor $5^3 = 125$, implying that an error of 1 arc second 4000 years ago can increase to approximately 2 arc minutes 20,000 years ago. Even if the error was greater than this, the accuracy is most probably sufficient for our purpose. Thus, R^2 is plotted in Fig. 35 for the time span 18,000 BC to AD 4000.

We find the maximum of R^2 in roughly the year 9800 BC. However, the pyramids do not seem to be that old. This is possibly a clue from the pyramid builders that points to a dramatic event in the remote past. This could be, e.g., the Noachian flood, the destruction of a planet on the orbit of today’s asteroid belt, or the previous encounter with Sedna at its closest approach (section 5.2.4).

Finally, by going back to Fig. 4 on page 4, it can be seen that the platform on top of the Pyramid of the Moon has roughly the same shape, size, and the same orientation as the barriers on the Avenue of the Dead. This means that the Pyramid of the Moon is just a bold marker on the scale representing the Sun (logarithm of the solar radius). Hence, this pyramid stands for the Sun and not for the Moon.

5.2.2 The temple of Quetzalcoatl

The next question is whether the temple of Quetzalcoatl (the Feathered Serpent Pyramid) can be integrated in this correlation. The logarithmic astronomical scale can be perfectly extended to the south, beyond the Rio San Juan. In the astronomical picture this would mean that we enter the wide trans-Neptunian space. Correspondingly, the Avenue of the Dead becomes immediately broader after passing the Rio San Juan (Neptune).

Before we continue, a special property of the astronomical distances is shown if they are plotted on a logarithmic scale. By replacing *a* in Eqs. (80a) and (80b) using Eq. (81a) and by multiplying *q* and *Q*, we obtain $q \cdot Q = b^2$. If we logarithmize this equation, we find

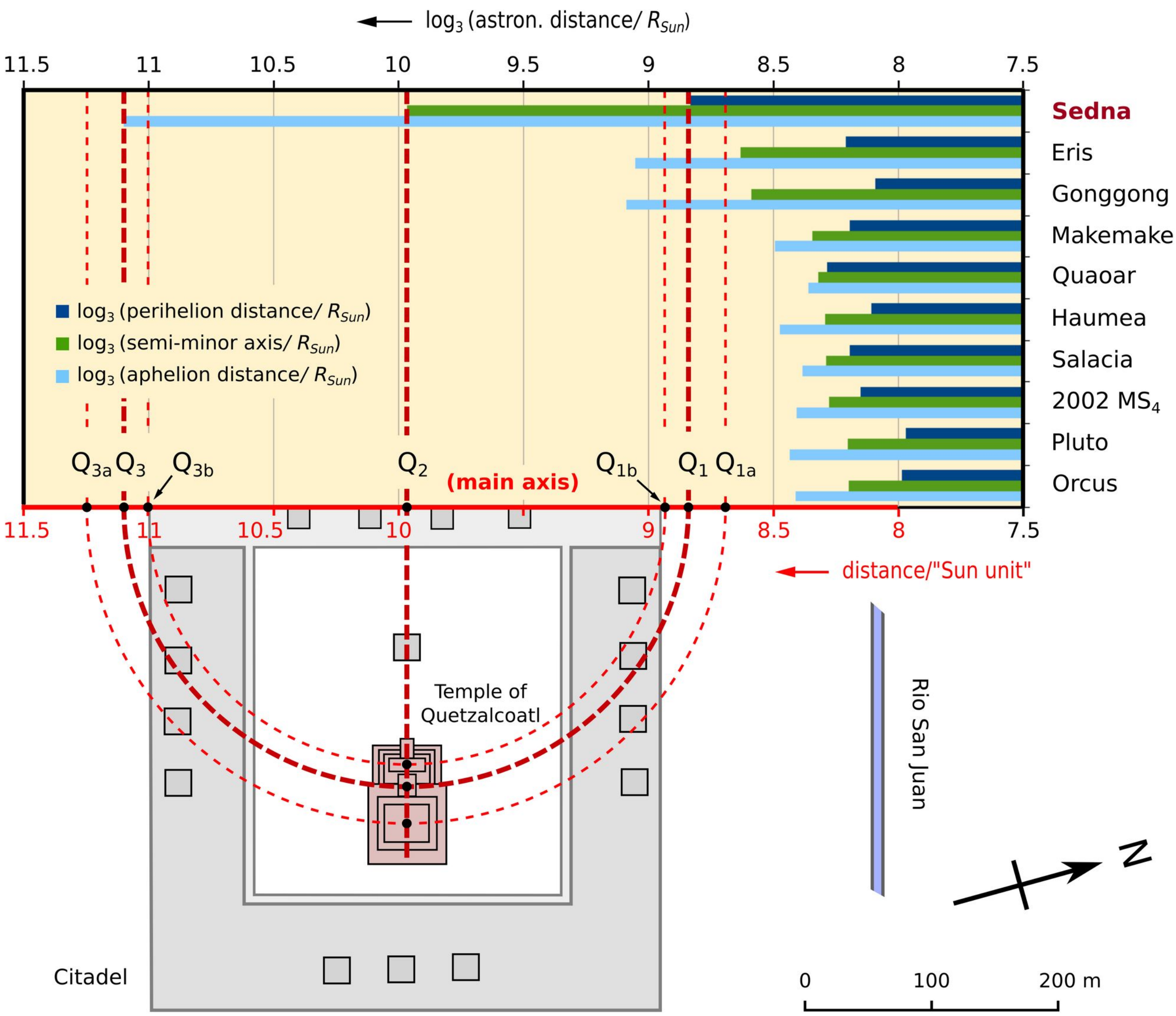
$$\log(b) = \frac{\log(q) + \log(Q)}{2} \tag{87}$$

Thus, log(*b*) is the arithmetic mean of log(*q*) and log(*Q*). This implies that log(*q*), log(*b*), and log(*Q*) follow each other at equal distances on the logarithmic scale. With these three positions on the scale, not only the size of the planetary orbit is fixed but also its shape. By pivoting the position of the temple of Quetzalcoatl by the same 90° angle (see Fig. 30), three positions (Q_1–Q_3) are defined on the scale with equal distances. The points Q_{1a}–Q_{3b} in Fig. 36 are alternative positions. The question is whether a trans-Neptunian object exists that exactly fits to these positions. The answer is provided in Fig. 36. Of the ten TNOs with diameters larger than or equal to 800 km, only Sedna fits almost perfectly. All of the other TNOs are out of range and located within the Kuiper belt, whereas Sedna orbits the Sun, effectively outside the solar system.

One might now ask why, as before, it is not only the perihelion distance that plays a role for Sedna. There is indeed a reason for this. In the region of the eight planets, the planets are clearly identified by their perihelion distances. There are no other planets and there is no doubt about their allocation. In the region beyond Neptune, however, the Kuiper Belt exists with a very large number of objects. Therefore, it makes sense to have more than only one parameter. As previously mentioned, with the three quantities *q*, *b*, and *Q*, the size and shape of the planetary orbit are precisely determined. This allows a possible celestial body to be accurately identified within the large number of objects.

From the several hundred smaller TNOs, determined properly in astronomy, around 99 % belong to the Kuiper belt and are located approximately between 8 and 9 on the given logarithmic axis. Of the four or five TNOs with an orbital size similar to that of Sedna, none fits like Sedna, and all of them are much smaller than Sedna [26].

Another question arises. In Fig. 34, the main axis runs straight on across the Rio San Juan. On the other hand, in Fig. 30, the main axis on the avenue has a parallel shift of several meters when passing the Rio San Juan because the Avenue becomes broader at this point. This shift is also illustrated on the satellite image in Fig. 37 (see positions R_1–R_3 and Q_1–Q_3). A possible reason for this is that Sedna does not have, e.g., a precisely fixed aphelion distance (keyword: osculating orbital elements) [77]. Sedna moves around the common barycenter of the solar system. In contrast, the position of Sedna is measured from the solar center, which oscillates around the barycenter. The latter is sometimes placed even outside the solar surface. It is planned to discuss this in more detail in a future publication.



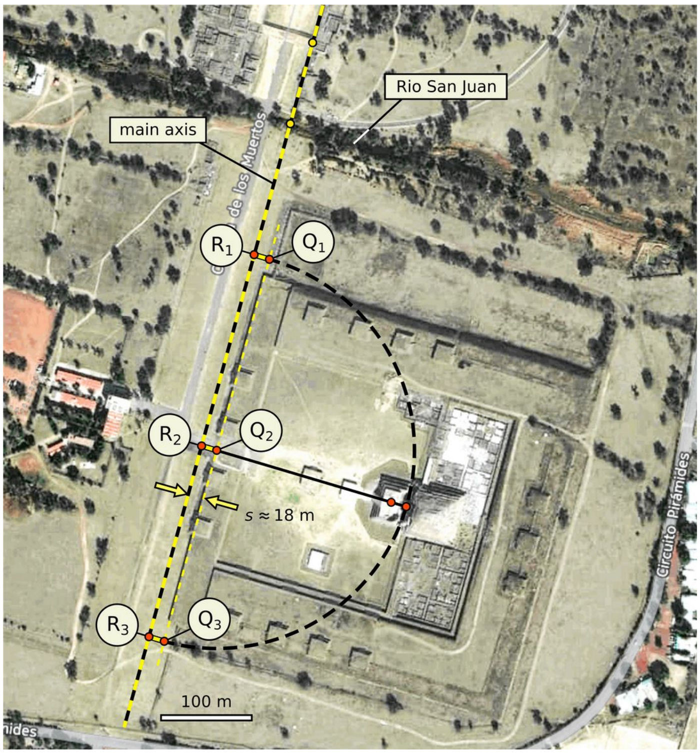


Figure 36 (above): *Orbital elements of TNOs larger than or equal to 800 km and comparison with the Teotihuacán site. Sedna fits surprisingly well (GPS data). The bold semicircle yields the best agreement with the astronomical data. The red scale is based on the Sun unit of 197 m. The data in the histogram were retrieved in October 2021 and listed in [26], whereas the data in Table 10 were updated in March 2024. Sedna and the Temple of Quetzalcoatl are not included in the P5 program, but they can easily be taken into account using spreadsheet analysis (MS Excel or LibreOffice Calc).*

Figure 37 (left): *The so-called Citadel, including the Temple of Quetzalcoatl (the Feathered Serpent Pyramid and the Adosada platform). The different positions and lines are explained in detail in [26]. Note that for the semicircle, a radius r of ca. 220.6 m was graphically used, based on former orbital data of Sedna. The later value of r = 223.18 m (distance between Q_1 and Q_2 in Tab. 8) does not yield much change. Satellite image: © 2017 HERE, 2014 DigitalGlobe, INEGI.*

If the different widths of the Avenue of the Dead are taken into account as described in [26, slide 41], the three positions according to Sedna can be perfectly included in the correlation diagram – see Fig. 38. By adapting the Sun unit (u_{Sun}) from 197 m to 199.08 m, being a minor change, the data points of the eight planets are placed almost precisely on the 45° diagonal and, thus, are in nearly perfect agreement with Eq. (86). Nevertheless, it is important to emphasize again that the correlation and especially the coefficient of determination do not depend on the choice of the units of length, on the logarithmic base, or on the zero position of the logarithmic scale (Pyramid of the Moon).

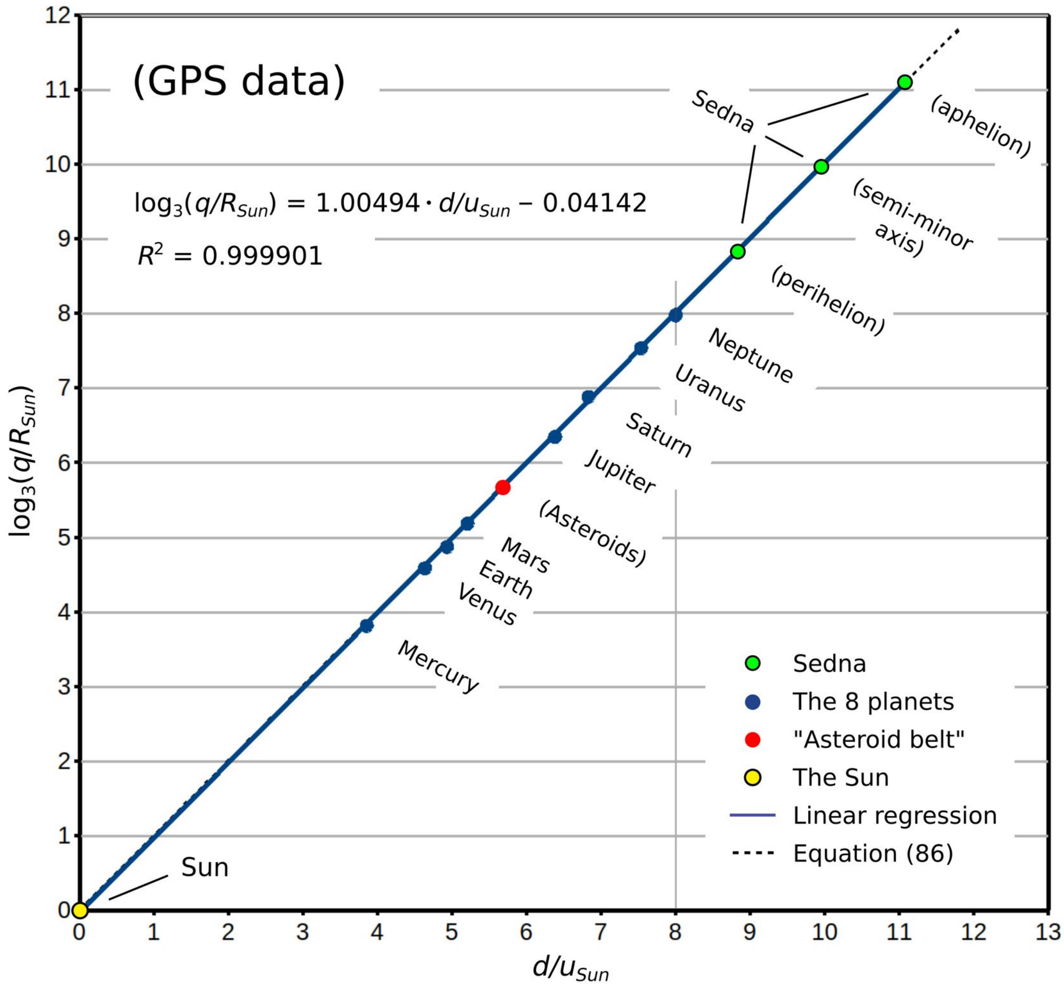


Figure 38: *Planetary correlation including the Sun and the trans-Neptunian object Sedna in AD 200. All of the points, except the red one, are used for the linear regression. The Sun unit, u_{Sun}, is adapted to 199.08 m to obtain an almost perfect agreement with Eq. (86). (This adjustment does not influence R^2.) The corresponding details are provided in [26].*

In view of Fig. 30, the position of Earth was moved from the main axis by a 90° rotation to the position of the Pyramid of the Sun. Thus, this pyramid seems to be a magnified symbol of Earth – like a pop-up window on the monitor. Analogously, the Feathered Serpent Pyramid, also rotated by 90°, is a blow-up of Sedna. Consequently, the huge rectangular citadel could be a blow-up of the Adosada platform (a companion of Sedna), because the platform is located almost in the center of the Citadel. This would then be a “blow-up within the blow-up.” Note: If the center of the semicircle in Fig. 37 is moved from the point Q_2 to R_2, the red point at the Feathered Serpent Pyramid moves exactly onto the Adosada platform. If Sedna has a companion, what could it be? A small moon? Or – considering the findings here – not a moon but a giant artificial space platform as an (extra-terrestrial) stopover for interstellar travel? ... Unusual results require unusual ideas.

In the following list, we repeat some important points and add some new aspects related to the planetary correlation. (Note: The “avenue” always refers to “Avenue of the Dead.”)

- Pyramids, barriers, river, and temple are all marks on the logarithmic scale (avenue).
- There are neither too many nor too few marks on the avenue or alongside the avenue.
- The full length of the avenue is used. The avenue is neither too long nor too short (Fig. 34).
- All of the main structures in Teotihuacán are included in the planetary correlation.
- The six barriers on the avenue do not make any sense except as marks on the axis.
- The Pyramid of the “Moon” represents the Sun. It is a bold marker on the scale for the Sun and thus the size relation between the Pyramid of the “Moon” (Sun) and the barriers (planets) is adequate. This explains also the stretched rectangular shape of the pyramid base area.
- The Pyramid of the “Sun” stands for the Earth and defines the position of Earth on the axis.
- So, instead of having a mark on the avenue, the Earth has a special treatment and is represented by the greatest pyramid. This makes sense because the whole site is located on Earth and the Earth is *our* planet.
- The Pyramid of the “Sun” also provides the position of Mercury, the first and innermost planet.
- The Rio San Juan is different to the other markers (barriers) and represents Neptune and the borderline (transition) between the planetary region and the huge trans-Neptunian space. Accordingly, the central avenue becomes broader at this position.
- In the diagrams of Eq. (86) in Figs. 33 and 38, the eighth planet, Neptune, has the number 8 on both diagram axes.
- The Sun is positioned in the origin of the same diagrams at the point (0, 0), which also makes sense because the real Sun forms the center of our solar system.
- Some further renaming seems reasonable. The Rio San Juan should be the “Rio Neptuno” and the central avenue should be the “Avenue of the planets” (Calzada de los Planetas).
- The “Feathered Serpent Pyramid” most probably stands for the trans-Neptunian object Sedna.
- Eq. (86) represents 10 equations and, if Sedna is included, 13 equations.
- In the main equation, the length units, the logarithmic base, and the origin (zero point) of the logarithmic scale can be chosen arbitrarily. The planetary correlation and especially the coefficient of determination, R^2, are unaffected by this.
- The question is not whether the calculations are correct or not. The calculations are correct within their small uncertainties and can easily be checked. The main question is: Are the findings altogether a big coincidence? According to $R^2 \geq 0.9998$, this is most probably not the case!
- The barrier according to the asteroid belt indicates the possibility that a planet existed on that orbit in the remote past with a perihelion distance of ca. 2.36 AU [26].
- Possibly, the Adosada platform represents a companion of Sedna. A hypothetical explanation of the extraordinary construction called the Citadel, is that it is an enlarged symbolic and technical picture of an extraterrestrial space platform orbiting Sedna (see next section).

If the interpretation of the pyramid site at Teotihuacán, representing a planetary correlation, is correct, the master builders obviously intended that we humans on Earth would one day solve the mystery. The whole site in Teotihuacán gives the impression of a location plan or a route that leads to Sedna. If they had planned to keep this information secret, they would not have constructed the pyramid site as it is.

Although the pyramids at Giza and Teotihuacán both describe a planetary correlation, the design and the construction techniques are very different. Thus, if the master builders of both pyramid areas are of extraterrestrial origin, they probably stem from different planets (exoplanets).

Finally, we present a tentative explanation for the huge Citadel, surrounding the Feathered Serpent Pyramid with dimensions of roughly 400 m $\times$ 400 m (see Fig. 37).

5.2.3 The Citadel

If we take the planetary correlation seriously, the logarithmic scale leads exactly to the dwarf planet Sedna. The huge Citadel has a volume much bigger than those of the Feathered Serpent Pyramid and the Adosada platform together. With the same amount of material necessary for the Citadel, another larger pyramid could have easily been built. What is the meaning of the Citadel?

The all-embracing platform does not look like a protective barrier or wall because a wall thickness of one or two meters would have been sufficient – and not a width (thickness) of about 70 m. Furthermore, it is easy to cross over this "wall." Explanations such as "cult purposes" or "religious cult" would not be acceptable because with such phrases, almost everything can be "explained." Pyramids can be found all over the world, but this huge structure is unique on this planet.

More than 30 years ago, I read some books about the UFO phenomenon. And in one book (I don't remember the title), it was written that we humans would be surprised if we knew that outside the solar system a huge extraterrestrial space platform exists. Additionally, it was mentioned that it would be interesting for human women because this is a good place to go shopping. Up to now, we cannot prove this statement to be true or not. Therefore, we tentatively take it as a hypothesis or conjecture.

If extraterrestrials plan to position a space station outside our solar system, where should they place it? Should they place it just somewhere in the empty outer space or should they let it orbit a celestial body? The latter option has several advantages.

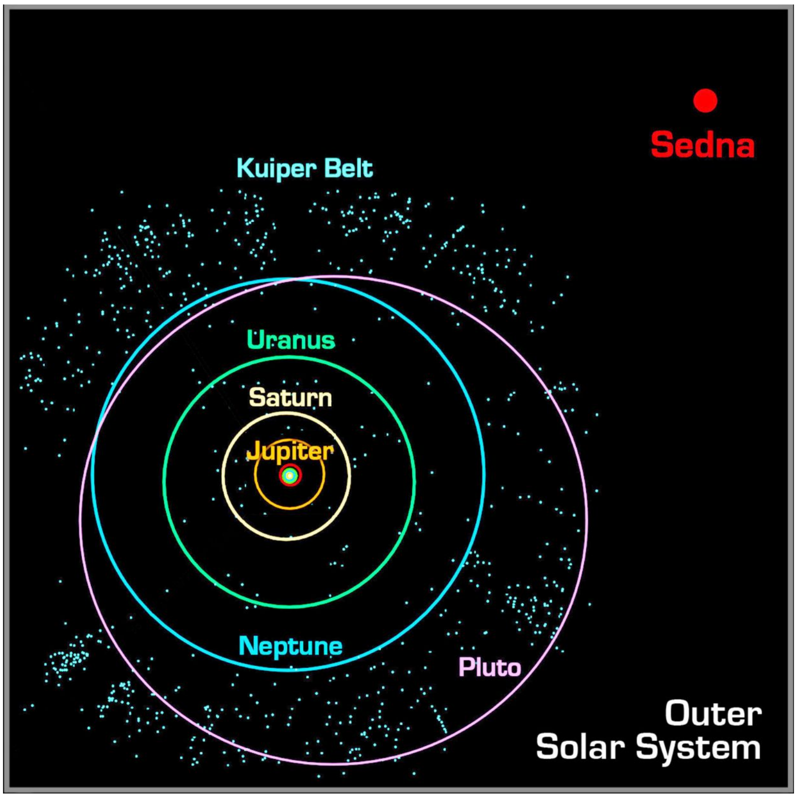


Figure 39: *The outer solar system including Sedna. Image courtesy of NASA / Robert L. Hurt, JPL-Caltech (subimage, Wikipedia).*

Such a space station is probably very large, e.g., with the dimensions of Los Angeles (size ca. 100 km), and could be detected within our solar system using earth-bound telescopes. Thus, some obvious advantages of an orbit around Sedna (Fig. 39) are:

- It is difficult to be detected from Earth because of its great distance.
- It simplifies navigation when approaching from outer space because the station is not within the milling crowd of the Kuiper belt and Sedna can be found easily because of its position, size, and slow motion.
- There is no risk of collision with objects in the Kuiper belt.
- Water (ice) and other resources are available, which is the most important issue.
- A psychological aspect: The position is not in totally empty space but close to a "home" planet.
- Excursions to the dwarf planet Sedna (in warm spacesuits) offer some variety in daily life.
- Construction of a station or habitat on Sedna is possible.

The Feathered Serpent Pyramid is not placed in the center of the Citadel, as can be seen in Fig. 40. Instead, the Adosada platform almost marks the center (geometric center or center of gravity). Therefore, corresponding to the main idea related to the Adosada platform, the Citadel could represent a magnification of the companion of Sedna – virtually another blow-up.

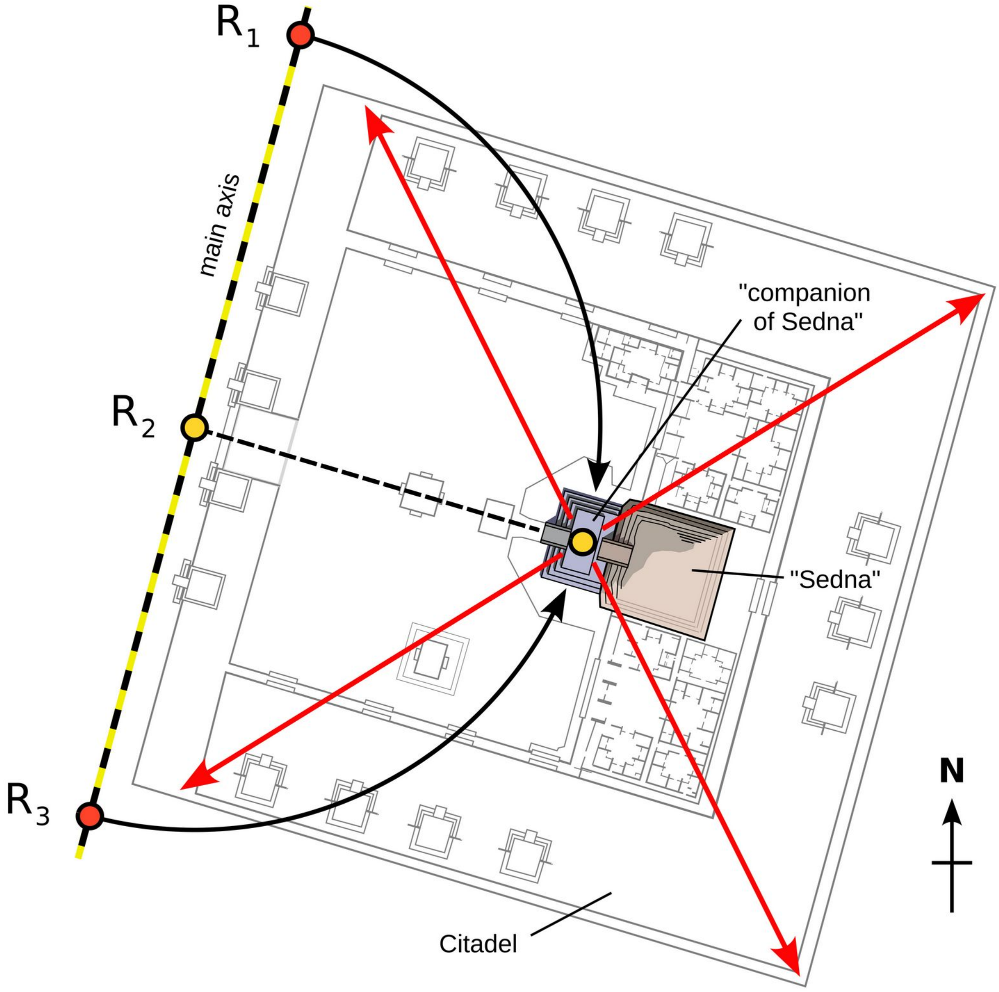


Figure 40: *True to scale representation of the temple of Quetzalcoatl and the Citadel. The Citadel seems to represent a magnified picture of the companion of Sedna, or perhaps more precisely, of an extraterrestrial space platform.*

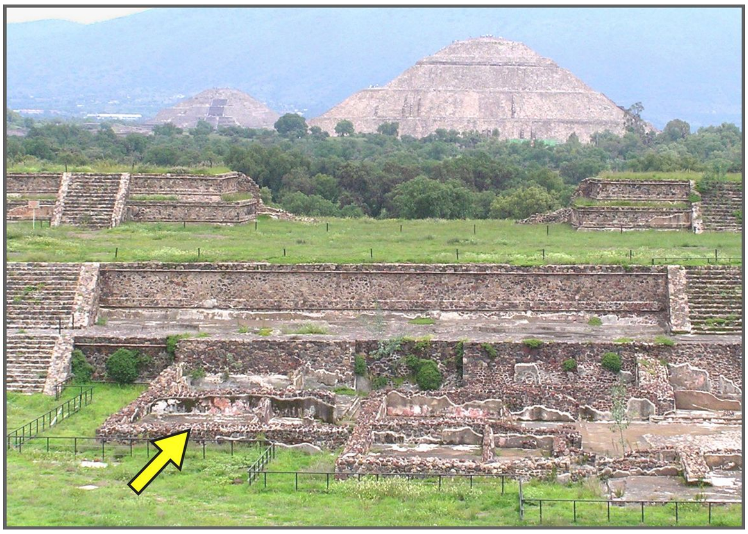


Figure 41: *In the foreground (arrow) walls within the Citadel.*

Before we go into detail, a remark is made concerning the walls to the north and south of the Feathered Serpent Pyramid. In these areas, the walls look like ancient ruins of houses or similar buildings. Indeed, this is not the case.

In Fig. 41, some of these walls are shown in the foreground. The walls are about 1 m high and 1 m thick. They all have nearly the same height and on their top surface, they have a nice flat pavement of natural stones (arrow). Therefore, it seems that these walls are not ruins but are more or less in their original state.

Fig. 41 shows the northwest edge of this area of walls (marked in Fig. 42 as “training center”). Thus this peculiar pattern of walls probably also has a special meaning. Following the main idea, it is possible to give a tentative interpretation of single constructional parts of the Citadel. The details concerning the Citadel as a symbolic technical representation of a spaceport are depicted in Fig. 42. Of course, the respective details should not be seen as completely serious, but the main idea seems to be reasonable against the background of the planetary correlation.

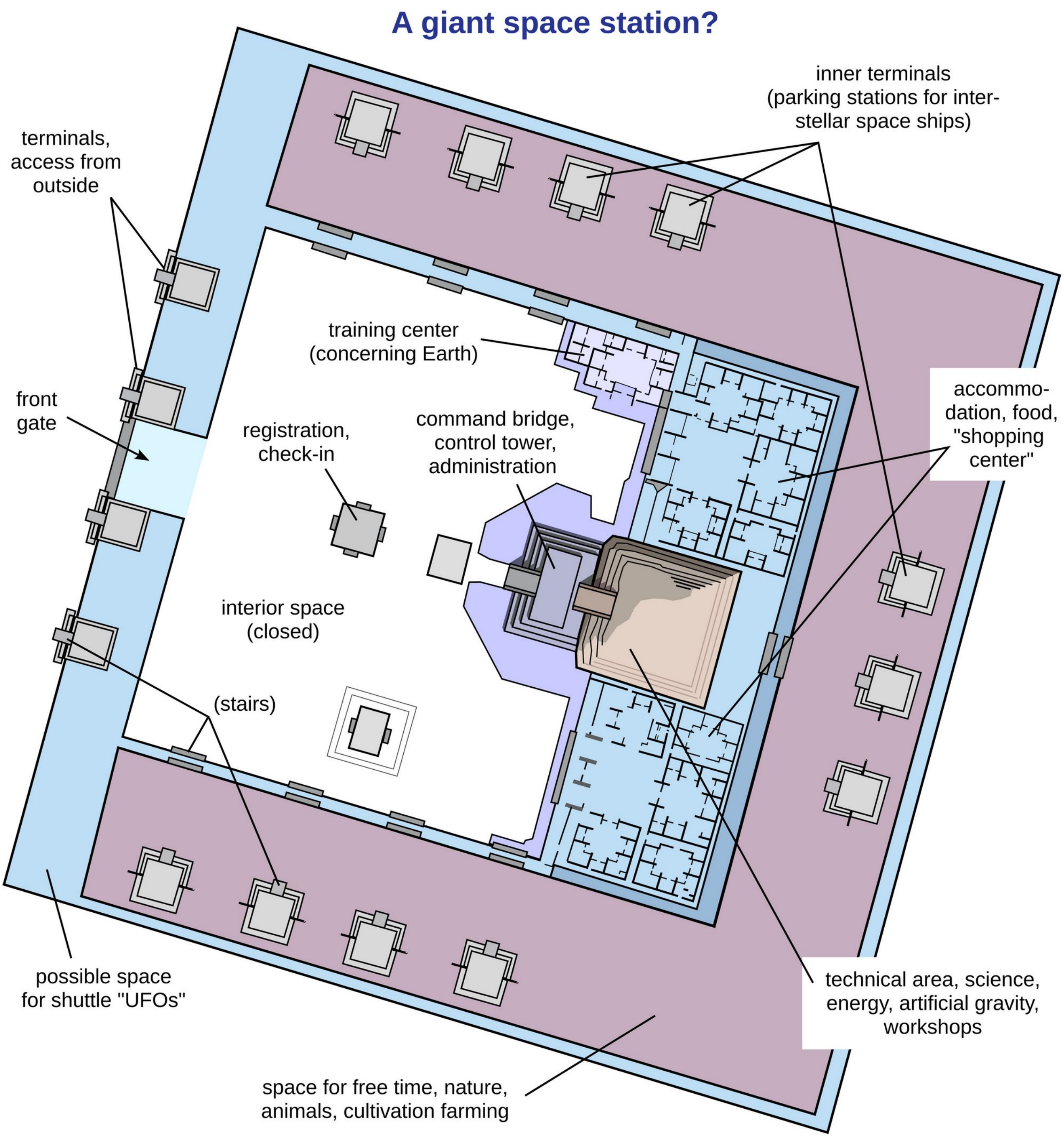


Figure 42: *The Citadel as a symbolic and technical representation of an extraterrestrial spaceport. The figure includes some interpretations of single sections of the Citadel as parts of the hypothetical space platform. The pyramid and the Adosada platform have a different meaning in this magnified picture, compared to Fig. 40.*

The planetary correlation at the pyramid site of Teotihuacán cannot be explained by a coincidence, because R^2 is very close to 1 and because of several other reasons. If someone has a better explanation for the pyramid area and the Citadel, we are open for alternatives.

5.2.4 An investigation of Sedna

This section has been included for two reasons. First, the pyramid site in Teotihuacán strongly points to the TNO Sedna, and second, Sedna – as was discovered in 2003 by M. E. Brown, C. Trujillo, and D. Rabinowitz [78] – has a highly eccentric orbit and will reach its closest approach to the Sun in 2076. Thus, we have a one-of-a-kind opportunity in the following one or few decades to find out more about Sedna and a potential companion. Generally, there are two different methods for investigating the dwarf planet. The first is a direct observation with an earth-bound or space telescope, and the important second method is a mission to Sedna by sending a space probe.

Concerning a telescope, the main problems are the large distance to Sedna, the small size of a companion of Sedna – if it exists – and its darkness. The amount of light coming from a star scales with the inverse square of the distance r of the star, meaning r^{-2}. If a remote celestial body does not shine by itself but is illuminated by the Sun, the factor r^{-2} applies twice, resulting in a factor of r^{-4}. Therefore, we will take a closer look at the technical parameters of the following three telescopes: the Hubble Space Telescope (HST), the James Webb Space Telescope (JWST), and the Extremely Large Telescope (ELT), which is still under construction. The latter two are shown in Fig. 43.

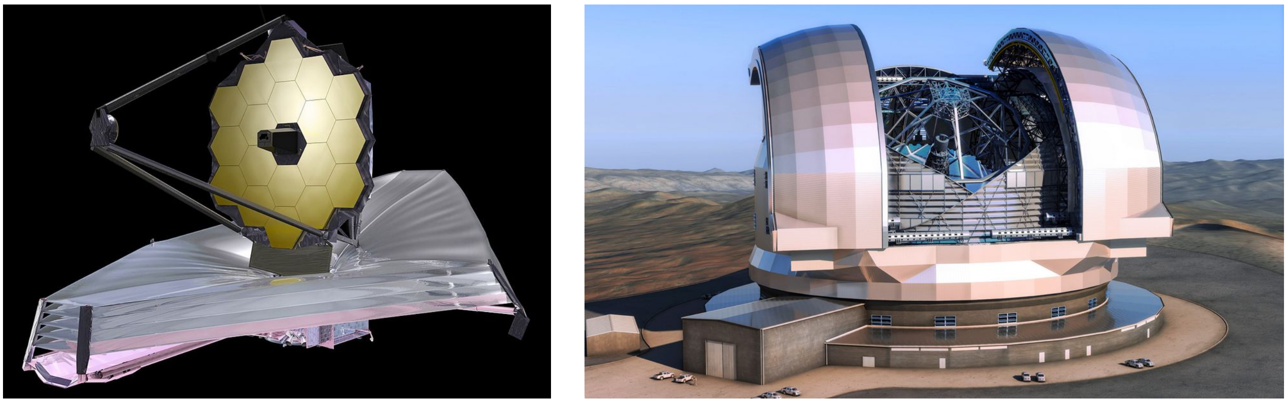


Figure 43: *Artist's impressions;* ***left:*** *James Webb Space Telescope (image: NASA, Wikipedia),* ***right:*** *Extremely Large Telescope, image: ESO/L. Calçada, (URL 16).*

The angular resolution θ for a telescope is $\theta = 1.22 \cdot \lambda / D$, where λ and D are the wavelength of the observed light and the diameter of the main mirror (effective aperture), respectively. With yellow light of 580 nm wavelength and a mirror diameter of 39.3 m, we obtain an angular resolution in arc seconds for the ELT of

$$\theta \;=\; 1.22 \cdot \frac{580 \cdot 10^{-9}\ \text{m}}{39.3\ \text{m}} \cdot \frac{360 \cdot 3600}{2\pi} \;=\; 0.0037'' \qquad (88)$$

This is a theoretical value. The official value is $\theta = 0.005''$, probably because of the remaining atmospheric disturbance or the assumed wavelength. If we suppose a companion of Sedna with an extension of 100 km, what is the size of the corresponding angle? At the nearest approach of Sedna, the (perihelion) distance amounts to $q = 76.37$ AU, where 1 AU ≈ $149.6 \cdot 10^6$ km. The resulting angle is

$$\theta_q \;=\; \frac{100\ \text{km}}{q} \quad \Rightarrow \quad \theta_q \;=\; \frac{100\ \text{km}}{76.37 \cdot 149.6 \cdot 10^6\ \text{km}} \cdot \frac{360 \cdot 3600}{2\pi} \;=\; 0.0018'', \qquad (89)$$

which is smaller than 0.005". It follows that the angular resolution of the ELT is not sufficient to directly identify any shape or structure of a hypothetical 100 km-sized object orbiting Sedna. The object would be a point source. However, the ELT will be capable of collecting more light than any other telescope to date. Due to the diameter of the primary mirror, and accounting for the shadow of the secondary mirror, the collecting areas are 4.0 m^2 (HST), 25.4 m^2 (JWST), and 978 m^2 (ELT). This means that the ELT will collect ca. 240 times more light than the HST. Thus, the ELT will most likely deliver an image of the companion of Sedna – if it exists – even though the image would be more or less a single point. (For point sources, the collecting area is relevant and not the focal ratio).

If we are interested in much more detail, the only other method would be to send a space probe to Sedna. The spacecraft New Horizons, which started its journey in January 2006, reached Pluto 9.5 years later. The current distance of Sedna is roughly twice the distance to Pluto and, therefore, a flight to Sedna will take ca. 20 years or more. A direct flight that is powered solely by conventional rocket propulsion would require too much technical expenditure and is unrealistic. A better solution is the iterative acceleration of the spacecraft by flyby maneuvers (gravity assists) at some of the planets in our solar system. An example is given in Fig. 44 using an EVEΔVEJSed trajectory [79]. The capital letters in EVEΔVEJSed stand for the planets used for the flyby. So, after starting from Earth on August 10, 2034, a flyby will take place at Venus and then Earth. After a brief deceleration phase (ΔV), the next flyby will again be at Earth and finally at Jupiter. The space probe will reach Sedna on August 9, 2064.

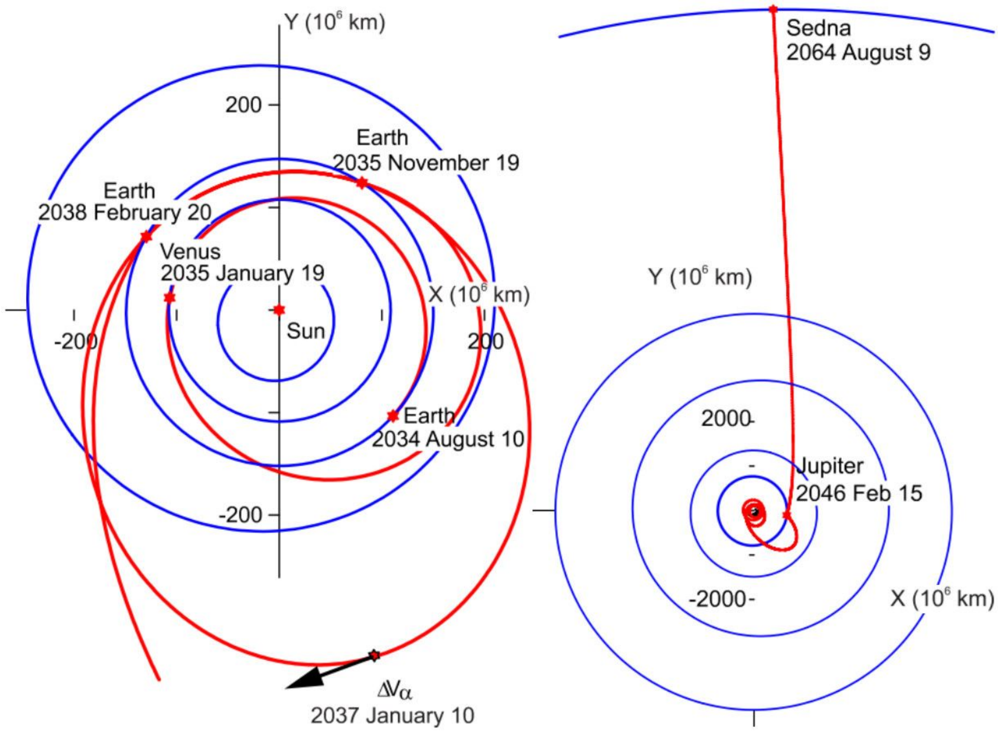


Figure 44: *Trajectory for a flight using the EVEΔVEJSed scheme. The blue orbits on the left belong to the planets Mercury to Mars, the orbits on the right to the outer planets Jupiter to Neptune and Sedna. The figure is taken from [79], with kind permission from V. A. Zubko.*

The problem is that Sedna's highly elliptical orbit has an eccentricity of ca. 0.859. After passing the perihelion, the distance of Sedna increases continuously, and after a few decades a flight to Sedna will become unfeasible. In ca. 6,000 years, the maximum distance is roughly 1,000 AU, meaning that even the light will need almost 6 days to reach the Earth from there. In addition to the optimum starting year of 2029 [79], other calculated dates, trajectories, and concepts exist [80–83]. So, if we miss this moment of closest approach between Sedna and the Sun, we will have to wait for a full orbital period of Sedna for the next flight opportunity, which means roughly another 12,000 years.

Concerning the details, the mission proposal TEASPOON (TransnEptuniAn Sedna PrObe for Outer exploratioN) has been available since 2022 [84]. It offers and defines payload composition, experiments, mass breakdown, power budget, radiation protection, collision avoidance, and a design of the link to Earth. So, a lot of preparatory work has already been done. Since Sedna is interesting because of many reasons including Teotihuacán (!), we should seize this unique opportunity. – How about sending two missions based on different technical strategies, including a Sedna orbiter [83]?

6. Summary and epilogue

Equations (1) – (3) suggest that the three pyramids of Giza represent the three inner planets of our solar system: Mercury, Venus, and Earth (Fig. 5). In all three equations, the Earth is related to the Cheops Pyramid. Venus belongs to the Chefren Pyramid, Mercury to the Mykerinos Pyramid, and the Sun to the light (speed of light). The equations are repeated here:

(numerators)		**(denominators)**
Cheops Pyramid and Earth	$\frac{S_{Cheops}}{c \cdot 1\,\text{s}} = \frac{V_{Earth}}{V_{Sun}}$	Light second and the Sun
Cheops Pyramid and Earth	$\frac{V_{Cheops}}{V_{Chefren}} = \frac{V_{Earth}}{V_{Venus}}$	Chefren Pyramid and Venus
Cheops Pyramid and Earth	$\frac{S_{Cheops}}{S_{Mykerinos}} = \frac{Q_{Earth}}{Q_{Mercury}}$	Mykerinos Pyramid and Mercury

S and *V* are the base length and volume of the pyramid, *Q* is the aphelion distance from the Sun, and *c* is the speed of light. The first equation, containing a second (s), is analyzed in detail in [5] and also in sections 4.10.6 and 4.10.7. Furthermore, the positions of the pyramids and the arrangement of the chambers in the Great Pyramid appear to correlate with the positions of the given three planets. The two points of time, when the positions match exactly, follow each other within a period of 44 days, which is half of the orbital period of Mercury. Between these two events, a conjunction of the four planets of Mercury, Venus, Earth, and Mars implies a "linear constellation" of five celestial bodies, namely the four planets and the Sun, which happens with a simultaneous transit of Mercury. This coincidence of the four planets being in conjunction ($dL_{min} < 5°$) and a simultaneous transit only happens more or less every 5000 years (see section 3.4.8). The basic chronology of the event in Terrestrial Time is as follows:

Apr. 17, 3088, 06:41:13 : Three inner planets in alignment of chambers, Mercury at perihelion

May 18, 3088, 19:20:59 : Transit of Mercury in front of solar disk (nearest approach) with *simultaneous* conjunction of Mercury, Venus, Earth, and Mars

May 31, 3088, 06:19:09 : Three inner planets in alignment of pyramids, Mercury at aphelion

Note: While we find the aphelion distance of Mercury in the third equation, Mercury is placed exactly in the aphelion at the "pyramids date" in 3088. Furthermore, the circumstances concerning the obliquity of the ecliptic and the Sun position in midwinter support the planetary correlation. (The constellations can be visualized, e.g., on the web page *Fourmilab* (URL 17) created by John Walker, or on *JPL's Solar System Dynamics, NASA* (URL 18) using UTC and simple 2-body kinematics.)

The question is not whether these three equations and the astronomical aspects are correct. They are correct within the given small uncertainties! The main question is whether these equations and correlations are all coincidentally valid or not. More precisely, the questions is: How great is the probability that all of these aspects are coincidental? In [5, pp. 87 ff.], a first mathematical estimate of the probability for this *simultaneous* coincidence was performed and it was found to be less than 1:1 million. It follows that these findings most probably are not a great coincidence. By including the additional results of this manual, the probability for the coincidence becomes even less!

If the hypothesis of the planetary correlation turns out to be true, some changes in the naming are possible. The Mykerinos Pyramid would be the "First Pyramid," the Chefren Pyramid would be the "Second Pyramid" and the Cheops Pyramid would be the "Third Pyramid," according to the sequence of the planets. In the same order, we could alternatively also call them "Mercury Pyramid," "Venus Pyramid," and "Earth Pyramid." The King's chamber, the Queen's chamber, and the subterranean chamber in the Cheops Pyramid could be renamed "Earth chamber," "Venus chamber," and "Mercury chamber," respectively. Furthermore, by continuing the sequence of the planets, the five "relieving chambers" above the King's chamber (Fig. 3) would be named after the five outer planets. Another interesting aspect is that the planetary correlation, calculated with VSOP87, yields a "Sun position" and a "Mars position" within the Cheops Pyramid. For many decades scientists have been searching for undetected chambers and corridors in the Cheops Pyramid, making these two positions potential good candidates for a new (secret) chamber.

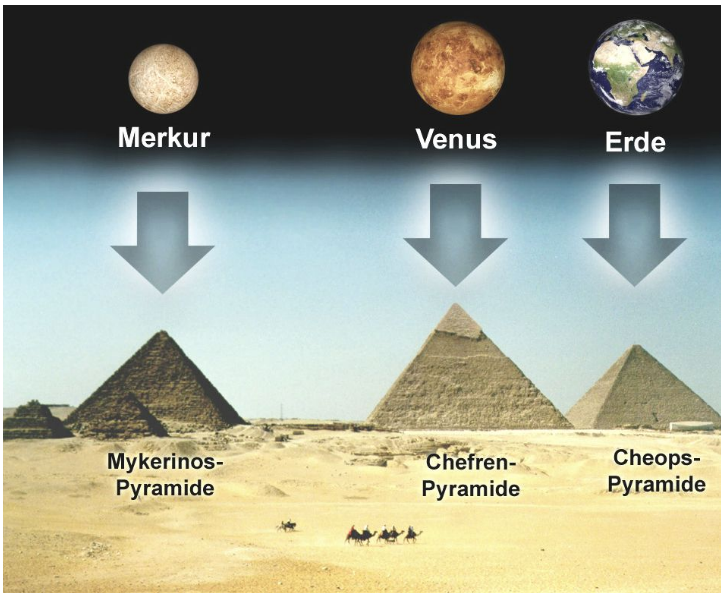


Figure 45: *Planetary correlation of the Giza pyramids. The pyramids are seen from the south (names in German).*

For those who believe that all of these mathematical and astronomical results are accidental coincidences, a technical phenomenon at many of the stone blocks on the Giza plateau was observed that cannot be explained by either ancient or modern technologies. This phenomenon[7] [5, 14] was found at blocks of limestone and granite and can be proven easily with current experimental methods. Some photos with larger magnification and an explanation of possible experimental tests are provided in "technical phenomenon" (URL 19).

The external and own references are listed and readily available. The P5 program, including the executable file, the source code, and associated data files, can be downloaded from the author's website (URL 1). All archaeological and astronomical data used here, as well as the calculations, can be checked by the reader. Most calculations were tested and verified in different ways. Nonetheless, if an error in the calculation or in the approach is found or if the reader has a suggestion for improvement, a note can be sent to: Hans Jelitto, Ewaldsweg 12, D-20537 Hamburg, Germany. If the reader plans to translate any part of this text or make any modifications to the P5 program, the author would appreciate receiving a copy of the results or an Internet link.

Apart from the planetary correlation, additional aspects can be found in [5, 14]. In the past, various speculations about mathematical peculiarities concerning the shapes and especially the casing angles of the pyramids were published. The algebraic approaches were classified in [5] and a new interpretation was found by combining the different hypotheses. This is supported by the structural conditions of the pyramids: the base area of the Cheops Pyramid not being exactly square, the different rectangular corner sockets for the original casing blocks at the four corners of the Cheops Pyramid, and the original granite casing of the Mykerinos Pyramid.

7 The technical effect: The original casing stones of the pyramids and, for instance, the granite stones at the valley temple of the Chefren Pyramid have very narrow joints between them with a width of approximately 0.1 to 0.5 mm. This is already known. The new phenomenon on some of the adjoining blocks is that natural structures, visible on the surface of the blocks, continue exactly across the joint from one block to the next without any misalignment. A "surface effect" due to weathering might – in principle – be possible for limestone, but not for granite. If a granite block of several tons is cut with a special machine today, then a gap of at least a few mm exists, and if these blocks are again moved together, then slant lines of natural structures have a displacement or shift of a few mm at the joint. This is not the case for many of the granite blocks in Giza. If this technical effect proves to be true, then it seems that the natural granite was originally "cut" without or nearly without any loss of material. For granite blocks, weighing tons, this is impossible at present, even using high-tech cutting techniques. In the first book [5], this phenomenon is called *fugenübergreifende Strukturen* in German. Translated to English, these could be called joint-exceeding structures or joint-transcending structures.

If only parts of the given results are true, the consequences for the current research in Egyptology are quite serious. It appears that some high-tech was involved when the pyramids of Giza were built. Since, to our knowledge, the ancient Egyptians did not have any high technology in terms of our present technical level, the next question arises: Did our planet Earth have extraterrestrial visitors in ancient times? For this, the theoretical possibilities of interstellar space travel are discussed in [5, pp. 218 ff.]. Furthermore, the current state of knowledge concerning the so-called exoplanets (extrasolar planets) – planets beyond our solar system – will be briefly reviewed considering this new viewpoint [14].

A detailed discussion of the archaeological measurements and more facets are included in [5, 14]. Some of the main points in [5] were published as articles in journals and as presentations (German) [3, 4, 85–88]. They can be downloaded from (URL 20) or using the links provided in the reference list. Although the main astronomical points of [14] concerning Giza are presented in this manual, the aspects will be described in more detail in that book and, of course, some new aspects will be presented. This, at least, is planned.

If the planetary correlation, symbolized in Fig. 45, is correct, it seems possible that the pyramid builders left some information or an artifact at the “Mars position” or “Sun position” inside or beneath the Great Pyramid. In this case, it seems important and evident that the information about new chambers, writings, artifacts, or whatever – if anything is found – would not only be for archaeologists or institutions, it would also be for the public, which means for everyone that is interested.

Switching to Teotihuacán: The central avenue (Avenue of the Dead) seems to represent a logarithmic astronomical scale and the pyramids, the river, the barriers, and the temple define markers on this scale. All of the eight planets, the asteroid belt, the Sun, and the trans-Neptunian object Sedna are included within this planetary correlation. The main equation (in its simplest form) is given by:

$$\log_3\left(\frac{q_i}{R_{Sun}}\right) = \frac{d_i}{u_{Sun}}$$

with $i = 0, ..., 9$. The planets are represented by the logarithms of their perihelion distances and the Sun by the logarithm of the solar radius. If Sedna is included, characterized precisely by the temple of Quetzalcoatl, the given formula represents not 10 but 13 equations. The presence of the Adosada platform raises the question of whether Sedna has a companion, such as a moon or something else. In this context, a hypothetical (!) interpretation of the Citadel as an enlarged view of an extraterrestrial space platform would perfectly fit the overall picture of the planetary correlation. Up to now, no object has been detected. Nevertheless, it is most likely that the new Extremely Large Telescope (ELT), under construction in Chile with the first light being expected for 2029, will be able to detect a companion if it exists. Possibly, the James Webb Space Telescope (JWST), which recently started operation, is also capable to clarify this question.

In addition, the barrier on the Avenue of the Dead corresponding to the asteroid belt allows for the calculation of the perihelion distance of approximately 2.36 AU [26]. This leads to the question: Did a former planet exist between Mars and Jupiter? A graphical overview is given in Figs. 10 and 34, and a conference lecture about Teotihuacán (video) [89] as well as the proceedings [90] are available (in German).

Writing this program description required less effort than writing the program code itself. When this manual was written, basically all the astronomical results and details concerning the Giza pyramids were known. When starting the programming, we had to start from scratch. From the viewpoint of natural sciences, the scientific context – meaning the astronomical and other calculations – is more or less (modern) basic knowledge. Nevertheless, when beginning any such project, both technical knowledge and completely new ideas are necessary – and there are still many unsolved archaeological questions. We hope that, in the future, more private and professional researchers will become interested in such questions in this new and young research area.

Assuming that the calculations make sense,
I hope the user has the same enjoyment I had,
when I wrote the program. (Hans Jelitto)

(Quetzalcoatl, God of Wind and Wisdom,
as depicted in the Aztec Codex Borbonicus,
taken from Wikipedia.)

Acknowledgments

The short version of VSOP87 and other subroutines (vsop1, vsop3, jdedate, and more) were created on the basis of the excellent book of Jean Meeus: *Astronomical Algorithms* (URL 21) (1991), Willmann-Bell Inc. (URL 22), Richmond, Virginia, USA. Additionally, the book *Transits* (URL 23) from Jean Meeus (same publisher) was valuable for developing and testing the transit computations. Special thanks goes to Dipl. Ing. Manfred Geerken (TUHH, Hamburg, Germany) for valuable help concerning the computer hardware and software used. I am also indebted to Dr. José Martínez Trinidad (Instituto Politécnico Nacional de México, Ciudad de México) for driving me to Teotihuacán in 2005. He accompanied me during the entire day at the pyramid site, and thus helped to make this work possible. My gratitude goes to Nicola Wilton for proofreading the English text as well as to Patrick Wackenhut and Thorsten Sander for their interesting suggestions concerning Giza. Many thanks also to Rémi Cousin (Columbia University, NYC) for permission to upload the WORLDBATH topography data within the TOPO program package. Additionally, my thanks go to the Instituto Nacional de Antropología e Historia (INAH, México) for their permission to use the ground plot of the pyramid site of Teotihuacán (Fig. 4) and to Dr. Vladislav A. Zubko (Space Research Institute, Russian Academy of Sciences, Moscow) for the image of the flight trajectory to Sedna (Fig. 44). Finally, I would like to thank the master builders of the pyramids in Giza and Teotihuacán – whoever they are – for having left us such fantastic mysteries that we must solve.

Appendix A1 – P5 Source Code

GFortran, free source form

The source code of the P5 program contains notes and comments providing additional technical information; it is intended mainly for scientists and programmers. Most (but not all) of the comments are written in German, however, the menus during program start and the output are provided in English. The version of the program is given by the calendar date at the beginning of the program head. If the source code should be compiled again, it is not necessary to take it from this text because it is available in the file `p5.f95`. Actually, the latter file is the reference! Note that the compiled P5 source code does not run alone. It requires the supplementary files that are specified in Table 1. The titles and rubrics of this appendix, provided in the Contents at the beginning of this manual, are not repeated here. Instead, the entire source code of the executable program is listed continuously. The reader should pay attention to the copyright notes on page 187 concerning the P5 program in general and particular subroutines.

The subroutine VSOP87 [1, 2] has been upgraded (→ VSOP87Z), as proposed by Bretagnon and Francou, so that the comprehensive VSOP87 data are read only once from the hard disk or SSD at program start. The subroutines of FITEX [16, 17] were converted to double precision and all program parts were updated to Fortran 95 standard (GFortran). In principle, the code is converted from the fixed to the free source form, although the length of the code lines is still not more than 72 characters. When a test was performed, not with GFortran but with the Intel® Fortran Compiler (ifort), available at the Computing Center of the TUHH (Hamburg University of Technology), the source file `p5.f95` had to be renamed to `p5.f90`. For the language standard, the script *Fortran 95 – Nachschlagewerk zur Fortran-Norm ISO/IEC 1539-1:1997* (RRZN, Leibniz Universität Hannover) was used. Unfortunately, this script is only sold to members and students of some universities in Germany, Austria, and Switzerland, and may only be used by them. Nevertheless, as Fortran is a standardized programming language, several other Fortran reference books are also useful, e.g., the manual *Using GNU Fortran* [91] (URL 24).

At the beginning of programming, the comments were written only for myself in order to later understand the logical configuration and meaning of the program. Now, I hope they are also helpful to the reader, if needed. By the way, this manual is also a workbook for me, useful for further research. For improved readability, the code is highlighted using the editor gedit (print to pdf). I must admit that the programming style is somewhat old-fashioned, e.g., the use of the implicit statement. Nevertheless, the program can be started easily, it runs fairly rapidly, and the results seem to be correct. For P4, two slightly different source codes for single- and multi-thread applications exist. Concerning P5, only one source code (p5.f95) is necessary, which can be used for single-thread as well as for multi-thread hardware. The corresponding commands for the compilation with the GNU Fortran compiler are provided in sections 2.2 and 4.2.6, respectively.

The program was developed from 1993 until today, but – of course – not continuously. From time to time, new ideas arose and were implemented into the program code over countless evenings and weekends. Concerning Giza, one of the last written subroutines was pos_angle (section 4.7.3) to calculate the position angles during a transit, because I was interested in what the transit of the year 3088 will look like. Finally, a planetary correlation of the pyramids in Teotihuacán was found and the program code was thus again extended by another main loop and a few subroutines.

Note that the correlation of Teotihuacán can be easily checked by hand, without this program. The only exception is the time scan, which requires the VSOP theory, because the astronomical data in books and on the Internet are available only for the present. However, the change in the astronomical parameters (*a* and *e*) over the ages is rather small and can be ignored for a general test of the planetary correlation. In the end, I hope the program and this manual will also be interesting to others.

```
!-------------------------------------------------------------------!
!                                                                   !
!                         P5 (GFortran)                             !
!                                                                   !
!                PLANETENKORRELATION DER PYRAMIDEN                  !
!                     IN GIZA UND TEOTIHUACAN                       !
!                                                                   !
!                                                                   !
!                              =                                    !
!                            =   =                                  !
!                          =       =                                !
!                        =    P 5    =                 =            !
!                      =    Programm   =             =   =          !
!                    =   fuer astronomi- =         =       =        !
!                  =   sche Berechnungen   =     =           =      !
!                =   zur Planetenkorrela-    =   =             =    !
!              =   tion in Giza und Teotihua-  =                 =!
! =          =   can. Grundlage sind Messungen   =                  !
!   =      =   namhafter Aegyptologen, die Aus-    =                !
!     =  =   wertung von Satellitenbildern und die   =              !
!       =   planetarische Theorie VSOP87 von Bretagnon  =           !
!         =   und Francou (IMCCE, Paris). Das Programm    =         !
!           =   ist eine Erweiterung der zweiten Edition    =       !
!             =   von P4. Die historische Abfolge ist daher   =     !
!               =   P3, P4 (1./2. Edition) und P5 (3./4. Ed.).  = = =!
!                 = = = = = = = = = = = = = = = = = = = = = = = = = !
! = = = = = = = = = =                                               !
!                                                                   !
!               Hans Jelitto, Hamburg, 30. August 2025              !
!                                                                   !
!                                                                   !
!                          Kurzbeschreibung                         !
!                                                                   !
!        Das Programm P5 berechnet fuer lange Zeitraeume die        !
!        Positionen der Planeten unseres Sonnensystems und er-      !
!        moeglicht einen praezisen Vergleich mit der Anordnung      !
!        der Giza-Pyramiden bzw. der Kammeranordnung innerhalb      !
!        der Cheops-Pyramide. Weiterhin berechnet es die Pha-       !
!        sen der Merkur- und Venustransite vor der Sonne und        !
!        bestimmt Zeitpunkte von "linearen" Planetenkonstella-      !
!        tionen (Syzygium) im Zusammenhang mit den Pyramiden.       !
!        Verschiedene Theorievarianten und eine Vielzahl von        !
!        Optionen ermoeglichen Quervergleiche.                      !
!                                                                   !
!        Eine weitere Planetenkorrelation wurde in Bezug auf        !
!        den Pyramidenbezirk in Teotihuacan (Mexiko) entdeckt.      !
!        Dies wurde hier ebenfalls implementiert. P5 reprodu-       !
!        ziert die astron. Berechnungen in den zwei Buechern:       !
!                                                                   !
!     1. "PYRAMIDEN UND PLANETEN - Ein vermeintlicher Messfeh-      !
!        ler und ein neues Gesamtbild der Pyramiden von Giza",      !
!        Wissenschaft und Technik Verlag, Berlin (1999),            !
!        ISBN 3-89685-507-7                                         !
!                                                                   !
!     2. "PYRAMIDEN UND PLANETEN II - Giza und Teotihuacan"         !
!        (in Vorbereitung)                                          !
!                                                                   !
!          -----------------------------------------------          !
!          *   COPYRIGHTS UND VERWENDUNG DES PROGRAMMS   *          !
!          -----------------------------------------------          !
!                                                                   !
```



```
!        Bezogen auf das Copyright von H. Jelitto stehen das        !
!        Programm P5 und die uebrigen Programmteile fuer wis-       !
!        senschaftliche, private, Ausbildungs- und paedagogi-       !
!        sche Zwecke zur freien Verfuegung, solange der Name        !
!        des (der) Urheber(s) ordnungsgemaess genannt wird, und     !
!        duerfen nicht fuer kommerzielle Zwecke irgendeiner         !
!        Art verwendet werden. Kommerzielle Nutzung bedarf der      !
!        schriftlichen Genehmigung. Fuer die anderen Programm-      !
!        teile (A. bis C.), die im Folgenden aufgezaehlt sind,      !
!        ist zu pruefen, ob eine Genehmigung der Urheber bzw.       !
!        Copyright-Inhaber erforderlich ist. (Informationen zur     !
!        Nutzung/Copyright der Datei "p5-manual-08-2025.pdf"        !
!        stehen zu Anfang jener Datei.)                             !
!                                                                   !
!        Das Programm P5 wird in der Hoffnung zur Verfuegung        !
!        gestellt, dass es fuer andere nuetzlich ist, jedoch        !
!        ohne irgendeine Art von Garantie oder Gewaehrleistung.     !
!                                                                   !
!        Die Copyright-Angaben A.-D. beziehen sich auf das Pro-     !
!        gramm P5, die Versionen von P4 (32-bit-, 64-bit- und       !
!        Multithread-Version), auf die Version P3, sowie alle       !
!        zugehoerigen Dateien. (Siehe z.B. Aufzaehlung unten.)      !
!                                                                   !
!     A. Unterprogramm VSOP87Z (Original: VSOP87) basierend auf     !
!        der Theorie "Variations Seculaires des Orbites Plane-      !
!        taires") und zugehoerige Datenfiles: P. Bretagnon und      !
!        G. Francou, Institut de mecanique celeste et de calcul     !
!        des ephemerides (IMCCE, since January 1, 2025: LTE),       !
!        77 Avenue Denfert-Rochereau, F-75014 Paris, France.        !
!                                                                   !
!     B. Programmpaket FITEX (bestehend aus 4 Unterprogrammen       !
!        im hinteren Programmteil): KIT, Karlsruhe Institute of     !
!        Technology (zuvor: FZK, Forschungszentrum Karlsruhe in     !
!        der Helmholtz-Gemeinschaft), Institut fuer Kernphysik,     !
!        Postfach 3640, D-76021 Karlsruhe. FITEX wurde von          !
!        G.W. Schweimer um 1972 entwickelt und erstmals ver-        !
!        oeffentlicht in: H.J. Gils: "The Karlsruhe Code MODINA     !
!        for Model Independent Analysis of Elastic Scattering       !
!        of Spinless Particles." KfK 3063, Nov. 1980, Kernfor-      !
!        schungszentrum Karlsruhe (KfK), Zyklotron Laboratorium,    !
!        und in: KfK 3063, 1. Supplement, Dec. 1983.                !
!                                                                   !
!     C. Umrechnung von "terrestrial time" (TT) in "universal       !
!        time" (UT) mittels  delta-T = TT - UT: Fred Espenak,       !
!        und Jean Meeus, NASA Eclipse Web Site, Polynomial          !
!        expressions for DELTA-T.                                   !
!                                                                   !
!     D. Das Hauptprogramm P5 und die uebrigen Programmteile,       !
!        einschliesslich der Modifikation des Unterprogramms        !
!        VSOP87 (--> "VSOP87Z"): (c) 2014-2025 Hans Jelitto,        !
!        Ewaldsweg 12, D-20537 Hamburg, Germany.                    !
!                                                                   !
!                   --------- Danksagung --------                   !
!                                                                   !
!        Das Unterprogramm jdedate zur Umrechnung von JDE in        !
!        ein Kalenderdatum basiert auf einem Algorithmus aus dem    !
!        Buch von Jean Meeus: "Astronomical Algorithms", 1991,      !
!        1st Engl. Ed., Willmann-Bell, Inc., Richmond, Virginia,    !
!        USA, S. 63. Dafuer und fuer die Auflistung der gekuerz-    !
!        ten Reihen der VSOP87D-Parameter gilt mein herzlicher      !
!        Dank! Ebenfalls war das Buch "Transits" von Jean Meeus     !
```

```
!       (derselbe Verlag) als Basis und zum Testen der Transit-     !
!       berechnungen aeusserst hilfreich.                           !
!                                                                   !
!                  ----------------------------                     !
!                                                                   !
!       Zum Programm P5 gehoeren die nachfolgenden 36 Dateien.      !
!       Eine ausfuehrbare 32-Bit-Version ist nur zum Programm       !
!       P4 verfuegbar (2. Edition, Juni 2015).                      !
!                                                                   !
!       Datei                    Kurzbeschreibung                   !
!   ----------------------------------------------------------      !
!       p5.f95  . . . . FORTRAN-95-Quellcode (dieser Text)          !
!       p5-64 . . . . . Exe-Datei, 64 bit, single-thread            !
!       p5-64-m . . . . Exe-Datei, 64 bit, multi-thread             !
!       p5-64.sh  . . . loescht Bildschirm u. startet p5-64         !
!       p5-64-m.sh  . . loescht Bildschirm u. startet p5-64-m       !
!       p5-manual-08-2025.pdf:  Bedienungsanleitung zu P5 und       !
!                       Beschreibung der Planetenkorrelationen      !
!       README-P5 . . . Kurzinformation zur Verwendung von P5       !
!       README-vsop87 . Kurzinformation zur Theorie VSOP87          !
!       vsop87.doc  . . Ausfuehrlichere Information zur Theo-       !
!                       rie "Planetary Solutions VSOP87"            !
!       out.txt . . . . Ergebnis-Datei. Wenn diese nicht be-        !
!                       reits existiert, wird sie bei entspre-      !
!                       chender Option vom Programm erstellt.       !
!                                                                   !
!       inedit.t  . . . Datei zum Editieren der Eingabeparame-      !
!                       ter --> Parametersatz fuer "inparm.t"       !
!       inparm.t  . . . Input gemaess Schnellstart-Optionen         !
!       ingiza.t  . . . Parameter f. FITEX, Kammer-Koordinaten      !
!                       in der Cheops-P. und Pyramiden-Koord.       !
!       inserie.t . . . Transitserien fuer Merkur und Venus         !
!       inteoti.t . . . GPS-Koordinaten, ... in Teotihuacan         !
!       invsop1.t . . . VSOP87D, gekuerzt, Meeus: Astr. Alg.        !
!       invsop3.t . . . Polynomdarstellung der Bahnelemente,        !
!                       berechn. aus VSOP82, Meeus: Astr. Alg.      !
!                                                                   !
!                       VSOP87A, kart. Koord. (Ekl. J2000.0)        !
!       VSOP87A.mer . . Merkur          (Diese und die folgen-      !
!       VSOP87A.ven . . Venus           den Dateien enthalten       !
!       VSOP87A.ear . . Erde            die Parameter zur           !
!       VSOP87A.mar . . Mars            VSOP87-Theorie voll-        !
!       VSOP87A.jup . . Jupiter         staendig - Version          !
!       VSOP87A.sat . . Saturn          April 2005.)                !
!       VSOP87A.ura . . Uranus                                      !
!       VSOP87A.nep . . Neptun                                      !
!       VSOP87A.emb . . Erde-Mond-Schwerpunktsystem                 !
!                                                                   !
!                       VSOP87C, kart. Koord. (Ekl. d. Epoche)      !
!       VSOP87C.mer . . Merkur                                      !
!       VSOP87C.ven . . Venus                                       !
!       VSOP87C.ear . . Erde                                        !
!       VSOP87C.mar . . Mars                                        !
!       VSOP87C.jup . . Jupiter                                     !
!       VSOP87C.sat . . Saturn                                      !
!       VSOP87C.ura . . Uranus                                      !
!       VSOP87C.nep . . Neptun                                      !
!                                                                   !
!       DATUM-2.f95 . . Separates Kalenderprogramm (Quellcode)      !
!       DATUM-2 . . . .     "        "   (ausfuehrbare Datei)       !
!   ----------------------------------------------------------      !
```



```
!                                                                     !
!    ----------------------------------------------------------       !
!                       DIE VERSCHIEDENEN OPTIONEN                    !
!    ----------------------------------------------------------       !
!                                                                     !
!             -->  Neue Optionen und Ergaenzungen:                    !
!                                                                     !
!          Die Aenderungen der Programmversion P4 (2. Ed.)            !
!          gegenueber der Ursprungsversion P3 wurden hier             !
!          ergaenzt durch neue Aenderungen und Erweiterungen          !
!          der vorliegenden Version P5 gegenueber P4.                 !
!                                                                     !
!      >   a)  Zu typischen Parameterkombinationen gibt es            !
!      >       jetzt 20 anstatt 15 Schnellstart-Optionen und          !
!      >       wie gehabt die Info-Option (111).                      !
!      >   b)  Verborgene Optionen: Ebenfalls Schnellstart-           !
!      >       optionen - aber nicht im Eingabe-Menue ange-           !
!      >       zeigt - existieren fuer die Resultate in den           !
!      >       Tabellen 39 bis 51 des Buches "Pyramiden und           !
!      >       Planeten" und fuer die Tabellen 17 bis 38 des          !
!      >       Buches 2, das sich in Vorbereitung befindet.           !
!      >       Die Tabelle 39 zum Beispiel besitzt drei Ab-           !
!      >       schnitte, die sich mit den Zahlen 390, 391             !
!      >       und 392 aufrufen lassen, zusammengesetzt aus           !
!      >       39 und 0 bis 2. Ebenso lassen sind Anschluss-          !
!      >       tabellen, wie z.B. 28.A und 28.B durch 280             !
!      >       und 281 berechnen. Das heisst, alle (verborge-         !
!      >       nen) Buchoptionen bestehen aus drei Ziffern!           !
!      >   c)  Spezialoption -804: Diese erzeugt die Liste            !
!      >       der JDE-Zahlen und Transit-Serien in einer             !
!      >       neuen Datei "inser-2.t". Wenn gewuenscht kann          !
!      >       diese Datei durch Umbenennen "inserie.t" er-           !
!      >       setzen (im Allgemeinen nicht erforderlich).            !
!      >   d)  Optional: Programmstart mit einer Input-Datei          !
!      >       "inedit.t", in der die Parameter manuell edi-          !
!      >       tiert werden koennen (Aufruf mit Option 999).          !
!      >   e)  Koordinaten der drei Kammern der Cheops-Pyra-          !
!      >       mide zum Positionsvergleich mit den Planeten.          !
!      >   f)  Positionsvorgabe durch die Mittelpunkte der            !
!      >       Kammern bzw. ihrer Ost- oder Westwaende.               !
!      >   g)  Sechs verschiedene moegliche Zuordnungen der           !
!      >       Planeten Erde, Venus und Merkur zu den drei            !
!      >       Kammern in der Cheops-Pyramide.                        !
!      >   h)  Perihelzeiten beim Merkur, Zeitpunkte nahe             !
!      >       der Periheldurchgaenge und freier Zeitpunkt.           !
!      >   i)  Automatische Erkennung und Markierung der              !
!      >       Planetenkonstellationen 1 bis 14 bei Verwen-           !
!      >       dung beliebiger Optionen.                              !
!      >   j)  Uebertragung der Positionen von Merkur bis             !
!      >       Neptun ins Pyramidengelaende auf Basis der             !
!      >       Pyramiden- bzw. Kammeranordnung (bei 3D-Be-            !
!      >       rechnung mit FITEX, Einzelberechnung, Konst.           !
!      >       1 bis 14). Geographische Koord. (GPS) nur bei          !
!      >       Konst. 12, fuer alle Etappen und Planeten.             !
!      >   k)  Kombination VSOP87-Kurzversion und -Voll-              !
!      >       version: Konstellationen, die mit der Kurz-            !
!      >       version gefunden wurden, werden automatisch            !
!      >       mit der Vollversion nachberechnet. Darueber            !
!      >       hinaus: "Zeitintervall um Aphel bzw. um Peri-          !
!      >       hel" auch fuer die Vollversion VSOP87 (sinn-           !
!      >       voll wegen schnellerer Mikroprozessoren und            !
```

```
!       >       der Programmoptimierung).                          !
!       >   l)  Ausser den beiden Optionen "Blick aus Richtung     !
!       >       ekl. Nordpol" und "ekl. Suedpol" sind jetzt        !
!       >       beide Optionen kombiniert moeglich.                !
!       >   m)  Zeitraeume werden nicht mehr mit der k-Nummer      !
!       >       des Aphel- bzw. Periheldurchgangs des Merkurs      !
!       >       angegeben, sondern mit der eher gebraeuchli-       !
!       >       chen Jahreszahl.                                   !
!       >   n)  Die Berechnungen mit VSOP87 wurde auf den Zeit-    !
!       >       raum 13000 v.Chr. bis 17000 n.Chr. begrenzt.       !
!       >       Ausnahme: "Orbital Elements" und Loesung der       !
!       >       Keplerschen Gl.: 30000 v.Chr. bis 30000 n.Chr.     !
!       >   o)  Syzygium: Merkur bis Erde bzw. Merkur bis Mars     !
!       >       in Konjunktion, d.h. 4 bzw. 5 Himmelskoerper       !
!       >       des Sonnensystems in einer Reihe: Sonne, Mer-      !
!       >       kur, Venus, Erde und optional auch Mars.           !
!       >   p)  Zusaetzlich werden Merkur- und Venustransite       !
!       >       vor der Sonnenscheibe registriert.                 !
!       >   q)  Zum Testen der Transit-Berechnung kann man         !
!       >       sich lueckenlos alle Transite von Merkur und       !
!       >       Venus anzeigen lassen, was einen Vergleich         !
!       >       mit Tabellen aus der Literatur bzw. aus dem        !
!       >       Internet ermoeglicht. In diesem Fall werden        !
!       >       Datum und Uhrzeit der Konjunktion, aufsteigen-     !
!       >       der bzw. absteigender Knoten und die Nummer        !
!       >       der jeweiligen Transitserie angegeben.             !
!       >   r)  Als Zeitpunkt fuer den Planetentransit gibt        !
!       >       es erstens das Kriterium "gleiche ekliptikale      !
!       >       Laengen", zweitens "minimale Separation zwi-       !
!       >       schen Sonne und Planet" (ohne Beruecksichti-       !
!       >       gung der Lichtlaufzeit) und drittens "Beginn,      !
!       >       Mitte und Ende des Transits", d.h. die genau-      !
!       >       en Kontaktzeitpunkte bzw. Phasen.                  !
!       >   s)  Bei der Phasenbestimmung gibt es die Option,       !
!       >       zusaetzlich die Positionswinkel des Planeten       !
!       >       waehrend der Phasen in Bezug auf die scheinba-     !
!       >       re Bewegungsrichtung der Sonne zu berechnen.       !
!       >       Hierbei ist eine Zeilenlaenge auf dem Monitor      !
!       >       von mindestens 148 Zeichen erforderlich.           !
!       >   t)  Fuer die Transitphasen gibt es die zwei Zeit-      !
!       >       systeme "terrestrial (dynamical) time" (TT)        !
!       >       und "universal time" (UT). Die Umrechnung mit      !
!       >       delta-T = TT - UT wird ueber analytische Glei-     !
!       >       chungen erreicht (F. Espenak und J. Meeus,         !
!       >       siehe NASA Eclipse Web Site).                      !
!       >   u)  Fuer die Angabe der Transitphasen von Merkur       !
!       >       und Venus wurde eine Datumsberechnung von          !
!       >       J. Meeus integriert. Hierbei gibt es die auto-     !
!       >       matische Kalenderwahl (julianischer bzw. gre-      !
!       >       gorianischer Kalender) oder es wird der grego-     !
!       >       rianische Kalender fuer alle Zeiten verwendet.     !
!       >       Die Datumsberechnung wurde derart modifiziert,     !
!       >       dass sie jetzt auch fuer negative JDE gilt.        !
!       >   v)  Die Berechnung der dezimalen Jahreszahl wurde      !
!       >       insofern verbessert, dass sie jetzt durch 2        !
!       >       lineare Funktionen dargestellt wird, die je-       !
!       >       weils fuer den Zeitraum des julianischen und       !
!       >       des gregorianischen Kalenders stehen (abhaen-      !
!       >       gig von der Kalenderwahl).                         !
!       >   w)  In Bezug auf den Pyramidenbezirk in Teotihua-      !
!       >       can koennen fuer die Wallabstaende auf der         !
```



```
!       >       Strasse der Toten und die Planetenabstaende        !
!       >       Korrelationskoeffizienten berechnet werden.        !
!       >       Dies ist fuer einen gegebenen Zeitpunkt als        !
!       >       auch fuer ein Zeitintervall in konstanten          !
!       >       Zeitschritten moeglich.                            !
!       >   x)  Die Option fuer die Programm-Ausgabe "Drucken"     !
!       >       im Programm "P3" wurde durch "in Datei" er-        !
!       >       setzt. Hierbei werden die Ergebnisse gleich-       !
!       >       zeitig auf den Bildschirm und in die Datei         !
!       >       "out.txt" geschrieben. Um die Resultate dauer-     !
!       >       haft zu speichern, muss die Datei "out.txt"        !
!       >       nach dem Programmlauf umbenannt werden. Sonst      !
!       >       kann sie beim naechsten Programmlauf ungewollt     !
!       >       ueberschrieben werden.                             !
!       >   y)  Ebenfalls wurde zur Anzeige der Ergebnisse         !
!       >       ein neues Format ergaenzt (special), das fuer      !
!       >       eine Konstellation (z.B. 12) einige spezielle      !
!       >       Parameter ausgibt. Damit lassen sich die we-       !
!       >       sentlichen Tabellen aus dem Buch 2, z.B. mit       !
!       >       den verborgenen Optionen (siehe oben Punkt b),     !
!       >       relativ einfach reproduzieren.                     !
!       >   z)  Optimierung der Rechengeschwindigkeit, unter       !
!       >       anderem durch Modifikation des Datenaufrufs im     !
!       >       VSOP87-Unterprogramm (neuer Name: VSOP87Z) und     !
!       >       Verbesserung der Programm-Ausgabe, z.B. durch      !
!       >       ausfuehrlichere Kopfzeilen, jetzt in Englisch.     !
!       >       Am Ende des Programmlaufs wird die benoetigte      !
!       >       Rechenzeit (CPU time) und Laufzeit (run time)      !
!       >       angegeben, die nach Multithread-Optimierung        !
!       >       sehr unterschiedlich sein koennen. Diese Opti-     !
!       >       mierung in P5 gilt fuer jede Thread-Anzahl.        !
!   -----------------------------------------------------------    !
!                                                                  !
!                                                                  !
!                  Optionen von P5 insgesamt:                      !
!                                                                  !
!       (Falls nicht mit "Teotihuacan" gekennzeichnet,             !
!       beziehen sich die Optionen meistens auf Giza.)             !
!                                                                  !
!   ---------- Schnellstart-Optionen: ------------------------     !
!       1-20    -->  Die wesentlichen astr. Berechnungen           !
!       21-22   -->  Mer./Ven.-Transite + Positionswinkel          !
!       111     -->  Information zu Autoren u. Copyrights          !
!       390-519 -->  Tabellen 39-51 in "Pyram. und Plan."          !
!       170-381 -->  Tabellen 17-38 ausser 29, Buch 2              !
!       999     -->  Input aus "inedit.t" (editierbar)             !
!      -804     -->  Erzeugung der Datei "inser-2.t"               !
!       (0)     -->  Startparameter fuer Einzelmenues              !
!                                                                  !
!   ---------- Pyramidenbezirke: -----------------------------     !
!       1.  Giza (Gizeh), Aegypten                                 !
!       2.  Teotihuacan, Mexiko (siehe weiter unten)               !
!                                                                  !
!   ---------- Planetenpositionen: ---------------------------     !
!       1.  Anordnung der 3 Pyramiden in Giza                      !
!       2.  Anordnung der 3 Kammern der Cheops-Pyramide            !
!       3.  Konjunktionen (Transit, Syzygium)                      !
!       4.  Planetenkorrelation in Teotihuacan                     !
!                                                                  !
!   ---------- VSOP87-Version: -------------------------------     !
!       1.  Kombination von Kurz- u. Vollversion VSOP87            !
```

```
!          2.  VSOP87 Kurzversion (Buch von J. Meeus)               !
!          3.  Keplersche Gleichung mit VSOP82 (Meeus)              !
!          4.  VSOP87 Vollversion (IMCCE, Internet)                 !
!                                                                   !
!   ---------- Koordinatensystem in VSOP87: -------------------     !
!          1.  Ekliptik der Epoche (VSOP87C, alle Vers.)            !
!          2.  J2000.0 (VSOP87A, nur Vollv. und Kepl. Gl.)          !
!                                                                   !
!   ---------- Umfang der Programm-Ausgabe: -------------------     !
!          1.  normal (eine Zeile pro Konstellation)                !
!          2.  detailliert (mehrere Zeilen pro Konstell.)           !
!                                                                   !
!   ---------- Zuordnung: Planeten <-> Kammern: --------------     !
!          1.-6.  Sechs moegl. Zuordnungen von Erde, Venus          !
!              und Merkur zu Koenigs-, Koeniginnen- und             !
!              Felsenkammer: 1. E-V-M (Standard), 2. E-M-V,         !
!              3. V-E-M,  4. V-M-E,  5. M-E-V,  6. M-V-E.           !
!                                                                   !
!   ---------- Zeitpunkte: ---------------------------------       !
!          1.  Apheldurchgang des Merkurs                           !
!          2.  Periheldurchgang des Merkurs                         !
!          3.  Aequidistante Abfolge von Zeitpunkten in             !
!              Zeitintervallen, die jeweils den Aphel-              !
!              durchgang des Merkurs enthalten                      !
!          4.  Aequidistante Abfolge von Zeitpunkten ana-           !
!              log um den Periheldurchgang des Merkurs              !
!          5.  Zeitpunkt voellig frei und Minimierung der           !
!              Abweichung zwischen Pyramiden und Planeten-          !
!              anordnung durch Variation des Zeitpunkts             !
!                                                                   !
!   ---------- "Sonnenposition": ----------------------------       !
!          1.  genau suedlich Mykerinos-Pyramide (1D)               !
!          2.  genau suedlich Chefren-Pyramide (1D)                 !
!          3.  unbestimmt (2D und 3D)                               !
!                                                                   !
!   ---------- Berechnung ("Sonnenposition" unbestimmt): ------     !
!          1.  2-dimensional, Projektion auf Hauptebene             !
!          2.  3-dimensional, durch lineares Gleichungs-            !
!              system und Uebertragung der Loesung                  !
!          3.  3-dimensional, Koordinatentransformation             !
!              mit Fit-Programm FITEX                               !
!                                                                   !
!   ---------- Referenzsystem bei 2D-Berechnung: --------------     !
!          1.  Ekliptikales System                                  !
!          2.  Merkurbahn-System, Transformtion A, B oder           !
!              C (Gerade "Sonne - Merkur-Aphel" = x-Achse,          !
!              Merkurbahn def. xy-Ebene, Ekl. der Epoche)           !
!          3.  Venusbahn-System, Transformation A, (Pro-            !
!              jektion "Aphel - Merkur" genau auf x-Achse,          !
!              Venusbahn def. xy-Ebene, Ekl. der Epoche)            !
!                                                                   !
!   ---------- "Polaritaet" bei Projektion (2D): --------------     !
!          1.  Blick vom ekliptikalen Nordpol                       !
!          2.  Blick vom ekliptikalen Suedpol                       !
!          3.  Beide Optionen 1. oder 2.                            !
!                                                                   !
!   ---------- Vorgegebene Hoehenlagen (3D, z-Koord.)): -------     !
!          1.  Grundflaechen der Pyramiden                          !
!          2.  Schwerpunkte    "      "                             !
!          3.  Spitzen         "      "                             !
!                                                                   !
```



```
!   ---------- Kammerpos. in Cheops-P. (3D, z-Koord.): --------     !
!          1.  Ostwaende der Kammern                                !
!          2.  Mitte      "     "                                   !
!          3.  Westwaende "     "                                   !
!                                                                   !
!   ---------- Zeitpunkt-Eingabe: -----------------------------     !
!          1.  Angabe der Konstellation (Nr. 1 bis 14)              !
!          2.  Jahr bzw. Jahresintervall (von ... bis ...)          !
!          3.  Aphel- bzw. Periheldurchgang (k-Nummer)              !
!          4.  Julian Ephemeris Day (JDE)                           !
!                                                                   !
!   ---------- Planeten in Konjunktion: ----------------------      !
!          1.  Alle Merkur-Transite in einem Zeitintervall          !
!          2.  Alle Venus-Transite   "   "         "                !
!          3.  Merkur bis Erde in einer Reihe (Syzygium)            !
!          4.  Merkur bis Mars   "   "     "   (    "   )           !
!          5.  Syzygium (3./4.) nur mit simultanem Transit          !
!                                                                   !
!   ---------- Transit-Bestimmung (geozentrisch): ------------      !
!          1.  Transite: gleiche eklipt. Laenge Planet/Erde         !
!          2.  Transite: minimale Separation Planet/Sonne,          !
!              1./2.: ohne Beruecksicht. der Lichtlaufzeit          !
!          3.  Phasen und minimale Separation von der Erde          !
!              aus gesehen, Lichtlaufzeit beruecksichtigt           !
!          4.  Phasen wie in 3. und Positionswinkel                 !
!                                                                   !
!   ---------- Kalendersystem: ------------------------------       !
!          1.  Gregorianischer Kalender fuer alle Zeiten            !
!          2.  Automatische Wahl des Kalenders                      !
!              (Greg. < 4712 BC < Julian. < 1582 AD < Greg.)        !
!                                                                   !
!   ---------- Zeitsysteme: --------------------------------        !
!          1.  "terrestrial dynamical time" (TT) bzw. JDE           !
!          2.  "universal time" (UT), basierend auf delta-T         !
!              (NASA Eclipse Web Site).                             !
!                                                                   !
!   ---------- Distanzen in Teotihuacan (Strasse der Toten): --     !
!          1.  berechnet aus GPS-Koordinaten [m]                    !
!          2.  vor Ort gemessen [m] oder Karte/Monitor [mm]         !
!                                                                   !
!   ---------- Lokale Laengeneinheit fuer Teotihuacan: --------     !
!          1.  mm (Karte/Monitor) oder m (real, vor Ort)            !
!          2.  "Sonne-Laengeneinheit" (Plaza de la Luna)            !
!                                                                   !
!   ---------- Astronomische Laengeneinheit (Teotihuacan): ----     !
!          1.  Kilometer                                            !
!          2.  Sonnenradius als Laengeneinheit                      !
!                                                                   !
!   ---------- Basis des Logarithmus (Teotihuacan): -----------     !
!          1.  Basis 10                                             !
!          3.  Basis 3   (Option 2 fehlt.)                          !
!          4.  beliebige Basis                                      !
!                                                                   !
!   ---------- Umfang der Ausgabe: ---------------------------      !
!          1.  einzeilige Datenausgabe pro Konstellation            !
!          2.  ausfuehrliche Datenausgabe                           !
!          3.  (Zeitpunkt oder Zeitintervall, Teotihuacan)          !
!                                                                   !
!   ---------- Ausgabegeraet: ------------------------------        !
!          1.  Monitor                                              !
!          2.  Monitor + in Datei gespeichert ("out.txt")           !
```

```
!          3.  Spezial-Programmausgabe (Monitor + Datei)              !
!          4.  Programm-Abbruch                                       !
!                                                                     !
!    -----------------------------------------------------------     !
!                                                                     !
!                         Anmerkungen:                                !
!                                                                     !
!    Die letztere Aufzaehlung (Optionen insgesamt) wurde der          !
!    Uebersichtlichkeit halber etwas vereinfacht. Sie entspricht      !
!    nicht immer dem Eingabe-Menue, das beim Programmstart mit        !
!    "detailed options (0)" abgefragt wird. Ausserdem sind nicht      !
!    alle Kombinationen der Optionen durchfuehrbar. Solche, die       !
!    nicht erlaubt sind, werden beim Programmstart gar nicht zur      !
!    Auswahl gestellt. Das Programm ist gegen inkorrekte Eingabe      !
!    weitestgehend abgesichert. Eine Kontrolle entfaellt nur, wenn    !
!    die Input-Parameter in der Datei "inedit.t" manuell editiert     !
!    werden und der Programmstart mit der Option 999 erfolgt.         !
!                                                                     !
!    Anstelle des FORTRAN-77-Compilers (IBM Professional Fortran      !
!    Compiler, Version 1.0, Ryan McFarland) wird jetzt unter          !
!    Ubuntu Linux der GNU-Compiler GFortran verwendet, der den        !
!    vollen Sprachumfang von Fortran 95 sowie die meisten Teile       !
!    von Fortran 2003 und Fortran 2008 enthaelt. Das feste Zeilen-    !
!    format wurde (im Prinzip) durch das freie Format ersetzt.        !
!                                                                     !
!    Zum Programmpaket FITEX:                                         !
!    Alle Real-Konstanten wurden mit Exponent "D" versehen, eben-     !
!    falls Funktionen wie DSQRT usw. eingefuehrt, sowie REAL(8)       !
!    und INTEGER(4). EPS wurde von 1.D-5 auf 1.D-8 gesetzt.           !
!                                                                     !
!    Zum Unterprogramm VSOP87 bzw. VSOP87Z:                           !
!    Die VSOP87-Routine wurde dahingehend modifiziert, dass die       !
!    umfangreichen Dateien der VSOP87-Theorie nur einmal gelesen      !
!    und im Rechenspeicher in ein Array geschrieben werden, was       !
!    die Rechengeschwindigkeit wesentlich erhoeht. Ausserdem wur-     !
!    de das Unterprogramm mit "OpenMP" weitgehend fuer eine be-       !
!    liebige Anzahl Threads parallelisiert (Fortran-95-Standard).     !
!                                                                     !
!    Bei den Konstellationen 13, 14, sowie den "quick start           !
!    options" 322 und 323 wird automatisch auch die jeweilige         !
!    Merkur-Aphelposition berechnet, da sich hierbei der Merkur       !
!    nicht im Aphel seiner Bahn befindet. Dies geschieht jedoch       !
!    nur bei Verwendung bestimmter Optionen, wie z.B. 3D/FITEX.       !
!                                                                     !
!    Dieses Quellprogramm enthaelt Abschnitte, die deaktiviert        !
!    wurden (durch "!c", "!h", "!t", "!f" bzw. "!v") und fuer         !
!    spezielle Zwecke gedacht sind. Das Aktivieren einiger Zeilen     !
!    durch Entfernen von z.B. "!h" am jeweiligen Zeilenanfang be-     !
!    wirkt das Einsortieren der Genauigkeiten Fpos in ein Array       !
!    (--> Histogramm: Fpos(0...5%) in Schritten von 0.05%).           !
!                                                                     !
!    Groessere Stellenanzahl in der Ergebnisausgabe (siehe "!f"):     !
!    Fuer einige Optionen koennen mehr Dezimalstellen angezeigt       !
!    werden. Dafuer sind entsprechende Format-Statements zu erset-    !
!    zen. Schnellstart-Opt. 4, 9: siehe Ende des Hauptprogramms;      !
!    3, 8: siehe Ende des Unterprogramms "plako" (durch Aktivie-      !
!    ren bzw. Deaktivieren jeweiliger Formatzeilen). Auch wenn ei-    !
!    nige Schnellstart-Optionen in dieser 4. Edition modifiziert      !
!    wurden, bleibt der theoretisch Hintergrund unveraendert.         !
!                                                                     !
!    Um bei Verwendung der Compiler-Option "-Wuninitialized" bzw.     !
```



```
!    "-Wall" Warnmeldungen zu vermeiden, wurden einige Variablen     !
!    zusaetzlich vorab initialisiert und mit "pre-init." markiert.   !
!                                                                    !
!--------------------------------------------------------------------!

!-----Module----------------------------------------------------------
      module base !    GRUNDLEGENDE VARIABLEN UND KONSTANTEN
      save        ! (Laengen in Metern, Zeiten in julian. Tagen)

      integer(4) :: lmax(15),jp(12,6),il(3)
      real(8)    :: xyr(37),re(78),pyr(40)
      real(8)    :: ax,ay,az,bx,by,bz,cx,cy,cz,ao,ai,at

      real(8), parameter :: pi   = 3.14159265358979320d0, &
        pidg = pi/180.d0,   zjd0 = 2451545.d0, &
        gdpi = 180.d0/pi,   c    = 299792458.d0, &
        tcen = 36525.d0,    AE   = 149597870700.d0, &
        tmil = 365250.d0,   z0   = 0.d0, &
!       ("Allen's Astrophys. Q.", R-Sonne: 695508 km bzw. 958,966",
!       Sonnenradius in "Transits", Meeus: 695990 km bzw. 959,63")
        R0   = 695508000.d0, & ! R-Sonne (Brown/Christensen-Dalsgaard)
        R3a  = 6378136.6d0,   R3p = 6356751.9d0, & ! R-Erde, IERS 2003
        pmer = 2451590.257d0, & ! Erste Merkur-Perihelzeit nach J2000
        ymer = 87.96934963d0  ! Merkur-Umlaufzeit: Perihel -> Perihel

      real(8), dimension(2), parameter :: &
!       Radien: Merkur 3,3629", Venus 8,41", Venusradius mit knapp
!       50 km Atmosphaere (ohne Atm. 6051000 m)
        Ra   = (/ 2439700.d0, 6099500.d0 /), & ! Radien (Mer., Ven.)
        tsid = (/  87.9693d0, 224.7008d0 /), & ! T-siderisch ( ", ")
        tsyn = (/ 115.8775d0, 583.9214d0 /), & ! T-synodisch ( ", ")

!       Theoretischer Massstabsfaktor (Planetenpositionen : Pyramiden-
        zthe = (/ 9.7073d7, 2.3614d9 /)      ! bzw. Kammerpositionen)

      real(8), dimension(14), parameter :: &
!       Nummern des Merkur-Apheldurchgangs der Konstellationen 1-14
        akon = (/-38912.d0,  -23134.d0,  -7356.d0,   8422.d0, &
                  24200.d0,  -24130.d0,  -8352.d0,   7426.d0, &
                  23204.d0,   38982.d0,  -4781.d0,   4519.d0, &
                  39313.9134336d0,  -20240.1362451d0 /)
!c                39313.91342804d0, -20240.136249887d0 /)
!                 (alte Werte, Konst. 13, 14, manuell und
!                 iterativ mit P3 bestimmt)
      end module

      module astro
      save
!     Parameter der VSOP87-Kurzversion nach Meeus
      real(8)    :: par1(69,6,3,12)
!     Parameter der VSOP87-Vollversion
      real(8)    :: par2(2048,3,0:5,3,9)
      integer(4) :: it2(0:5,3,9),in2(0:5,3,9),iv2(9)
!     zur Berechnung mittels Keplerscher Gleichung
      real(8)    :: par3(4,6,8,2)
!     zur Bestimmung der Transit-Serie
      real(8), parameter :: t13BC = -3027093.d0, t17AD = 7930183.d0
      real(8), dimension(2), parameter :: cc=(/16802.20d0,88756.13d0/)
      integer(4), dimension(4), parameter :: jj = (/-150,154,-6,19/)
      integer(4), dimension(2), parameter :: ji = (/15,7/)
```

```fortran
      real(8)    :: ser(-180:170,2),ase(-180:170),zstart
      integer(4) :: ise(-180:170),isflag,ismax
!     zur Berechnung der Planetenkorrelation in Teotihuacan
      character(20) :: tname(0:17); character(1) :: q(0:17),st(0:17)
      real(8)       :: teot(0:17,4),comp(0:8,4),bmas(2,3)
      real(8)       :: alin(3),blin(3),phdis(3)
      end module

      program P5
!-----Hauptprogramm-----------------------------------------------------

!-----Deklarationen und Initialisierungen
      use base; use astro
      implicit double precision (a-h,o-z)
      dimension :: res(12),rp(3,4),md(0:9),pan(5),sd(2),zjda(4)
      dimension :: df(6),diff(9),r(6),rku(3),rk(12)
      dimension :: x(7),e(7),iw(100),f(9),y(9),z(9),w(1000)
      dimension :: x0(7),iw0(4),w0(3),zmem(78),inum(0:4)
      dimension :: ida(7),da(7),id5(5,7),da5(5,7),iw1(8),iw2(8)
      dimension :: xx(5),yy(5),test(10),ort(0:9,4),rcm(3),acm(3)
!h    dimension :: ihis(100)  !h
      character(1) :: t1(3),tra(2),tr,dp,ts,sl
      character(2) :: dd,dn,ds,dss,kon
      character(3) :: dk,pla(0:9)
      character(5) :: dmo,dmo5(5)
      character(7) :: emp
      character(8) :: str,str2,str3
      character(10) :: plan(0:9),zdate,ztime,zzone
      character(20) :: dummy
      character(23) :: text(0:9),tt(2)
      character(49) :: titab
      real(8)       :: lbase(4)          !  Teotihuacan
      character(27) :: tluna(2)          !       "
      character(11) :: trsun(2),di(3)    !       "
      character(5)  :: tdi(3),str4       !       "
      character(14) :: di2(3,2)          !       "
      character(40) :: di3(2)            !       "
      data diff/0.d0,12.19d0,21.41d0,0.d0,-34.784d0,145.d0,60.4d0, &
               168.d0,21.41d0/,pla/'Sun','Mer','Ven','Ear','Mar', &
              'Jup','Sat','Ura','Nep','E-M'/
      data titab/'body            x[m]      y[m]      z[m]    dr[m]'/
      data tt/  '  (pyramid positions)  ','  (chamber positions)  '/
      data text/'                       ','  of the "planets"     ', &
             7*'                       ','       barycenter   -->'/
      data plan/'Sun       ','Mercury   ','Venus     ','Earth     ', &
               'Mars      ','Jupiter   ','Saturn    ','Uranus    ', &
               'Neptune   ','Earth-Moon'/
      data str/'    --- '/,str2/'   --   '/,str3/'    --  '/
      data emp/'   --- '/,dn/'  '/,ds/' *'/,dss/' <'/,dp/':'/
      data di2/'GPS dist. [m] ','real dist. [m]','Map dist. [mm]', &
         ' GPS distance ',' real distance',' Map distance '/  ! Teoti.
      data di3/'log(per./km)   log(a/km)    log(aph./km)', &  !   "
               'log(per./Rs)   log(a/Rs)    log(aph./Rs)'/    !   "
      data di/'- R^2 (GPS)',' R^2 (real)','- R^2 (Map)'/      !   "
      data tdi/'"GPS"','dist.','dist.'/                       !   "
      data lbase/10.d0,0.d0,3.d0,0.d0/                        !   "
      data tluna/'normal (mm or m)           ', &             !   "
                 'Sun unit (Plaza de la Luna)'/               !   "
      data trsun/'normal (km)','Sun radius '/,str4/' --- '/   !   "
      data zjde0/0.d0/,ifitrun/0/,zjdelim/0.d0/,izmin/0/ ! pre-init.
```



```fortran
!-----Input-Daten und Programmstart
      call inputdata(ipla,ilin,imod,imo4,ikomb,io,lv,ivers, &
       itran,isep,iuniv,ical,ika,iaph,iamax,step,ison,ihi,irb,ijd, &
       zmin,zmax,ak,zjde1,dwi,dwikomb,dwi2,dwi3,nurtr,iek,iop0,iout)
      if (iout==4) then; write(6,*); go to 1000; endif
      call cpu_time(zia)
      call date_and_time(zdate,ztime,zzone,iw1)
      write(6,'(/''  <P5>   Computation started ...'')')

! . . Die Input-Parameter werden in die Datei "inedit.t" geschrieben.
!     Man kann sie dann gegebenenfalls manuell an geeigneter Stelle in
!     "inparm.t" (Liste der Schnellstart-Optionen) einfuegen, wobei
!     allerdings im Unterprogramm "inputdata" die Schnellstart-
!     Optionen angepasst werden muessen. Ausserdem suche --> iop0!
      if (iop0/=999 .and.iop0/=-804) then
        call inputfile(ipla,ilin,imod,imo4,ikomb,io,lv,ivers,itran, &
         isep,iuniv,ical,ika,iaph,iamax,step,ison,ihi,irb,ijd,zmin, &
         zmax,ak,zjde1,dwi,dwikomb,dwi2,dwi3,nurtr,iek,iop,2,iout)
      endif

! . . Parameter fuer Spezial-Output (Konst. 12) --> is12 = 1
      is12 = 0
      if (((ipla==1 .and.iaph==1).or.(ipla==2 .and. &
          iaph==2 .and.ika==1 .and.ijd==12)).and.imod==2 .and. &
          ikomb==0 .and.iuniv==1 .and.io==2 .and.ison==5 .and. &
          (ijd==12 .or.ijd==14).and.iout==3) is12 = 1

! . . Erstellung weiterer Parameter
      if (iout==1) then
        ix = 6
      else
        ix = 1
        open(unit=ix,file='out.txt')
        write(6,'(9x,''Output file: "out.txt"'')')
      endif
   10 write(6,*); kmin = 0; kmax = 0
      if (ipla<=2) then
        if (ijd>=1 .and.ijd<=14) then
          ak = akon(ijd)
          if (ipla==2 .and.iek==1) ak = ak - 1.d0
          call ephim(0,iaph,ipla,ical,ak,iak,zjde1,zjahr,delt)
        endif
        if (ijd==15 .and.imod==2 .and.iaph<=2) &
          call ephim(0,iaph,ipla,ical,ak,iak,zjde1,zjahr,delt)
      endif
      if (ipla==3 .or.(ipla<=2 .and.ijd==15 .and. &
        (imod/=2 .or.(imod==2 .and.(iaph==3 .or.iaph==4))))) then
        call ephim(2,iaph,ipla,ical,ak,kmin,zjdemin,zmin,delt)
        call ephim(2,iaph,ipla,ical,ak,kmax,zjdemax,zmax,delt)
        if (ipla==3) izmin = idint(zmin)
      endif

! . . Parameter fuer Transit-Pruefung
      if (ipla<=3) then
        if (ilin==1) then
          itransit=1; il(1)=1; il(2)=3; il(3)=2
        elseif (ilin==2) then
          itransit=2; il(1)=2; il(2)=3; il(3)=1
        else
          itransit=0; il(1)=1; il(2)=4; il(3)=1
        endif
```

```
      endif

!-----Einlesen der Startwerte und Parameter fuer FITEX
!     sowie der Koordinaten der Pyramiden bzw. Kammern
      i0 = 0; j0 = 0; if (ipla==1) j0 = 18
      if (ipla==3) e(1) = 1.d-6
      if (ipla==1 .or.ipla==2) then
        open(unit=10,file='ingiza.t')
        do i=1,8+j0; read(10,*); enddo
        read(10,*) dummy,(x0(i),i=1,7)
        read(10,*) dummy,(e(i),i=1,7)
        read(10,*)
        read(10,*) dummy,(iw0(i),i=1,4)
        read(10,*) dummy,(w0(i),i=1,3)
        read(10,*)
        read(10,*) dummy,iter
        read(10,*); read(10,*)
!       Indizes von rp, k: Pyr./Kammern, i: Koordinaten und "Hoehe"
        do k=1,3; read(10,*) dummy,(rp(k,i),i=1,4); enddo
        read(10,*)
        if ((ison==2 .or.ipla==2).and.is12==0) then
          read(10,*) dummy,diff(2),diff(3)
        else
          read(10,*)
        endif
        do i=1,22-j0; read(10,*); enddo
        do i=1,4
          read(10,*) dummy,zjda(i)
        enddo; close(10)
        if (ipla==2 .and.imod/=3) call chambers(ika,rp)
      endif

!-----Einlesen der Transitserien zum Festlegen der Startnummer(n)
      if (ilin<=2 .and.ipla<=3) then
        do i=-180,170
          ase(i) = z0; ise(i) = i0
          if (.not.(iop0==-804 .and.ilin==2)) ser(i,1) = z0
          ser(i,2) = z0
        enddo
        if (iop0/=-804) then
          open(unit=10,file='inserie.t')
          do i=1,5; read(10,*); enddo
          do i=-150,150,5; read(10,*)idummy,(ser(i+j,1),j=0,4); enddo
          do i=1,4; read(10,*); enddo
          do i=-10,15,5; read(10,*)idummy,(ser(i+j,2),j=0,4); enddo
          close(10)
        endif
        ismax = -10000; zstart = 99.99d0
      endif

!-----Einlesen der Daten fuer Teotihuacan
      if (ipla==4) then
       open(unit=10,file='inteoti.t')
       do i=1,19;read(10,*); enddo
       do i=0,17;read(10,*)tname(i),q(i),st(i),(teot(i,j),j=1,4);enddo
       close(10)
      endif

!-----Weitere Initialisierungen
      do i=0,4; inum(i) = i0; enddo
      isflag = i0; ifl = i0
```



```
      iflag1 = i0; iflag2 = i0
      ipos = i0; nfit = 7; mfit = 9
      ipar = i0; if (isep==4) ipar = 2
      indx = 1; iekk = iek; lid5 = -50000
      prec = z0; lu = 10; delt = z0
      if (ipla<=3) step = step/24.d0
      diff1 = diff(2); diff2 = diff(3)
      zamax = dfloat(iamax); zjdevor = -1.d10
      do i=0,9; md(i) = 1; enddo
!h    do i=1,100; ihis(i) = i0; enddo  !h

!     Initialisierung zur Berechnung fuer die Datei "inserie.t",
!     (--> "inser-2.t", danach manuelles Kopieren nach "inserie.t")
      if (iop0==-804) then
        if (ilin==1) is = -177  ! fuer Merkur, Jahre -18000 bis 18000
        if (ilin==2) is = -6    ! fuer  Venus, Jahre -30000 bis 30000
      endif

! . . Berechnung des Zeitsprungs fuer die Option "Linearkonstell.";
!     "tsprung" ist ein Zeitintervall in Tagen, das nach dem Ablauf
!     einer Konjunktion von Venus und Erde uebersprungen wird. Dieses
!     darf nicht zu gross sein, um alle Ereignisse zu erfassen. Das
!     erste Ereignis im Intervall der Jahre -13000 bis 17000 geht ver-
!     loren fuer tsy = 577 Tage (tsprung = 557 Tage, dwin = 5 Grad),
!     d.h. "tsprung" waere zu gross. Darueber hinaus ergab sich je-
!     weils als groesster zulaessiger Wert fuer tsy (Version Kepl.):
!
!        dwin    tsy   tsprung        dwin    tsy   tsprung
!       [Grad] [Tage]  [Tage]        [Grad] [Tage]  [Tage]
!      ------------------------     ------------------------
!          5     576     557           20     577     510
!         10     578     543           45     578     430 (not used)
!         15     578     527           90     575     286 (not used)
!      ------------------------     ------------------------
!
!     Die Gleichung fuer tsprung (siehe unten) ist sinnvoll, da alle
!     tsy-Werte etwa gleich gross sind, was auch fuer die Optionen
!     "Kurzv." und "Kombi." gilt. Zur Sicherheit wurde tsy = 570 Tage
!     festgelegt (synodische Umlaufzeit der Venus: 583.9 Tage).
      if (ipla==3 .and.ison==5) step = 1.d0
      dwi0 = dwi;  tsy = 570.d0     ! (fuer Syzygien)
      if (ilin==1) tsy = 115.7d0   ! (Merkur, optim.)
      if (ilin==2) tsy = 582.7d0   ! (Venus, optim.)
      if (ipla==3 .and.ikomb==i0) dwi = dwi + 1.d0
      dwin = dwi
      if (ilin<=2) tsprung = tsy
      if (ilin>=3) tsprung = dnint(tsy*(1.d0-dwin/180.d0))
      if (tsprung<1.d0) tsprung = 1.d0
      if (ipla==4) go to 30

! . . Blickrichtung von der suedlichen ekliptikalen Hemisphaere
      if (iek==2 .and.ipla<=2) then
        diff1 = -diff1; diff2 = -diff2
        do i=1,9; diff(i) = -diff(i); enddo
      endif
      if (ipla==3) go to 20
      if (ipla==4) go to 30

!-----Pyramidenabstaende und Winkel
!     Indizes von "pyr":
!        1 bis 5: leer
```

```fortran
!        6: leer      7: pdx       8: pdy       9: pdz      10: leer
!       11: pax      12: pbx      13: pcx      14: pay      15: pby
!       16: pcy      17: paz      18: pbz      19: pcz      20: leer
!       21: pa       22: pb       23: pc       24: pb/pa oder pbx/pax
!       25: pc/pa oder pby/pay  26: pc/pb oder pby/pbx  27: alpha
!       28: beta     29: gamma    30: leer     31: alpha1   32: alpha2
!       33: alpha3  34: pax/2    35: pay/2    36: pbx/2    37: pby/2
!       38: (pax+pbx)/2   39: (pay+pby)/2   40: leer
!     Indizes 11-19 und 21-29 bei "pyr" und "xyr" entsprechen sich.
!
! . . Anpassung der Koordinaten fuer Grundflaeche, Schwerpunkt und
!     Spitze der Pyramiden bzw. Ostwand, Mitte und Westwand der
!     Kammern.
      if (ihi==2) then
        cm = 0.25d0; if (ipla==2) cm = 0.5d0
        do i=1,3; rp(i,4) = rp(i,4) * cm; enddo
      endif
      if (ihi==2 .or.ihi==3) then
        do i=1,3; rp(i,3) = rp(i,3) + rp(i,4); enddo
      endif
! . . Abstaende der Pyramiden bzw. Kammern und weitere Groessen.
      pyr(11) = rp(2,1)-rp(3,1);  pyr(12) = rp(1,1)-rp(3,1)
      pyr(14) = rp(2,2)-rp(3,2);  pyr(15) = rp(1,2)-rp(3,2)
      pyr(17) = rp(2,3)-rp(3,3);  pyr(18) = rp(1,3)-rp(3,3)
      pyr(13) = pyr(12)-pyr(11);  pyr(16) = pyr(15)-pyr(14)
      pax = pyr(11);  pay = pyr(14);  paz = z0
      pbx = pyr(12);  pby = pyr(15);  pbz = z0
      pcx = pyr(13);  pcy = pyr(16);  pcz = z0
      if (ison==3) then
        pyr(31) = - datan(pyr(14)/pyr(11))
        pyr(32) = - datan(pyr(15)/pyr(12))
        pyr(33) = - datan(pyr(16)/pyr(13))
        pyr(34) = pyr(11)*0.5d0
        pyr(35) = pyr(14)*0.5d0
        pyr(36) = pyr(12)*0.5d0
        pyr(37) = pyr(15)*0.5d0
        pyr(38) = (pyr(11)+pyr(12))*0.5d0
        pyr(39) = (pyr(14)+pyr(15))*0.5d0
      endif
!     Koordinaten des gemeinsamen Zentrums "rcm" der drei Pyramiden
!     bzw. Kammern und mittlerer Abstand zu den Pyramiden bzw. Kammern
!     "dmi" (zur Fehlerberechnung von "Sonnen-","Planeten- und Aphel-
!     positionen" in Giza in den Subroutinen "sonpos", "aphelko" und
!     "plako")
      do i=1,3; rcm(i) = (rp(1,i) + rp(2,i) + rp(3,i))/3.d0; enddo
      do i=1,3
        acm(i) = dsqrt((rp(i,1)-rcm(1))**2 + (rp(i,2)-rcm(2))**2 &
                     + (rp(i,3)-rcm(3))**2)
      enddo
      dmi = (acm(1) + acm(2) + acm(3))/3.d0
!c    do i=1,8
!c      write(6,'(5f12.6)') (pyr(5*(i-1)+j),j=1,5)
!c    enddo
! . . Zusaetze zur 3-dim. Berechnung
      if (ison>=4) then
        pyr(19) = pyr(18) - pyr(17)
        paz = pyr(17); pbz = pyr(18)
        pcz = pyr(19)
!c      write(6,'('' x: '',3f12.3)') (pyr(i),i=11,13)
!c      write(6,'('' y: '',3f12.3)') (pyr(i),i=14,16)
!c      write(6,'('' z: '',3f12.3)') (pyr(i),i=17,19)
```



```fortran
! . . . Erzeugung eines Vektors pd, der auf pa und pb senkrecht steht.
        pdx = pby * paz - pay * pbz
        pdy = pax * pbz - pbx * paz
        pdz = pbx * pay - pax * pby
        aba = dsqrt(pax*pax + pay*pay + paz*paz)
        abb = dsqrt(pbx*pbx + pby*pby + pbz*pbz)
        abd = dsqrt(pdx*pdx + pdy*pdy + pdz*pdz)
        dfakt = (abb + aba) * 0.5d0/abd
        pyr(7) = pdx * dfakt
        pyr(8) = pdy * dfakt
        pyr(9) = pdz * dfakt
! . . . Modellwerte fuer FITEX
        if (ison==5) then
          z(1) = z0;   z(2) = z0;   z(3) = z0
          z(4) = pax;  z(5) = pay;  z(6) = paz
          z(7) = pbx;  z(8) = pby;  z(9) = pbz
        endif
      endif
! . . Laengen, Laengenverhaeltnisse, Winkel
      if (ison<=2) then
        pyr(24) = pbx/pax
        pyr(25) = pby/pay
        pyr(26) = pby/pbx; if (iek==2) pyr(26) = -pyr(26)
      else
        pyr(21) = dsqrt(pax*pax + pay*pay + paz*paz)
        pyr(22) = dsqrt(pbx*pbx + pby*pby + pbz*pbz)
        pyr(23) = dsqrt(pcx*pcx + pcy*pcy + pcz*pcz)
        pyr(24) = pyr(22)/pyr(21)
        pyr(25) = pyr(23)/pyr(21)
        pyr(26) = pyr(23)/pyr(22)
        pyr(27) = dacos((pax*pbx+pay*pby+paz*pbz)/(pyr(21)*pyr(22)))
        pyr(28) = dacos((pax*pcx+pay*pcy+paz*pcz)/(pyr(21)*pyr(23)))
        pyr(29) = dacos((pbx*pcx+pby*pcy+pbz*pcz)/(pyr(22)*pyr(23)))
      endif

!-----Einlesen aller Parameter der VSOP87D-Kurzversion (Meeus)
   20 if (imod==1) then
        open(unit=10,file='invsop1.t')
        read(10,*)
        do n=1,12
          read(10,*); read(10,*) lmax(n)
          read(10,*) (jp(n,j),j=1,lmax(n))
          do m=1,lmax(n)
            read(10,*)
            do j=1,jp(n,m)
              read(10,*) idummy,(par1(j,m,i,n),i=1,3)
            enddo
          enddo
        enddo
        close(10)
      endif

!-----Bahnparameter als Polynome 3. Grades aus VSOP82 (Meeus)
   30 if (io==2 .or.irb/=1 .or.imod==3 .or.ipla>=3) then
        open(unit=10,file='invsop3.t')
        do ll=1,2
          do n=1,3; read(10,*); enddo
          do k=1,8
            do n=1,2; read(10,*); enddo
            do j=1,6; read(10,*) (par3(i,j,k,ll),i=1,4); enddo
          enddo
```

```
          enddo
          close(10)
        endif

!-----Titelzeilen (Giza-Pyramiden)
        if (ipla<=3) then
          do iu=ix,6,5
            call titel1(iaph,ijd,iu,ison,ipla,ilin,isep,nurtr, &
              iuniv,is12,iop0)
            call titel2(iu,imod,ivers,irb,ipla, &
              ison,ihi,iek,ijd,ika,iaph,ilin,ical,ak,zjde1,zjahr,delt, &
              dwi,dwikomb,dwi0,dwi2,dwi3,iamax,step,ikomb,zmin,zmax)
! . . . . Tabellenkopf
            call tabe(iaph,imod,iek,iu,io,ison,ipla,ilin,itran,is12, &
              iop0,iout)
          enddo
        endif
        if (iaph==5) go to 200
        if (ipla==3) go to 300
        if (ipla==4) go to 800

!     Anmerkung: In jedem Programmlauf wird nur eine
!     der vier folgenden Hauptschleifen verwendet.

!=================================================================
!------------------------- 1. Hauptschleife -------------------------
!=================================================================

!-----1. Hauptschleife (Pyramiden- und Kammerpositionen--------------
!     sowie Aphel- und Perihelzeitpunkte des Merkur)
        k = kmin
  100 zk = dfloat(k)
        if (imod==2 .and.ijd==15 .and.iaph<=2) zk = ak
        isw = 1; if (iaph<=2 .and.iout==3) isw = 2
        jmax = i0
        ncount = i0

!.....JDE-Zeitpunkt (Merkur im und ausserhalb des Aphels)
  120 zjde = zjde1
        if (ijd==15 .or.iaph==3 .or.iaph==4) then
          ik = k
          if (isw==1 .or.(isw==2 .and.iaph<=2)) then
            if (ijd==15 .and.(imod/=2 .or. &
              (imod==2 .and.(iaph==3 .or.iaph==4)))) ak = zk
            if (ijd==15) then
              call ephim(i0,iaph,ipla,ical,ak,iak,zjde,zjahr,delt)
            else
              call ephim(1,iaph,ipla,ical,ak,iak,zjde,zjahr,delt)
            endif
          else
            acount = dfloat(ncount)
            if (ijd==15) then
              ak = zk + step * (acount - zamax * 0.5d0)/ymer
              call ephim(i0,iaph,ipla,ical,ak,iak,zjde,zjahr,delt)
            else
              zjde = zjde1 + step * (acount - zamax * 0.5d0)
              call ephim(1,iaph,ipla,ical,ak,iak,zjde,zjahr,delt)
            endif
          endif
        endif
        if (ijd==i0) call ephim(1,iaph,ipla,ical,ak,iak,zjde,zjahr,delt)
```



```
        ik = idnint(ak)
        time = (zjde - zjd0)/tcen
        tau  = (zjde - zjd0)/tmil
        if (ison==5) then
          do i=1,4; iw(i) = iw0(i); enddo
          do i=1,3; w(i) = w0(i); enddo
          do i=1,7; x(i) = x0(i); enddo
          do i=4,6; x(i) = x(i) * pidg; enddo
        endif
        inum(1) = inum(1) + 1

!.....Variante 1 (VSOP87D, Kurzversion aus "Meeus", multiple threads)
        if (imod==1) then
!$omp   parallel do shared(tau,re) private(i,resu)
          do i=1,9; call vsop1(i,tau,resu); re(i) = resu; enddo
!$omp   end parallel do
        endif

!.....Variante 2 (VSOP87A/C, Vollversion)
  140 if (imod==2) then
          do i=1,3; ii = 3*(i-1)
            call vsop2(zjde,ivers,i,md,ix,prec,lu,r,ierr,rku)
            do j=1,3; re(ii+j) = rku(j); enddo
          enddo
        endif

!.....Variante 3 (Kepl. Gleichung, Polynome 3. Grades nach VSOP82)
        if (io==2 .or.irb/=1 .or.imod==3) then
          immax = 3; if (io==2) immax = 4
          do i=1,immax; ii = 6*i
            call vsop3(lv,i,ix,ir,time,res)
            if (ir/=i0) go to 1000
            re(25+ii) = res(1);  re(28+ii) = res(5)
            re(26+ii) = res(2);  re(29+ii) = res(4)
            re(27+ii) = res(3);  re(30+ii) = res(6)
            if (imod==3 .and.i<=4) re(3*i-2) = res(11)
          enddo
        endif

!.....Koordinaten-Transformation und Bestimmung von F-pos
        if (irb>=2 .or.imod/=3) call kartko(ison)
        if (irb>=2) call transfo(irb,rku)
        if (irb>=2 .or.imod/=3) &
           call relpos(ipla,ison,ijd,iek,iekk,ika)

!.....Korrelation der Positionen pruefen, Output
        ic   = i0
        err3 = z0; err4 = z0
        dif1 = re(1) - re(4); call reduz(dif1,i0,i0)
        dif2 = re(1) - re(7); call reduz(dif2,i0,i0)
        if (ison<=2) then
          err1 = dif1 - diff1; call reduz(err1,i0,i0)
          err2 = dif2 - diff2; call reduz(err2,i0,i0)
          if (iek==3) then
            err3 = dif1 + diff1; call reduz(err3,i0,i0)
            err4 = dif2 + diff2; call reduz(err4,i0,i0)
          endif
!.......Hauptbedingung pruefen (ison = 1, 2) . . . . . . . . . . . . .
          if ((dabs(err1)<dwi.and.dabs(err2)<dwi).or.ijd/=15 &
            .or.(iek==3 .and.dabs(err3)<dwi.and.dabs(err4)<dwi) &
            .or.(ijd==15 .and.imod==2 .and.ikomb==i0)) then
```

```
          if (ikomb==1 .and.imod==1) then
            imod = 2
            dwi = dwikomb
            go to 140
          endif
          if (iek==3) then
            iekk = 1
            if (dabs(err3)<dwi.and.dabs(err4)<dwi) iekk = 2
          endif
          inum(2) = inum(2) + 1
          ic = 1
!         Resultat Output
          call konst(ik,kon)
          dd = dn
          if (iek==2 .or.iekk==2) dd = ds
          do iu=ix,6,5
            if (imod/=3) then
              if (iek==3 .and.iekk==1) then
                write(iu,56)kon,ik,zjde,zjahr,re(1), &
                  dif1,dif2,err1,err2,dd
              elseif (iek==3 .and.iekk==2) then
                write(iu,56)kon,ik,zjde,zjahr,re(1), &
                  dif1,dif2,err3,err4,dd
              else
                write(iu,55)kon,ik,zjde,zjahr,re(1), &
                  dif1,dif2,err1,err2,xyr(36)
              endif
            else
              if (iek==3 .and.iekk==2) then
                write(iu,56)kon,ik,zjde,zjahr,re(1), &
                  dif1,dif2,err3,err4,dd
              else
                write(iu,56)kon,ik,zjde,zjahr,re(1), &
                  dif1,dif2,err1,err2,dd
              endif
            endif
          enddo
        endif
      else
        if ((iaph==3 .or.iaph==4).and.isw==1 .and.ijd==15) then
          ifl = i0
          if (xyr(36)<dwi2) ifl = 1
        endif
!.......Hauptbedingung pruefen (ison = 3, 4, 5). . . . . . . . . . . .
        if (((isw==1 .or.(isw==2 .and.iaph<=2)).and. &
           (xyr(36)<dwi.or.ijd/=15 .or. &
           (imod==2 .and.ikomb==i0.and.iaph<=2))).or. &
           (isw==2 .and.((ifl==1 .and.xyr(36)<dwi3.and. &
           ijd==15).or.ijd/=15))) then
          if (ikomb==1 .and.imod==1) then
            imod = 2
            dwi = dwikomb
            go to 140
          endif
          inum(2) = inum(2) + 1
!         Sonnenposition
          call sonpos(ison,iek,ix,rp(3,1),rp(3,2),rp(3,3),rcm,dmi, &
                      iter,iw,ke,mfit,nfit,f,x,e,w,y,z)
          ic = 1; dd = dn
          if (iek==2) dd = ds
          do isun=1,4; ort(i0,isun) = xyr(30+isun); enddo
```



```
!         Resultat Output
          if (isw==1) then
            call konst(ik,kon)
            do iu=ix,6,5
              if (ison==5) then
                if (ipla==2) then
                  write(iu,184)kon,ik,zjahr,dif1,dif2,ke,iw(3), &
                    (xyr(30+i),i=1,4),dd,xyr(36)
                else
                  write(iu,165)kon,ik,zjahr,dif1,dif2,ke,iw(3), &
                    (xyr(30+i),i=1,4),dd,xyr(36)
                endif
              elseif (ison==3) then
                write(iu,67)kon,ik,zjahr,re(1),dif1,dif2, &
                  xyr(31),xyr(32),emp,xyr(34),dd,xyr(36)
              else
                if (ipla==2) then
                  write(iu,85)kon,ik,zjahr,re(1),dif1,dif2, &
                    (xyr(30+i),i=1,4),dd,xyr(36)
                else
                  write(iu,65)kon,ik,zjahr,re(1),dif1,dif2, &
                    (xyr(30+i),i=1,4),dd,xyr(36)
                endif
              endif
            enddo
          else
            if (((xyr(36)<dwi2.or.iaph<=2).and.ijd==15).or. &
                ijd/=15 .or.imod==2) then
              if (iout==3) then
                call konst(ik,kon); delh = delt * 24.d0
                call reduz(x(5),1,i0)
                if (ipla==1) then
                  xma = xyr(35)*1.d-7;dxy=dsqrt(xyr(31)**2+xyr(32)**2)
                  sonne = datan((xyr(33)-rp(3,3))/dxy)*gdpi
                else
                  xma = xyr(35)*1.d-9; dxr = xyr(31)-rp(3,1)
                  dyr = xyr(32)-rp(3,2); dzr = xyr(33)-rp(3,3)
                  sonne = datan(dyr/dsqrt(dxr*dxr + dzr*dzr))*gdpi
                  if (dxr>0.d0) sonne = 180.d0 - sonne
                  call reduz(sonne,i0,i0)
                endif
                do iu=ix,6,5
                  if (iaph==3 .or.iaph==4) then
                    if (ipla==2) then
                      write(iu,275)zjde,delh,x(5)*gdpi,xma, &
                        sonne,(xyr(30+i),i=1,4),dd,xyr(36)
                    else
                      write(iu,255)zjde,delh,x(5)*gdpi,xma, &
                        sonne,(xyr(30+i),i=1,4),dd,xyr(36)
                    endif
                  elseif (iaph<=2) then
                    if (ipla==2) then
                      write(iu,276)kon,ik,zjahr,x(5)*gdpi,xma, &
                        sonne,(xyr(30+i),i=1,4),dd,xyr(36)
                    else
                      write(iu,256)kon,ik,zjahr,x(5)*gdpi,xma, &
                        sonne,(xyr(30+i),i=1,4),dd,xyr(36)
                    endif
                  endif
                enddo
              else
```

```
!               Pruefung zur Signifikanz --> dk
                dk = '   '
                zf = dabs((xyr(35)-zthe(ipla))/zthe(ipla))
                if (zf<=2.d-2 .and.xyr(36)> 0.5d0) dk = 'M  '
                if (zf> 2.d-2 .and.xyr(36)<=0.5d0) dk = 'F  '
                if (zf<=2.d-2 .and.xyr(36)<=0.5d0) dk = 'FM '
                if (zf<=1.d-3 .and.xyr(36)<=0.1d0) dk = '>>>'
                do iu=ix,6,5
                  if (ison==5) then
                    if (ipla==2) then
                      write(iu,386)dk,ik,zjde,xyr(35),ke,iw(3), &
                        (xyr(30+i),i=1,4),dd,xyr(36)
                    else
                      write(iu,366)dk,ik,zjde,xyr(35),ke,iw(3), &
                        (xyr(30+i),i=1,4),dd,xyr(36)
                    endif
                  elseif (ison==3) then
                    write(iu,367)dk,ik,zjde,xyr(35),ncount-iamax/2, &
                      xyr(31),xyr(32),emp,xyr(34),dd,xyr(36)
                  else
                    if (ipla==2) then
                      write(iu,384)dk,ik,zjde,xyr(35),ncount-iamax/2,&
                        (xyr(30+i),i=1,4),dd,xyr(36)
                    else
                      write(iu,365)dk,ik,zjde,xyr(35),ncount-iamax/2,&
                        (xyr(30+i),i=1,4),dd,xyr(36)
                    endif
                  endif
                enddo
              endif
            endif
          endif
!h        call histogramm(xyr(36),ihis)  !h
        endif
      endif

!.....Weiterer Output
      do iu=ix,6,5
        if (ic==1 .and.imod/=3 .and.io==2 .and.is12==0) then
          call linie(iu,2)
          write(iu,57) (re(i),i=1,9)
          do i=1,3
            t1(i) = ' '; if (xyr(3+i)<z0) t1(i) = '-'
          enddo
          write(iu,54) (xyr(i),i=1,3),t1(1),dabs(xyr(4)), &
            t1(2),dabs(xyr(5)),t1(3),dabs(xyr(6)),(xyr(i),i=7,9)
          write(iu,'(1x,6f9.6,f22.8,'' %'')') xyr(11),xyr(12), &
            xyr(14),xyr(15),xyr(17),xyr(18),xyr(36)
          call linie(iu,2)
        endif
        if (is12/=0) call linie(iu,1)
        if (is12==0 .and.ic==1.and.imod==3.and.io==2) call linie(iu,2)
        if (ic==1 .and.io==2 .and.is12==0) then
          if (imod/=3) then
            if (ivers==3) then
              write(iu,'(''  ascending node (M/V/E/Ma): '',2f12.6, &
               & ''       ---   '',f12.6)')re(34),re(40),re(52)
            else
              write(iu,'(''  ascending node (M/V/E/Ma): '',4f12.6)') &
               (re(28+6*i),i=1,4)
            endif
```



```
            write(iu,'(''  inclination i  (M/V/E/Ma): '',4f12.6)') &
              (re(29+6*i),i=1,4)
            write(iu,'(''  perihelion pi  (M/V/E/Ma): '',4f12.6)') &
              (re(30+6*i),i=1,4)
            if (imod/=3 .and.irb/=1) &
            write(iu,'(''  ang. par. (omega, i, tau): '',3f12.6)') &
              ao*gdpi,ai*gdpi,at*gdpi
            if (ison==5) then
              write(iu,'(''  transl. X1, X2, X3; del-t: '',3f12.6, &
                & f9.3,'' days'')') (x(i),i=1,3),delt
              do i=4,6; call reduz(x(i),1,i0); enddo
              write(iu,'(''  Euler angl. X4, X5, X6; M: '',3f12.6, &
                & f13.0)') (x(i)*gdpi,i=4,6),xyr(35)
!c            write(6,'(''  X7: '', f12.6)') x(7)
            endif
          else
            do i=5,8; ii = 6*i
              call vsop3(lv,i,ix,ir,time,res); if (ir/=i0) go to 1000
              re(25+ii) = res(1);  re(28+ii) = res(5)
              re(26+ii) = res(2);  re(29+ii) = res(4)
              re(27+ii) = res(3);  re(30+ii) = res(6)
            enddo
            call elements(iu,ivers,pla)
          endif
          if ((ison==3 .and.ijd>=1 .and.ijd<=10).or.ison==4) write( &
            & iu,'(''  scale factor M           : '',f13.0)')xyr(35)
          call linie(iu,1)
        endif
      enddo

!.....Output: Koordinaten aller Planeten einschliesslich Neptun und
!     des Schwerpunktsystems Erde-Mond, letzteres nur fuer VSOP87A,
!     sowie transformierte "planetarische" Koordinaten in Giza
      if ((imo4==1 .and.iaph<=2 .and.is12==0 .and.io==2) &
          .or.is12/=0) then
        call plako(diff,ipla,ijd,ik,ison,ipos, &
         rcm,x,y,ort,rp,dd,dn,dss,pla,plan,emp,text,tt,titab, &
         is12,dmi,zjda,zjde,ivers,md,ix,prec,lu,r,ierr,rku)
      endif

! . . Ruecksprung fuer Aphel-Umgebung
      if (ikomb==1 .and.imod==2) then
        imod = 1; dwi = dwi0
      endif
      if (iaph==3 .or.iaph==4) then
        ncount = ncount + 1
        if (ncount>jmax) then
          ncount = i0
          if (isw==1) then
            if (ijd==15 .and.ifl==i0) go to 190
            isw = 2; jmax = iamax; go to 120
          endif
        else
          go to 120
        endif
      endif

! . . Standardruecksprung
  190 k = k + 1
      if (k<=kmax) go to 100
```

```
!.....Aphelposition der Merkurbahn fuer Konstellation 13 bzw. 14
!     (Pyramidenpos./Aphel) sowie "quick start option" 322 und 323
      if (ipla==1) call aphelko(imod,ivers,iaph,ipla, &
         ison,ijd,io,iop0,ix,rp(3,4),x,y,rcm,dmi)

!-----Ende der 1. Hauptschleife (Pyramiden- und Kammerpositionen)-----
      go to 900

!===================================================================
!------------------------ 2. Hauptschleife -------------------------
!===================================================================

!-----2. Hauptschleife (freier Zeitpunkt und Minimierung von Fpos-----
!     fuer Pyramiden- und Kammeranordnung, Tabelle 51 in "Pyramiden
!     und Planeten" und Tabelle 20 (?) im zweiten Buch)
  200 zjde = zjdemin
      dfe = 0.3d0; eep = e(1); irestart = i0; x36 = z0
!     VORSICHT: "zfact" und "zstep" nicht zu gross waehlen. Sonst ge-
!     hen beim Ruecksprung (s.u.) Konstellationen verloren. Standard-
!     werte fuer Pyramiden: 0.5/ 1.0 und fuer die Kammern: 0.1/ 0.2
      if (ipla==1) then
        zfact = 0.5d0; zstep = 1.d0
      else
!       (optimiert fuer alle Kammerzuordnungen)
        zfact = 0.1d0; zstep = 0.2d0
      endif

!.....Startparameter fuer "fitmin"
  220 ifitrun = i0; itin = i0
      imodus = 1; iflag = i0
      ke = 1; indx = 1; nu = i0
      ddx1 = 1.d0; ddx2 = 1.d0
      do i=1,10; test(i) = z0; enddo
      do i=1,5; xx(i) = z0; yy(i) = z0; enddo
      xx(1) = zjde; go to 250
  240 call ephim(1,iaph,ipla,ical,ak,iak,zjde,zjahr,delt)
  250 tau = (zjde - zjd0)/tmil
      if (ison==5) then
        do i=1,4; iw(i) = iw0(i); enddo
        do i=1,3; w(i) = w0(i); enddo
        do i=1,7; x(i) = x0(i); enddo
        do i=4,6; x(i) = x(i) * pidg; enddo
      endif
      inum(1) = inum(1) + 1

!.....Variante 1 (VSOP87D, Kurzversion aus "Meeus", multiple threads)
      if (imod==1) then
!$omp   parallel do shared(tau,re) private(i,resu)
        do i=1,9; call vsop1(i,tau,resu); re(i) = resu; enddo
!$omp   end parallel do
      endif

!.....Variante 2 (VSOP87A/C, Vollversion)
      if (imod==2) then
        do i=1,3
          ii = 3*(i-1)
          call vsop2(zjde,ivers,i,md,ix,prec,lu,r,ierr,rku)
          do j=1,3; re(ii+j) = rku(j); enddo
        enddo
      endif
```



```
!.....Koordinaten-Transformation und Bestimmung von F-pos
      call kartko(ison)
      call relpos(ipla,ison,ijd,iek,iekk,ika)
      if (ison==5) yy(indx) = xyr(36)

! . . zjde so lange erhoehen, bis relativer Fehler nicht mehr steigt.
!c    write(6,'('' zjde,irestart,xyr(36),dwi,imod = '',f18.7,i3, &
!c      & 2f9.3,i3)') zjde,irestart,xyr(36),dwi,imod
      if (xyr(36)>10.d0) imod = 1
      if (irestart==1) then
        if (xyr(36)>x36) then
          go to 290
        else
          zjdelim = zjde
        endif
      endif
      irestart = i0

! . . Bedingung zum Aufruf von fitmin pruefen
      if (xyr(36)>dwi.and.ifitrun==i0) go to 290
      if (ikomb==1) imod = 2

! . . Minimierung des relativen Fehlers F-pos mit "fitmin"
      ifitrun = 1; imodus = 1
      if (ddx1<dfe.or.ddx2<dfe) imodus = 2
      call fitmin(imod,imodus,iaph,ke,xx,yy,eep,step,nu,iflag, &
             ddx1,ddx2,test,itin,indx,ix)
      zjde = xx(indx)
      if (ke==1) go to 240
      irestart = 1

! . . verhindert, dass fitmin endlos ins vorherige Minimum faellt
      if (dabs(zjde-zjdevor)<=0.1d0) then
        zjde = zjdelim; go to 290
      endif
      zjdevor = zjde

!.....Hauptbedingung pruefen (ison = 5). . . . . . . . . . . . . . .
      if (xyr(36)>=dwikomb) go to 290
      inum(2) = inum(2) + 1

! . . Sonnenposition und Output
      call ephim(1,iaph,ipla,ical,ak,iak,zjde,zjahr,delt)
      call konst(iak,kon)
      call sonpos(ison,iek,ix,rp(3,1),rp(3,2),rp(3,3), &
                rcm,dmi,iter,iw,ke,mfit,nfit,f,x,e,w,y,z)
      dd = dn; if (iek==2 .or.iekk==2) dd = ds
      xma = xyr(35) * 1.d-9
      if (ipla==1) xma = xyr(35) * 1.d-7
      call reduz(x(5),1,i0)
      do iu=ix,6,5
        if (iout==3) then
          if (ipla==1) then
            write(iu,405)kon,iak,zjahr,delt,x(5)*gdpi,xma, &
              (xyr(30+i),i=1,3),dd,xyr(36)
          else
            write(iu,406)kon,iak,zjahr,delt,x(5)*gdpi,xma, &
              (xyr(30+i),i=1,3),dd,xyr(36)
          endif
        else
          if (ipla==1) then
```

```
              write(iu,407)kon,iak,zjde,zjahr,ke,iw(3), &
                (xyr(30+i),i=1,4),dd,xyr(36)
            else
              write(iu,408)kon,iak,zjde,zjahr,ke,iw(3), &
                (xyr(30+i),i=1,4),dd,xyr(36)
            endif
          endif
        enddo
!h      call histogramm(xyr(36),ihis)  !h

! . . Standardruecksprung
  290 zjump = xyr(36)*zfact + zstep
      zjde = zjde + zjump
      x36 = xyr(36)
      if (zjde<=zjdemax) go to 220

!-----Ende der 2. Hauptschleife (freier Zeitpunkt)--------------------
      go to 900

!====================================================================
!------------------------- 3. Hauptschleife -------------------------
!====================================================================

!-----3. Hauptschleife (Suche von Linearkonstellationen)-------------
!     Syzygium von Sonne, Merkur, Venus, Erde und Mars,
!     sowie Bestimmung der Transite von Merkur und Venus.

!     "zfact" und "zstep" wie in 2. Hauptschleife (nicht zu gross)
  300 zfact = 0.025d0 * (1.d0 + (21.d0-dwi)/20.d0)
      if (dwi>=21.d0) zfact = 0.025d0
      zstep = 0.01d0
      sz = (1.d0 + 10.d0*zfact)
      iabsatz = 3; if (iop0==21) iabsatz = 2 ! --> Leerzeile
      zjde = zjdemin; dfd = 5.d0; dfe = 0.5d0
      izp = 1; icv = 0
  310 zjdestep = zjde
      if (ilin==2 .and.inum(0)>1 .and.iop0/=-804) dfd = 0.02d0
      call ephim(1,iaph,ipla,ical,ak,iak,zjde,zjahr,delt)
      ik = idnint(ak)
      inum(0) = inum(0) + 1
      if (ilin>=3) itransit = i0
      do i=1,2; tra(i) = ' '; enddo
      if (ison==5) ifitrun = i0
      if (ilin<=2) ifitrun = 1
!.....Startparameter fuer "fitmin", "sekante" und "ringfit"
  320 if (ison==5) then
        iflag = i0; ke = 1; indx = 1; nu = i0
        ddx1 = dfd; ddx2 = ddx1; itin = i0
        do i=1,10; test(i) = z0; enddo
        do i=1,5
          xx(i) = z0; yy(i) = z0
        enddo
        xx(1) = zjde
      endif
      go to 340
  330 zjde = xx(indx)
      call ephim(1,iaph,ipla,ical,ak,iak,zjde,zjahr,delt)
  340 time = (zjde - zjd0)/tcen
      tau  = (zjde - zjd0)/tmil
      inum(1) = inum(1) + 1
```



```
!.....Variante 1 (VSOP87D, Kurzversion aus "Meeus", multiple threads)
      if (imod==1) then
!$omp   parallel do shared(tau,re) private(i,resu)
        do i=1,12; call vsop1(i,tau,resu); re(i) = resu; enddo
!$omp   end parallel do
        if (ilin<=2) then
          call kartko(ison)
          do i=1,9; rk(i) = xyr(i); enddo
        endif
      endif

!.....Variante 2 (VSOP87A/C, Vollversion)
  350 if (imod==2) then
        do i=il(1),il(2),il(3); ii = 3*(i-1)
          call vsop2(zjde,ivers,i,md,ix,prec,lu,r,ierr,rku)
          do j=1,3
            re(ii+j) = rku(j)
            if (ilin<=2) rk(ii+j) = r(j)
          enddo
        enddo
      endif

!.....Variante 3 (Keplersche Gleichung, Polyn. 3. Grades nach VSOP82)
      if (imod==3) then
        do i=1,4
          ii = 6*i
          call vsop3(lv,i,ix,ir,time,res)
          if (ir/=i0) go to 1000
          re(25+ii) = res(1);  re(28+ii) = res(5)
          re(26+ii) = res(2);  re(29+ii) = res(4)
          re(27+ii) = res(3);  re(30+ii) = res(6)
          if (i<=4) re(3*i-2) = res(11)
        enddo
      endif

!.....Korrelation der Positionen pruefen
      ic  = i0
      iwo = i0
      df(1) = re(1)-re(4);   df(2) = re(1)-re(7)
      df(3) = re(1)-re(10);  df(4) = re(4)-re(7)
      df(5) = re(4)-re(10);  df(6) = re(7)-re(10)
      do i=1,6; call reduz(df(i),i0,i0); enddo
      if (ilin==3) difm = dmax1(dabs(df(1)),dabs(df(2)),dabs(df(4)))
      if (ilin==4) difm = dmax1(dabs(df(1)),dabs(df(2)),dabs(df(3)), &
          dabs(df(4)),dabs(df(5)),dabs(df(6)))
      if (isep==1) then
        if (itransit==1) difm = df(2)
        if (itransit==2) difm = df(4)
      else
        if (itransit==1 .or.itransit==2) then
          call sepa(itransit,2,rk,sep1)
          difm = dabs(sep1)
        endif
      endif
      if (ison==5) yy(indx) = difm
! . . Test-Ausdruck (--> !t)
!t    difr = re(7)-re(1)
!t    call reduz(difr,i0,i0)
!t    do iu=ix,6,5;write(iu,'(''imod,ifit,dt,Le-Lm,jde,difm = '',2i2,&
!t     &f5.1,f6.1,f18.7,f13.7)')imod,ifitrun,step,difr,zjde,difm;enddo
```

```
!.....Hauptbedingung pruefen . . . . . . . . . . . . . . . . . . . . . . .
      if (difm>=dwi.and.ifitrun/=1) go to 370
! . . Ruecksprung fuer ikomb = 1
      if (ikomb==1 .and.imod==1 .and.ilin>=3) then
        ifitrun = 1; imod = 2; dwi = dwikomb
        go to 350
      endif

! . . Minimierung des Gesamtwinkels difm mit "fitmin" fuer ison = 5
!     (Das heisst, "ison" hat hier eine andere Funktion und bedeutet
!     Minimumsuche.)
      if (ison==5) then
        ifitrun = 1; step = 1.d0
        if (ilin>=3 .and.itransit==i0) then
          call fitmin(imod,1,iaph,ke,xx,yy,e(1),step,nu, &
          iflag,ddx1,ddx2,test,itin,indx,ix); zjde = xx(indx)
        endif
        if (itransit==1 .or.itransit==2) then
          if (isep==1) then
            xj2 = xx(indx); yy2 = yy(indx); indx = 2
            call ringfit(xj1,xj2,xj3,yy1,yy2,yy3, &
              1.d-6,1.d-2,nu,50,ix,ke)
            xx(2) = xj2; zjde = xj2
          else
            eep = e(1)
            if (ikomb==1 .and.imod==1 .and.isep>=3) eep=1.d2*e(1)
            imodus = 1
            if (ddx1<dfe.or.ddx2<dfe) imodus = 2
             call fitmin(imod,imodus,iaph,ke,xx,yy,eep,dfd,nu, &
               iflag,ddx1,ddx2,test,itin,indx,ix)
               zjde = xx(indx)
          endif
        endif
        if (ke==1 .or.(isep==1 .and.ke==5)) go to 330
      endif

! . . Spezialtest fuer ikomb = 0 (imod = 1, 3)
!     Anmerkung: Aufgrund der Zeitschritte (1 Tag) ist es moeglich,
!     dass das Minimum des Winkelintervalls (difm) fuer die eklipti-
!     kalen Laengen der Planeten genau zwischen zwei Zeitpunkten er-
!     reicht wird. Falls die Schwelle (dwi0) so knapp unterschritten
!     wird, dass sie an den Zeitpunkten davor und danach schon wieder
!     ueberschritten wird, wuerde das Ereignis verloren gehen. Des-
!     halb wird die Schwelle (dwi) zuvor um 1 Grad erhoeht, dann das
!     Winkelintervall minimiert und anschliessend geprueft, ob die
!     urspruengliche Schwelle (dwi0) unterschritten wurde.
      if (ikomb==i0.and.ilin>=3) then
        if (difm<dwi0) go to 360
        go to 370
      endif
! . . Gegebenenfalls Sprung von der oberen zur unteren Konjunktion.
!     Bei Minimierung der Winkelseparation (isep 2,3,4) wuerden ab
!     einem gewissen Zeitpunkt nur noch obere Konjunktionen berech-
!     net werden. Das wird durch die folgende if-Abfrage behoben.
  360 if (isep>=2 .and.((itransit==1 .and.dabs(df(2))>170.d0) &
                  .or.(itransit==2 .and.dabs(df(4))>170.d0))) then
          zjde = zjde + tsy*0.5d0
          go to 320
      endif
      if (ikomb/=1 .or.(ikomb==1 .and.(difm<dwikomb.or. &
          ilin<=2))) then
```



```
        if (itransit==i0.and.nurtr==1) inum(2) = inum(2) + 1
        ic = 1
        if (ic==1 .and.icv==0 .and.ison/=5 .and.ilin>=3) then
          inum(3) = inum(3) + 1
          do iu=ix,6,5
            write(iu,'(i12,''. syzygy'')') inum(3)
          enddo
        endif
        call konst(ik,kon)
! . . . Pruefen des Transits (nur bei imod = 1, 2)
        if (itran==1 .and.ison==5) then
          if (itransit==i0.or.ilin<=2) call memo(zjde,zjahr, &
              delt,df(1),df(2),df(3),difm,zmem,iak,imem)
          if (itransit==1 .or.itransit==2) then
            call transit(itransit,ikomb,imod,ipla,ilin,iaph,ivers, &
             isep,ical,iuniv,tr,sep1,itt,sep,zjde,id5,da5,dmo5, &
             zjahr,rk,md,ddx1,ddx2,dfd,test,itin,is,irs,ix,pan,sd,sl,&
             iop0,inum)
            tra(itransit) = tr
          endif
! . . . . Ereignis evtl. mit Transit, Output (ohne Transit bei imod=3)
          if ((ilin>=3 .and.itransit==2).or. &
              (ilin<=2 .and.tr/=' ').or.imod==3) then
            if (ikomb==1 .and.imod==1 .and.ilin<=2) then
              imod = 2; go to 320
            endif
            if (nurtr==1 .or.(nurtr==2 .and. &
              (tra(1)/=' '.or.tra(2)/=' '))) then
              if (ilin<=2 .or.nurtr==2) inum(2) = inum(2) + 1
              iwo = 1
              if (ilin>=3) then
                do iu=ix,6,5
                  if (dabs(zmem(5))<1.d-4) then
                    zmem(5) = dabs(zmem(5))
                    write(iu,456)kon,' ',tra(1),tra(2),imem, &
                      (zmem(i),i=1,7)
                  elseif (dabs(zmem(6))<1.d-4) then
                    zmem(6) = dabs(zmem(6))
                    write(iu,457)kon,' ',tra(1),tra(2),imem, &
                      (zmem(i),i=1,7)
                  else
                    write(iu,455)kon,' ',tra(1),tra(2),imem, &
                      (zmem(i),i=1,7)
                  endif
                enddo
              else
                if (iop0==-804 .and.(zjahr<=-13000.d0 .or. &
                    zjahr>=17000.d0)) go to 390; ts = ' '
                if (tra(ilin)/='M'.and.tra(ilin)/='V') ts=tra(ilin)
                if (iuniv==2) call delta_T(zjde)
                call jdedate(zjde,ical,ida,da,dmo)
                if (ida(3)>=izmin) then
                 do iu=ix,6,5
                   if (isep==4 .and.((ilin==2 .and.lid5/=-50000 .and.&
                     id5(3,3)-lid5>50).or.(ilin==1 .and.mod(inum(2) +&
                      iabsatz,4)==0))) write(iu,*) ! --> Leerzeile
                   if (izp<=3) call zwizeile(iu,io,zmem(1), &
                     ilin,imod,isep,ical,izp)
                   if ((isep<=3 .and. zmem(1)<=-1566122.5d0).or. &
                      (isep==4 .and.(zmem(1)<=-1931365.0d0 .or. &
                                     zmem(1)>= 5373485.0d0))) then
```

```
            if (isep<=2) then
              write(iu,458)kon,ts,imem,da(7),dmo,ida(3), &
              (ida(i),dp,i=4,5),ida(6),(zmem(i),i=3,6),sep,irs
            else
             if (isep==3) then
              if (itt==3) &
               write(iu,459)kon,ts,da5(3,7),dmo5(3),id5(3,3),&
               ((id5(l,i),dp,i=4,5),id5(l,6),l=1,5),sep,sl,irs
              if (itt==2) &
               write(iu,461)kon,ts,da5(3,7),dmo5(3),id5(3,3),&
               ((id5(l,i),dp,i=4,5),id5(l,6),str2,l=1,3,2), &
               (id5(5,i),dp,i=4,5),id5(5,6),sep,sl,irs
              if (itt==1) &
               write(iu,471)kon,ts,da5(3,7),dmo5(3),id5(3,3),&
               str2,str2,(id5(3,i),dp,i=4,5),id5(3,6), &
               str2,str2,sep,sl,irs
             else
              if (itt==3) &
               write(iu,659)kon,ts,da5(3,7),dmo5(3),id5(3,3),&
               ((id5(l,i),dp,i=4,5),id5(l,6),l=1,5),sep,sl, &
               (pan(i),i=1,5),sd(1),sd(2),irs
              if (itt==2) &
               write(iu,661)kon,ts,da5(3,7),dmo5(3),id5(3,3),&
               ((id5(l,i),dp,i=4,5),id5(l,6),str2,l=1,3,2), &
               (id5(5,i),dp,i=4,5),id5(5,6),sep,sl,pan(1), &
               str3,pan(3),str3,pan(5),sd(1),sd(2),irs
              if (itt==1) &
               write(iu,671)kon,ts,da5(3,7),dmo5(3),id5(3,3),&
               str2,str2,(id5(3,i),dp,i=4,5),id5(3,6), &
               str2,str2,sep,sl,str3,str3,pan(3), &
               str3,str3,sd(1),sd(2),irs
             endif
             if (itt==i0.and.iu==6) inum(2) = inum(2) - 1
            endif
           else
            if (isep<=2) then
              write(iu,558)kon,ts,imem,da(7),dmo,ida(3), &
              (ida(i),dp,i=4,5),ida(6),(zmem(i),i=3,6),sep,irs
            else
             if (isep==3) then
              if (itt==3) &
               write(iu,559)kon,ts,da5(3,7),dmo5(3),id5(3,3),&
               ((id5(l,i),dp,i=4,5),id5(l,6),l=1,5),sep,sl,irs
              if (itt==2) &
               write(iu,561)kon,ts,da5(3,7),dmo5(3),id5(3,3),&
               ((id5(l,i),dp,i=4,5),id5(l,6),str2,l=1,3,2), &
               (id5(5,i),dp,i=4,5),id5(5,6),sep,sl,irs
              if (itt==1) &
               write(iu,571)kon,ts,da5(3,7),dmo5(3),id5(3,3),&
               str2,str2,(id5(3,i),dp,i=4,5),id5(3,6), &
               str2,str2,sep,sl,irs
             else
              if (itt==3) &
               write(iu,759)kon,ts,da5(3,7),dmo5(3),id5(3,3),&
               ((id5(l,i),dp,i=4,5),id5(l,6),l=1,5),sep,sl, &
               (pan(i),i=1,5),sd(1),sd(2),irs
              if (itt==2) &
               write(iu,761)kon,ts,da5(3,7),dmo5(3),id5(3,3),&
               ((id5(l,i),dp,i=4,5),id5(l,6),str2,l=1,3,2), &
               (id5(5,i),dp,i=4,5),id5(5,6),sep,sl,pan(1), &
               str3,pan(3),str3,pan(5),sd(1),sd(2),irs
```



```
              if (itt==1) &
               write(iu,771)kon,ts,da5(3,7),dmo5(3),id5(3,3),&
               str2,str2,(id5(3,i),dp,i=4,5),id5(3,6), &
               str2,str2,sep,sl,str3,str3,pan(3), &
               str3,str3,sd(1),sd(2),irs
             endif
             if (itt==i0.and.iu==6) inum(2) = inum(2) - 1
            endif
           endif
           if (isep<=2 .and.iu==6) then
            if (ts=='m'.or.ts=='v') inum(3) = inum(3) + 1
            if (ts=='C'.or.ts=='c') inum(4) = inum(4) + 1
           endif
          enddo
         else
           ic = i0; iwo = i0; inum(2) = inum(2) - 1
         endif
         lid5 = id5(3,3) ! --> Leerzeile
        endif
       endif
      endif
      if (itransit==i0.or.ilin<=2) zjde0 = zjde
!t    read(*,*)  !t
! . . . Ereignis ohne Transit-Pruefung (z.B. imod = 3), Output
     else
      do iu=ix,6,5
       if (dabs(df(2))<1.d-4) then
        write(iu,456)kon,' ',tra(1),tra(2),ik, &
         zjde,zjahr,delt,(df(i),i=1,3),difm
       elseif (dabs(df(3))<1.d-4) then
        write(iu,457)kon,' ',tra(1),tra(2),ik, &
         zjde,zjahr,delt,(df(i),i=1,3),difm
       else
        write(iu,455)kon,' ',tra(1),tra(2),ik, &
         zjde,zjahr,delt,(df(i),i=1,3),difm
       endif
      enddo
      call memo(zjde,zjahr,delt,df(1),df(2),df(3),difm,zmem, &
       iak,imem)
     endif
    endif
! . . Ruecksprung fuer Transit-Pruefung
 370 if (itran==1 .and.ison==5 .and.ilin>=3) then
     if (itransit/=i0) zjde = zjde0
     if (ison==5 .and.ic==1 .and.ilin>=3 .and.imod/=3) &
       itransit = itransit + 1
     if (itransit==1 .or.itransit==2) go to 320
    endif

! . . Bedingung fuer Zeitsprung zur Verkuerzung der Rechenzeit
    if (ilin>=3 .and.dwin<=21.d0) then
     iflag2 = iflag1; iflag1=i0
     if (dabs(df(4))<=dwin) iflag1=1
    endif; ifitrun = i0

! . . Weiterer Output
    do iu=ix,6,5
     if (((ilin<=2 .and.(tra(1)/=' '.or.tra(2)/=' ').and. &
        ((isep<=2 .or.(isep>=3 .and.itt/=0)).and. &
        ida(3)>=izmin)).or.(ic==1 .and.ilin>=3)).and. &
       io==2 .and.iwo==1) then
```

```
            if (imod/=3) then
              call linie(iu,2+ipar); write(iu,57) (zmem(i),i=11,19)
              write(iu,57) (zmem(i),i=20,22); call linie(iu,2)
            endif
            if (ic==1 .and.imod==3 .and.io==2) call linie(iu,2)
            immin = 1; if (imod==3) immin = 5
            immax = 4; if (ilin>=3) immax = 8
            if (immin<=immax) then
              do i=immin,immax; ii = 6*i
                call vsop3(lv,i,ix,ir,time,res); if (ir/=i0) go to 1000
                zmem(25+ii) = res(1);  zmem(28+ii) = res(5)
                zmem(26+ii) = res(2);  zmem(29+ii) = res(4)
                zmem(27+ii) = res(3);  zmem(30+ii) = res(6)
              enddo
            endif
            if (ilin<=2) then
              if (ivers==3) then
                write(iu,'(''  ascending node (M/V/E/Ma): '',2f12.6, &
                 & ''      ---   '',f12.6)')zmem(34),zmem(40),zmem(52)
              else
                write(iu,'(''  ascending node (M/V/E/Ma): '',4f12.6)') &
                  (zmem(28+6*i),i=1,4)
              endif
              write(iu,'(''  inclination i  (M/V/E/Ma): '',4f12.6)') &
                (zmem(29+6*i),i=1,4)
              write(iu,'(''  perihelion pi  (M/V/E/Ma): '',4f12.6)') &
                (zmem(30+6*i),i=1,4)
            else
              do i=31,78; re(i) = zmem(i); enddo
              call elements(iu,ivers,pla)
            endif
            call linie(iu,1+ipar)
          endif
        enddo
    390 if (ikomb==1 .and.imod==2) then; imod = 1; dwi = dwi0; endif

  ! . . Bedingter groesserer Zeitsprung
        if (ilin<=2 .or.(dwin<=21.d0 .and.((iflag2==1 .and.iflag1==i0) &
          .or.(ison==5 .and.ifitrun==i0.and.(ke==i0.or.ke==3))))) then
          zjde = zjde + tsprung; iflag1 = i0
        else
          zjde = zjdestep
          if (ison==5 .or.(ison/=5 .and.dabs(difm)>dwin*sz)) then
            stepl = difm*zfact + zstep; if (ic==1) stepl = 0.9d0*ymer
            zjde = zjde + stepl
          else
            zjde = zjde + step
          endif
        endif
        icv = ic
        if (zjde<=zjdemax) go to 310
  ! . . Ergaenzung (Tabellenkopf fuer Transit-Test mit inum(2)=0)
        if (ilin<=2 .and.inum(2)==0) then
          do iu=ix,6,5
            call zwizeile(iu,io,zmem(1),ilin,imod,isep,ical,izp)
          enddo
        endif

  !-----Ende der 3. Hauptschleife (Linearkonstellation, Transit)--------
        go to 900
```



```
  !====================================================================
  !------------------------- 4. Hauptschleife -------------------------
  !====================================================================

  !-----4. Hauptschleife (Teotihuacan)---------------------------------
  !     Wallabstaende auf der "Strasse der Toten", in Google Maps
  !     linear gemessen bzw. ueber die geographischen Koordinaten
  !     (GPS) berechnet und Vergleich mit den Logarithmen der Pla-
  !     netenabstaende. "Teot" enthaelt alle Ortsdaten und "comp"
  !     nur die Daten fuer die lineare Regression.
  !
  !     Array "teot(0..17,i)" (Index 0..17: Reihenfolge in "inteoti.t")
  !     i=1:  geogr. Breite (Wallposition)
  !     i=2:  geogr. Laenge (     "      )
  !     i=3:  Wallabstaende [m] (vorab eingegeben oder berechnet)
  !     i=4:  Wallabstaende von Karte oder Bildschirm in Millimetern
  !
  !     Array "comp(0..9,i)" (von "compare")
  !     i=1:  ausgewaehlte Wallabstaende (vorgegeben oder berechnet)
  !     i=2:  Periheldistanz   (bzw. Sonnenradius)
  !     i=3:  Grosse Halbachse ( "         "      )
  !     i=4:  Apheldistanz     ( "         "      )

  !-----Wallabstaende berechnet in Metern aus den GPS-Koordinaten
  !     (ilin = 1), vorgegeben in Metern (ilin = 2) oder gemessen von
  !     Karte/Bildschirm z.B. in Millimetern (ilin = 3)
    800 continue
        if (ilin==1) then  ! Distanzen gemaess GPS-Koordinaten
          hcorr = (6375.726d0+2.304d0)/6375.726d0 !Hoehenkorrekturfaktor
          do i=0,17
            if (i/=4 .and.i<=10) then
              call distance(0,i,dis)
            elseif (i==4) then
              call distance(0,2,dis1)
              call distance(2,4,dis2); dis = dis1 + dis2
            elseif (i==11) then
              call distance(0,14,dis1)
              call distance(11,14,dis2); dis = dis1 - dis2
            elseif (i==12) then
              call distance(12,14,dis3); dis = dis1 - dis3
            elseif (i==13) then
              call distance(13,14,dis4); dis = dis1 - dis4
            elseif (i==14) then; dis = dis1
            elseif (i==15) then; dis = dis1 + dis4
            elseif (i==16) then; dis = dis1 + dis3
            elseif (i==17) then; dis = dis1 + dis2
            endif
            teot(i,3) = dis * hcorr ! mit linearer Hoehenkorrektur
          enddo
        endif
        kk = 4; if (ilin<=2) kk = 3; comp(0,1) = teot(0,kk)
        do i=1,4; comp(i,1) = teot(i+1,kk); enddo
        do i=5,8; comp(i,1) = teot(i+2,kk); enddo

  !-----Program output 1: Ausdruck der Eingabe-Daten
        lbase(4) = dwi
        do iu=ix,6,5
          write(iu,820)'Planetary Correlation', &
            & 'of the Pyramids at Teotihuacan','< P5-option',iop0,' >'
          write(iu,'(4x,a13//4x,a8,17x,a27,a5,a15)') '1. INPUT DATA', &
            & 'Position',' GPS lat.     GPS long.    ',tdi(ilin),  &
```

```
            & ' [m]     d [mm]'; call linie(iu,1)
          do i=0,14; write(iu,815) tname(i),teot(i,1),q(i),teot(i,2), &
            q(i),teot(i,3),st(i),teot(i,4),st(i)
            if (i==10) call linie(iu,2); enddo
          do i=15,17; write(iu,816) tname(i),str4,str4,teot(i,3),st(i),&
            & teot(i,4),st(i); enddo; call linie(iu,1)
          write(iu,'(40x,a38)') '(* pyramid/temple position - off-axis)'
          write(iu,'(40x,a38)') '(+ sum or difference of two distances)'
          if (ilin==1) &
           write(iu,'(40x,a38)')'(Data in column "GPS" are GPS-results)'

!-----Program output 2: Tabellenkopf
          write(iu,'(/4x,a18//4x,a25,3x,a27/4x,a25,3x,a11)') &
            & '2. CALCULATED DATA', &
            & 'Teotihuacan, length unit:',tluna(isep), &
            & 'astronomical length unit:',trsun(iuniv)
          if (lbase(ical)>=9.99995d0) then
            write(iu,'(4x,a25,f10.4)') &
              & 'logarithmic base (astr.):',lbase(ical)
          else
            write(iu,'(4x,a25,f9.4)') &
              & 'logarithmic base (astr.):',lbase(ical)
          endif
          if (io==1) then
            write(iu,'(/26x,18(''-''),a11,1x,18(''-''))')di(ilin)
            write(iu,*)'       Julian year       (per. dist', &
              & 'ance)        (a)       (aph. distance)'
            call linie(iu,1)
          else
            write(iu,'(/4x,a4,9x,a14,7x,a40)')'Body',di2(ilin,isep), &
                di3(iuniv)
          endif
        enddo

!-----Spezielle Laengeneinheit (Distanz vom Zentrum der "Mondpyramide"
!     zur Mitte des "Plaza de la Luna")
        if (isep==2) then
          do i=0,8; comp(i,1) = comp(i,1)/teot(1,kk);enddo
          teot(6,kk) = teot(6,kk)/teot(1,kk)
        endif

!-----Bahnelemente der Planeten (VSOP3) und Logarithmieren
        xlog = dlog10(lbase(ical))
        do i=2,4  ! (Sonne)
          comp(0,i) = dlog10(R0*0.001d0)/xlog
          if (iuniv==2) comp(0,i) = 0.d0
        enddo
        time = zmin*0.01d0
  810   do i=1,8  ! (Planeten)
          call vsop3(lv,i,ix,ir,time-20.d0,res); if (ir/=i0) go to 1000
          if (iuniv==2) res(2) = res(2)/(R0*0.001d0) ! spezielle Einheit
          comp(i,3) = dlog10(res(2)*AE*0.001d0)/xlog
          comp(i,2) = comp(i,3) + dlog10(1.d0-res(3))/xlog
          comp(i,4) = comp(i,3) + dlog10(1.d0+res(3))/xlog
        enddo

!-----Berechnung fuer Periheldistanz, gr. Halbachse u. Apheldistanz
        do i=1,3
! . . . Bestimmtheitsmass (R^2)
          call rcoef2(i,9,bmas)
```



```
! . . . Lineare Regression --> Steigung a und Ordinatenabschnitt b
          call lintrend(i,9,alin(i),blin(i))

! . . . Distanz des hypothetischen Planeten "Phaeton" (logarithmisch)
          phdis(i) = alin(i)*teot(6,kk) + blin(i)
        enddo

!-----Program output 2: Berechnete Daten
!     (Die drei Distanzen "aphel, a und perihel" gelten alternativ
!     und enthalten den fiktiven Planeten "Phaeton" zwischen Mars
!     und Jupiter.)
        do iu=ix,6,5
         if (io==1) then
           write(iu,850) time*100.d0,(bmas(1,i),i=1,3)
         else
           call linie(iu,1)
           if (isep==1) then
             do i=0,8; write(iu,835) plan(i),(comp(i,j),j=1,4); enddo
             call linie(iu,2); write(iu,'(2x,a10,f15.2,6x,3f14.4)') &
               & ' (Phaeton)',teot(6,kk),phdis
           else
             do i=0,8; write(iu,830) plan(i),(comp(i,j),j=1,4); enddo
             call linie(iu,2); write(iu,'(2x,a10,f16.4,5x,3f14.4)') &
               & ' (Phaeton)',teot(6,kk),phdis
           endif
           call linie(iu,2)
           write(iu,'(2x,a23,9x,a2,f13.8,2f14.8)') &
                    '  linear fit, f(x)=ux+v','u:',alin
           write(iu,'(34x,a2,f13.8,2f14.8)')'v:',blin
           call linie(iu,2)
           write(iu,840)'Julian year:',time*100.d0,'R^2:', &
             (bmas(1,i),i=1,3)
           write(iu,841)'adj. R^2:',(bmas(2,i),i=1,3)
         endif
        enddo

!-----Ruecksprung
        if (step>0.d0) then
          time = time + step*0.01d0
          if (time<=zmax*0.01d0+1.d-8) go to 810
        endif

!-----Ende der 4. Hauptschleife (Teotihuacan)------------------------

!==================================================================
!--------------------- Ende der Hauptschleifen ---------------------
!==================================================================

  900 do iu=ix,6,5; if (io/=2) call linie(iu,1+ipar); enddo

! . . Ruecksprung bei Option -804 und Speichern von "inser-2.t"
        if (iop0==-804) then
          if (ilin==1) then
            ilin = 2
            zmin = -30000.d0
            zmax =  30000.d0
            go to 10
          endif
          call save_ser
        endif
```

```
!-----Endzeilen
      call cpu_time(zib)
      call date_and_time(zdate,ztime,zzone,iw2)
      call comtime(1,zia,zib,iw1,iw2,ihour,imin,sec)
      call comtime(2,zia,zib,iw1,iw2,ihour2,imin2,sec2)
      do iu=ix,6,5
        call endzeile(ipla,imod,ilin,iaph,isep,ison,ijd,ipos,io, &
          iu,inum,ihour,imin,sec,ihour2,imin2,sec2,is12,iop0)
!h      if (ipla<=2.and.imod<=2.and.ison>=3) then                 !h
!h        write(iu,'(7x,a24,a33)') 'Frequency of deviations ', &  !h
!h          & ' Fpos(0 to 5%) in steps of 0.05%:'                 !h
!h        call linie(iu,1)                                        !h
!h        do i=0,4;write(iu,'(2(3x,10i3))') (ihis(j+i*20),j=1,20) !h
!h        enddo; call linie(iu,1); write(iu,*); endif             !h
        close(iu)
      enddo
 1000 continue

!-----Ende des Hauptprogramms-----------------------------------------
      stop
   54 format(1x,3f9.6,3(a1,f7.6),3f9.6)
   55 format(1x,a2,i7,f14.5,f10.3,f8.3,4f8.3,f6.1)
   56 format(1x,a2,i7,f15.5,f11.3,f9.3,4f8.3,a2)
   57 format(1x,3(f9.4,f8.4,f9.6))
   65 format(1x,a2,i7,f10.3,3f8.3,3f7.1,f5.1,a2,f7.3)
   67 format(1x,a2,i7,f10.3,3f8.3,2f7.1,a7,f5.1,a2,f7.3)
   85 format(1x,a2,i7,f10.3,3f8.3,3f7.2,f5.2,a2,f7.3)
  165 format(1x,a2,i7,f10.3,2f8.3,i3,i4,3f7.1,f6.1,a2,f7.3)
  184 format(1x,a2,i7,f10.3,2f8.3,i3,i4,3f7.2,f6.2,a2,f7.3)
  255 format(1x,f14.5,f7.1,f7.2,f7.3,f7.2,3f7.1,f6.1,a2,f7.3)
  256 format(1x,a2,i7,f10.3,f8.2,f7.3,f7.2,4f7.1,a2,f7.3)
  275 format(1x,f14.5,f7.1,f7.2,f7.3,f7.2,3f7.2,f6.2,a2,f7.3)
  276 format(1x,a2,i7,f10.3,f8.2,f7.3,f8.2,3f7.2,f6.2,a2,f7.3)
  365 format(1x,a3,i8,f13.3,f12.0,i6,1x,3f7.1,f5.1,a2,f7.3)
  366 format(1x,a3,i8,f13.3,f12.0,i2,i4,3f7.1,f6.1,a2,f7.3)
  367 format(1x,a3,i8,f13.3,f12.0,i6,1x,2f7.1,a7,f5.1,a2,f7.3)
  384 format(1x,a3,i8,f13.3,f12.0,i6,1x,3f7.2,f5.2,a2,f7.3)
  386 format(1x,a3,i8,f13.3,f12.0,i2,i4,3f7.2,f6.2,a2,f7.3)
  405 format(1x,a2,i7,f11.3,f8.3,f9.3,f9.4,f8.1,2f7.1,1x,a2,f7.3)
  406 format(1x,a2,i7,f11.3,f8.3,f9.3,f9.4,f8.2,2f7.2,1x,a2,f7.3)
  407 format(1x,a2,i7,f15.5,f11.3,i3,i4,f8.1,2f7.1,f6.2,a2,f6.3)
  408 format(1x,a2,i7,f15.5,f11.3,i3,i4,f8.2,2f7.2,f6.2,a2,f6.3)
  455 format(1x,a2,3a1,i7,f15.5,f11.3,5f8.3)
  456 format(1x,a2,3a1,i7,f15.5,f11.3,2f8.3,f6.1,f10.3,f8.3)
  457 format(1x,a2,3a1,i7,f15.5,f11.3,3f8.3,f6.1,f10.3)
  458 format(1x,a2,a1,i7,f5.0,a5,i6,i3,2(a1,i2),4f8.3,f7.1,i5)
  459 format(1x,a2,a1,f4.0,a5,i6,i3,2(a1,i2),4(i4,2(a1,i2)),f7.1, &
             a1,i4)
  461 format(1x,a2,a1,f4.0,a5,i6,i3,2(a1,i2),2(a10,i4,2(a1,i2)), &
             f7.1,a1,i4)
  471 format(1x,a2,a1,f4.0,a5,i6,1x,a8,2x,a8,i4,2(a1,i2),2(2x,a8), &
             f7.1,a1,i4)
  558 format(1x,a2,a1,i7,f5.0,a5,i5,i4,2(a1,i2),4f8.3,f7.1,i4)
  559 format(1x,a2,a1,f4.0,a5,i5,5(i4,2(a1,i2)),f7.1,a1,i3)
  561 format(1x,a2,a1,f4.0,a5,i5,i4,2(a1,i2),2(a10,i4,2(a1,i2)), &
             f7.1,a1,i3)
  571 format(1x,a2,a1,f4.0,a5,i5,2a10,i4,2(a1,i2),2a10,f7.1,a1,i3)
  659 format(1x,a2,a1,f4.0,a5,i6,5(i4,a1,i2,a1,i2),f8.1,2x,a1, &
             2x,5f8.2,3x,2f8.2,i6)
  661 format(1x,a2,a1,f4.0,a5,i6,i4,2(a1,i2),2(a10,i4,2(a1,i2)), &
             f8.1,2x,a1,2x,f8.2,a8,f8.2,a8,f8.2,3x,2f8.2,i6)
```



```
  671 format(1x,a2,a1,f4.0,a5,i6,2a10,i4,2(a1,i2),2a10,f8.1,2x,a1, &
             2x,2a8,f8.2,2a8,3x,2f8.2,i6)
  759 format(1x,a2,a1,f4.0,a5,i5,1x,5(i4,a1,i2,a1,i2),f8.1,2x,a1, &
             2x,5f8.2,3x,2f8.2,i6)
  761 format(1x,a2,a1,f4.0,a5,i5,1x,i4,2(a1,i2),2(a10,i4,2(a1,i2)), &
             f8.1,2x,a1,2x,f8.2,a8,f8.2,a8,f8.2,3x,2f8.2,i6)
  771 format(1x,a2,a1,f4.0,a5,i5,1x,2a10,i4,2(a1,i2),2a10,f8.1,2x,a1,&
             2x,2a8,f8.2,2a8,3x,2f8.2,i6)
! . . Teotihuacan
  815 format(4x,a20,1x,f13.6,1x,a1,f12.6,1x,a1,f10.2,1x,a1,f9.1,1x,a1)
  816 format(4x,a20,7x,a5,9x,a5,2x,f12.2,a2,f9.1,a2)
  820 format(/30x,a21/25x,a30/32x,a11,i4,a2/)
  830 format(4x,a10,f14.4,5x,3f14.4)
  835 format(4x,a10,f13.2,6x,3f14.4)
  840 format(4x,a12,f11.2,5x,a4,f13.8,2f14.8)
  841 format(27x,a9,f13.8,2f14.8)
  850 format(5x,f13.2,4x,3f17.10)

! . . Giza: Ausgabe einer groesseren Stellenanzahl zur Feinabstimmung
!     bzw. Minimierung von F[%] fuer die Schnellstart-Optionen 4 u. 9.
!     Dies wurde verwendet fuer Buch 1.
!     Suche in der Umgebung des Merkur-Aphels bzw. Merkur-Perihels
!f255 format(1x,f14.5,f8.2,f7.2,f8.4,f6.1,3f7.1,f5.1,a2/65x,f14.8)  !f
!f275 format(1x,f14.5,f8.2,f7.2,f7.3,f7.2,3f7.2,f5.1,a2/65x,f14.8)  !f
      end program P5

      subroutine inputdata(ipla,ilin,imod,imo4,ikomb,io,lv,ivers, &
        itran,isep,iuniv,ical,ika,iaph,iamax,step,ison,ihi,irb,ijd, &
        zmin,zmax,ak,zjde1,dwi,dwikomb,dwi2,dwi3,nurtr,iek,iop0,iout)
!-----Inputdaten und Programmstart------------------------------------
      implicit double precision (a-h,o-z)
      character(36) :: com
      iy = 6; ipla = 1; itran = 1; io = 0; ire = 0; z0 = 0.d0
      write(iy,'(//29x,23(''-''))')
      write(iy,'(30x,''PLANETARY CORRELATION'')')
      write(iy,'(30x,''P5 Program, Aug. 2025'')')
      write(iy,'(29x,23(''-''))')

! . . Schnellstart-Menue
      write(iy,'(/4x,a13,6x,a17,5x,a15,5x,a11/1x,78a1/5(2x,2(a17,4x),&
      & a16,4x,a14/),1x,78a1)') &
      'Giza pyramids','Great P. chambers','transits syzygy', &
      'Teotihuacan', &
      ('-',i=1,78), &
     '3D Mer at aph (1)','3D Mer at per (6)','Mercury tr  (11)',&
     'GPS m km  (16)', &
     '2D Mer at aph (2)','Keplers equ   (7)','Venus tr    (12)',&
     'Map mm km (17)', &
     'constell 3088 (3)','constell 3088 (8)','syzygy 3 pl (13)',&
     'GPS log3  (18)', &
     '1.5 days 3088 (4)','1.5 days 3088 (9)','syzygy 4 pl (14)',&
     'Map log3  (19)', &
     'near aphelion (5)','F minimized  (10)','TYMT test   (15)',&
     '24000 y.  (20)', &
      ('-',i=1,78)
      do
        do
         write(iy,'(8x,a10,3x,a20,3x,a26)',advance='no')'info (111)',&
          'detailed options (0)','(1..20 or book options) : '
         read(*,*,iostat=iox) iop0
         if (iox==0) exit
```

```
          call emes(ire,com,dm)
         enddo
         iop=iop0
         if (iop==0) then; write(iy,*); go to 10; endif
         if (iop==111) then; call info; iout=4; return; endif

!  . . . Verborgene Optionen fuer Tabellen aus beiden oben genannten
!        Buechern, s.a. im Programmkopf unter "Neue Optionen, b)"
         if ((iop>=0 .and.iop<=22).or. &
!          1. "Pyramiden und Planeten", Tab. 39-51
           (iop>=390 .and.iop<=392).or.(iop>=400 .and.iop<=402).or. &
           (iop>=410 .and.iop<=432).or.(iop>=440 .and.iop<=442).or. &
           iop==450 .or.(iop>=460 .and.iop<=461).or.(iop>=470 .and. &
           iop<=471).or.(iop>=480 .and.iop<=481).or.(iop>=490 .and. &
           iop<=492).or.(iop>=500 .and.iop<=502).or.(iop>=510 .and. &
           iop<=512).or.(iop>=517 .and.iop<=519).or. &
!          2. Buch 2, Tab. 17-38 ausser 29
           iop==170 .or.iop==171 .or.iop==180 .or.iop==181 .or. &
           (iop>=190 .and.iop<=195).or.(iop>=200 .and.iop<=202).or. &
           (iop>=210 .and.iop<=213) .or.iop==220 .or.iop==221 .or. &
           (iop>=230 .and.iop<=232).or.(iop>=240 .and.iop<=242).or. &
           iop==250 .or.iop==251 .or.iop==260 .or.iop==270 .or. &
           iop==271 .or.iop==280 .or.iop==281 .or.iop==300 .or. &
           iop==301 .or.iop==310 .or.iop==311 .or.(iop>=320 .and. &
           iop<=323).or.(iop>=330 .and.iop<=335).or.iop==338 .or. &
           (iop>=370 .and.iop<=373).or.(iop>=380 .and.iop<=381).or. &
           iop==999 .or.iop==-804) exit
         ire = 1; call emes(ire,com,dm)
       enddo

!  . . Auswertung der eingegebenen Option
       if (iop<0 .or.iop>20) then
         id = mod(iop,10); ita = (iop-id)/10

!        Buch 1 (Parameter fuer Datei "inparm.t")
         if (ita==39) iop = 21 + id
         if (ita==40) iop = 24 + id
         if (ita==41 .or.ita==42) then
           iop = 27 + id
           if (id==7) iop = 3
           if (id>=8) iop = 26 + id
         endif
         if (ita==43) iop = 36 + id
         if (ita==44) iop = 28 + 3*id
         if (ita==45) iop = 2
         if (ita==46 .or.ita==47) iop = 39 + id
         if (ita==48) iop = 41 + id
         if (ita==49) iop = 3
         if (ita==49 .and.id>=1) iop = 33 + id
         if (ita==50) iop = 1
         if (ita==50 .and.id>=1) iop = 42 + id
         if (ita==51) iop = 45 + id
         if (ita==51 .and.id>=7) iop = 89 + id

!        Buch 2 (Parameter fuer Datei "inparm.t")
         if (ita==17) iop = 48 + id
         if (ita==18) iop = 50 + id
         if (ita==19) iop = 52 + id
         if (ita==20) iop = 58 + id
         if (ita==21) iop = 61 + id
         if (ita==22) iop = 65 + id
```



```
         if (ita==23 .and.id==0) iop = 8
         if (ita==23 .and.id>=1) iop = 66 + id
         if (ita==24 .and.id==0) iop = 3
         if (ita==24 .and.id>=1) iop = 68 + id
         if (ita==25) iop = 71 + id
         if (ita==26) iop = 14
         if (ita==27) iop = 73 + id
         if (ita==28) iop = 75 + id
         if (ita==30) iop = 77 + id
         if (ita==31) iop = 79 + id
         if (ita==32) iop = 81 + id ! Bei iop0=322, 323 s.a. "aphelko".
         if (ita==33 .and.id<=5) iop = 85 + id
         if (ita==33 .and.id==8) iop = 91
         if (ita==37) iop = 16 + id
         if (ita==38 .and.id==0) iop = 20
         if (ita==38 .and.id==1) iop = 99
         if (iop0==-804) iop = 94 ! Erzeugung der Datei "inser-2.t"
         if (iop0==21 .or.iop0==22) iop = 91 + id ! V/M-Tra. + Pos-win.
       endif                                      !  (Sonderoptionen)

!  . . Einlesen der Parameter aus "inparm.t"
       call inputfile(ipla,ilin,imod,imo4,ikomb,io,lv,ivers, &
         itran,isep,iuniv,ical,ika,iaph,iamax,step,ison,ihi,irb,ijd, &
         zmin,zmax,ak,zjde1,dwi,dwikomb,dwi2,dwi3,nurtr,iek,iop,1,iout)
       return

!.....Menues fuer Einzeleingabe der Parameter........................

!  . . Planetenpositionen (Parameter: ipla)
    10 do
         write(iy,'(''  >>>  Giza pyramids  (1), GP chambers (2),''/ &
                  &''       conj./transits (3), Teotihuacan (4)  : ''&
           & )',advance='no')
         read(*,*,iostat=iox) ipla
         if (ipla>=1 .and.ipla<=4 .and.iox==0) exit
         call emes(ire,com,dm)
       enddo

!  . . Linearkonstellation (ilin)  --- Transite ---
       ilin = 4
       if (ipla==3) then
        do
         write(iy,'(''  Tr. Mer.(1), Ven.(2), 3-co.(3), 4-co.(4)  : ''&
           & )',advance='no')
         read(*,*,iostat=iox) ilin
         if (ilin>=1 .and.ilin<=4 .and.iox==0) exit
         call emes(ire,com,dm)
        enddo
       endif

!  . . VSOP, Theorie-Variante (imod)
!      Es erfolgt hier eine Aenderung des Parameters 'imod' (s.u.).
!      Eingabe :  VSOP87  Kombi.(1), Kurzv.(2), Kepl.(3), Vollv.(4)
!      intern  :  VSOP87  Kurzv.(1), Vollv.(2), Kepl.(3)
       ikomb = 0; imod  = 3
       if (ipla<=3) then
        do
         if (ipla<=2) then
          write(iy,'(''  VSOP87     combi. (1), short version (2),''/ &
                  &''          Kepl. equ. (3),  full version (4)  : ''&
            & )',advance='no')
```

```
          read(*,*,iostat=iox) imod
          if (imod>=1 .and.imod<=4 .and.iox==0) exit
         else
          if (ilin>=3) then
           write(iy,'(''  VSOP87  combi.(1), short v.(2), '', &
             & ''Kepl.(3)  : '')',advance='no')
           read(*,*,iostat=iox) imod
           if (imod>=1 .and.imod<=3 .and.iox==0) exit
          else
           write(iy,'(''  VSOP87-version  full v.(1),  '', &
             & ''short v.(2)  : '')',advance='no')
           read(*,*,iostat=iox) imod
           if (imod>=1 .and.imod<=2 .and.iox==0) exit
          endif
         endif
         call emes(ire,com,dm)
        enddo
!       Aendern des Parameters "imod"
!       (imo4 wird eingefuehrt, da imod wechselt, falls ikomb = 1 ist.)
        imo4 = 0
        if (imod==1) ikomb = 1
        if (imod==2) imod = 1
        if (imod==4) then; imod = 2; imo4 = 1; endif
       endif

! . . Version von VSOP87 (lv)
!      (Bei Transits u. J2000: geringe Abw. zu Meeus => keine Option
!      bzw. ipla <= 2.)
       lv = 1; ivers = 3
       if (ipla<=3) then
        if (imod/=1 .or.(imod==1 .and.ikomb==1 .and.ipla<=2)) then
         do
          write(iy,'(''  System   ecl. of epoch (1),  J2000.0 (2)'', &
            & ''  : '')',advance='no')
          read(*,*,iostat=iox) lv
          if ((lv==1 .or.lv==2).and.iox==0) exit
          call emes(ire,com,dm)
         enddo
         if (lv==2) ivers = 1
        endif
       endif

! . . Merkur- und Venustransite vor Sonne pruefen bei VSOP-Vollversion
!      (Diese Option wird nicht mehr abgefragt, da nach Optimierung der
!      VSOP87-Routine der Geschwindigkeitsvorteil durch Weglassen der
!      Transit-Pruefung nur noch gering ist, d.h., itran ist stets 1.)
!c     if (ipla==3.and.ikomb==1.and.ilin>=3) then
!c       do
!c        write(iy,'(''  Check planetary transit  yes (1), no (2)'', &
!c          & ''  : '')',advance='no')
!c        read(*,*,iostat=iox) itran
!c        if ((itran==1.or.itran==2).and.iox==0) exit
!c        call emes(ire,com,dm)
!c       enddo; if (itran==2) io = 1
!c     endif

! . . Transit-Pruefung bei gleicher ekl. Laenge, minimaler Separation
!      oder Berechnung der Phasen, optional mit Positionswinkeln (isep)
       isep = 1
       if (itran==1 .and.ilin<=2 .and.ipla<=3) then
         do
```



```
          write(iy,'(''  Date  equ.L.(1), nearest (2), phases (3),''/ &
                & ''              phases and position angles (4)  : ''&
            & )',advance='no')
          read(*,*,iostat=iox) isep
          if (isep>=1 .and.isep<=4 .and.iox==0) exit
          call emes(ire,com,dm)
         enddo
       endif

! . . Julian/Gregorian calendar: Automatic choice of calender or
!      only Gregorian calendar (ical)
       ical = 0
       if (ipla<=3) then
        do
         write(iy,'(''  Calendar  only Greg. (1), Jul./Greg. (2)  : ''&
           & )',advance='no')
         read(*,*,iostat=iox) ical
         if ((ical==1 .or.ical==2).and.iox==0) exit
         call emes(ire,com,dm)
        enddo
       endif

! . . Terrestrial Time bzw. Universal Time (iuniv)
       iuniv = 1
       if (itran==1 .and.ilin<=2 .and.isep>=3 .and.ipla<=3) then
         do
          write(iy,'(''  Time system          JDE/TT (1),  UT (2)'', &
            & ''  : '')',advance='no')
          read(*,*,iostat=iox) iuniv
          if ((iuniv==1 .or.iuniv==2).and.iox==0) exit
          call emes(ire,com,dm)
         enddo
       endif

! . . Zuordnung der Planeten Erde (E), Venus (V) und Merkur (M) zu
!      Koenigs-, Koeniginnen- und Felsenkammer, diese Reihenfolge (ika)
       ika = 0
       if (ipla==2 .and.imod/=3) then
        do
         write(iy,'(''  Planets  E-V-M (1), E-M-V (2), V-E-M (3),''/ &
                & ''         V-M-E (4), M-E-V (5), M-V-E (6)  : ''&
           & )',advance='no')
         read(*,*,iostat=iox) ika
         if (ika>=1 .and.ika<=6 .and.iox==0) exit
         call emes(ire,com,dm)
        enddo
       endif

! . . Zeitpunkte im/um Aphel bzw. Perihel oder freier Zeitpunkt (iaph)
       iaph = 1; iamax = 0
       step = 24.d0
       if (ipla<=2) then
        do
         if (imod<=2 .and.ikomb==0 .and.imo4==0) then
          write(iy,'(''  Passage aph./per. area of aph./per. free''/ &
                & ''          (1)  (2)          (3)  (4)   (5)  : ''&
            & )',advance='no')
          read(*,*,iostat=iox) iaph
          if (iaph>=1 .and.iaph<=5 .and.iox==0) exit
         elseif (imod<=2 .and.ikomb==1 .and.imo4==0) then
          write(iy,'(''  Passage   aph. (1),  per. (2),  free (5)'', &
```

```
         & ''  : '')',advance='no')
        read(*,*,iostat=iox) iaph
        if ((iaph==1 .or.iaph==2 .or.iaph==5).and.iox==0) exit
       elseif (imod<=2 .and.ikomb==0 .and.imo4==1) then
        write(iy,'(''  Passage    aph./ per.  area of aph./ per.''/ &
               & ''                (1)   (2)          (3)   (4)'', &
         & ''  : '')',advance='no')
        read(*,*,iostat=iox) iaph
        if (iaph>=1 .and.iaph<=4 .and.iox==0) exit
       else
        write(iy,'(''  Passage    aphelion (1),  perihelion (2)'', &
         & ''  : '')',advance='no')
        read(*,*,iostat=iox) iaph
        if ((iaph==1 .or.iaph==2).and.iox==0) exit
       endif
       call emes(ire,com,dm)
      enddo
      if (iaph==3 .or.iaph==4) then
       do
        write(iy,'(''  Steps per Mercury passage : '')',advance='no')
        read(*,*,iostat=iox) iamax
        if (iamax>0 .and.iamax<=200000 .and.iox==0) exit
        call emes(ire,com,dm)
       enddo
       do
        write(iy,'(''  Step width  (hours, real) : '')',advance='no')
        read(*,*,iostat=iox) step
        if (step>z0.and.step<=9999.9994d0 .and.iox==0) exit
        call emes(ire,com,dm)
       enddo
       if (imod==2) io = 1
      endif
     endif

! . . Sonnenposition (ison)
     ison = 1
     if (ipla<=2) then
      do
       if (ipla==1 .and.iaph<=2) then
        if (imod<=2) then
        write(iy,'(''  Sun pos.  Myk.(1),  Chefr.(2),  free (3)'', &
         & ''  : '')',advance='no')
        else
        write(iy,'(''  Sun pos. south of   Myk.(1),   Chefr.(2)'', &
         & ''  : '')',advance='no')
        endif
        read(*,*,iostat=iox) ison
       else
        if (imod<=2) ison = 3
       endif
       if (((imod<=2 .and.ison>=1 .and.ison<=3).or. &
           (imod==3 .and.(ison==1 .or.ison==2))).and.iox==0) exit
       call emes(ire,com,dm)
      enddo
     endif

! . . Freie Sonnenposition, Berechnung 2- oder 3-dimensional (ison2)
     if (iaph==5) ison = 5
     if (ison==3) then
      do
       if (ipla==1) then
```



```
        write(iy,'(''  Sun     2D (1), 3D/SLE (2), 3D/FITEX (3)'', &
         & ''  : '')',advance='no')
       else
        write(iy,'(''  Sun (three-dim.):    SLE (2),  FITEX (3)'', &
         & ''  : '')',advance='no')
       endif
       read(*,*,iostat=iox) ison2
       if (((ipla==1 .and.ison2>=1 .and.ison2<=3).or. &
           (ipla==2 .and.(ison2==2 .or.ison2==3))).and.iox==0) exit
       call emes(ire,com,dm)
      enddo
      if (ison2==2) ison = 4
      if (ison2==3) ison = 5
     endif

! . . Hoehenlage der Pyramiden-Grundflaechen bzw. -Schwerpunkte (ihi)
     ihi = 0
     if (ipla<=2 .and.ison>=4) then
      do
       if (ipla==1) then
        write(iy,'(''  z-coord.    base (1),  C-M (2),  top (3)'', &
         & ''  : '')',advance='no')
       else
        write(iy,'(''  Wall      east (1), middle (2), west (3)'', &
         & ''  : '')',advance='no')
       endif
       read(*,*,iostat=iox) ihi
       if (ihi>=1 .and.ihi<=3 .and.iox==0) exit
       call emes(ire,com,dm)
      enddo
     endif

! . . Grundebene Ekliptik, Merkur- oder Venusbahn (irb)
     irb = 1
     if (ipla<=2 .and.imod<=2 .and.ison==1) then
      do
       write(iy,'(''  Coord.   ecl.(1),   Mer.(2-4),   Ven.(5)'', &
         & ''  : '')',advance='no')
       read(*,*,iostat=iox) irb
       if (irb>=1 .and.irb<=5 .and.iox==0) exit
       call emes(ire,com,dm)
      enddo
     endif

! . . Angabe bzw. Berechnung von JDE (ijd)
     ijd = 15
     if (ipla<=2 .and.ikomb==0 .and.iaph/=5) then
      do
       if (imod==2 .and.iaph<=2) then
        write(iy,'(''  Constell. (1..14),  k-No. (15),  JDE (0)'', &
         & ''  : '')',advance='no')
       else
        write(iy,'(''  Constell. (1..14),  years (15),  JDE (0)'', &
         & ''  : '')',advance='no')
       endif
       read(*,*,iostat=iox) ijd
       if (ijd>=0 .and.ijd<=15 .and.iox==0) exit
       call emes(ire,com,dm)
      enddo
     endif
     ak = z0
```

```
          zmin = z0
          zmax = z0
          if (ipla<=3) then
           if (ijd==15) then
            if (imod==2 .and.iaph<=2 .and.ipla/=3) then
              do
               write(iy,'(''     k (real): '')',advance='no')
               call pcheck(1,ak,2,dm,imod,ire)
               if (ire==0) exit
              enddo
            else
              do
               write(iy,'(''     from  year (real): '')',advance='no')
               call pcheck(1,zmin,1,dm,imod,ire)
               if (ire==0) exit
              enddo
              do
               write(iy,'(''     until year (real): '')',advance='no')
               call pcheck(1,zmax,1,dm,imod,ire)
               if (zmin>=zmax.and.ire==0) then
                call emes(ire,com,dm)
                ire = 1
               endif
               if (ire==0) exit
              enddo
            endif
           endif
           if (ipla==3) then
            step = z0
            if (ilin>=3 .and.ikomb==0) then
             do
             write(iy,'(''  Step width [hrs] (min.-search 0.) (real)'', &
               & ''  : '')',advance='no')
             read(*,*,iostat=iox) step
             if (step>=z0.and.iox==0) exit
             call emes(ire,com,dm)
             enddo
            endif
           endif
           if (step==z0) ison = 5
           if (ipla==3 .and.step/=z0) io = 1
           zjde1 = z0
           if (ijd==0) then
            do
              write(iy,'(''     JDE (real) : '')',advance='no')
              call pcheck(1,zjde1,3,dm,imod,ire)
              if (ire==0) exit
            enddo
           endif
          endif

     ! . . Winkelintervall bzw. relativer Fehler (dwi ... dwikomb)
          dwi = z0
          dwi2 = z0; dwi3 = z0
          dwikomb = z0; dm = 99.99d0
          if (ipla<=2 .and.ijd==15 .and.(imod/=2 .or. &
             (imod==2 .and.(iaph==3 .or.iaph==4)))) then
           if (ikomb==0 .and.iaph/=5) then
            do
             if (ison<=2) then
              if (imod/=3) dm = 10.d0
```



```
               write(iy,'(''  Tolerance ecl. long. Venus, Earth (real)'', &
                 & ''  : '')',advance='no')
              else
               write(iy,'(''  Max. F-pos at aphelion/ per.  (real) [%]'', &
                 & ''  : '')',advance='no')
              endif
              call pcheck(2,dwi,1,dm,imod,ire)
              if (ire==0) exit
            enddo
           else
            do
             if (ison<=2) then
              if (imod/=3) dm = 10.d0
              write(iy,'(''  Tolerance ecl. long. VSOP short   (real)'', &
                & ''  : '')',advance='no')
             else
              if (iaph/=5 .or.(iaph==5 .and.ikomb==1)) then
               write(iy,'(''  Max. F-pos    VSOP short ver. (real) [%]'',&
                 & ''  : '')',advance='no')
              else
               write(iy,'(''  Max. F-pos, VSOP short, start fitmin [%]'',&
                 & ''  : '')',advance='no')
              endif
             endif
             call pcheck(2,dwi,1,dm,imod,ire)
             if (ire==0) exit
            enddo
            do
             if (ison<=2) then
              if (imod/=3) dm = 10.d0
              write(iy,'(''      "      "     "   VSOP full    (real)'', &
                & ''  : '')',advance='no')
             else
              if (iaph/=5 .or.(iaph==5 .and.ikomb==1)) then
               write(iy,'(''   "     "      VSOP  full ver. (real) [%]'',&
                 & ''  : '')',advance='no')
              else
               write(iy,'(''   "     "    VSOP short, final range  [%]'',&
                 & ''  : '')',advance='no')
              endif
             endif
             call pcheck(2,dwikomb,1,dm,imod,ire)
             if (ire==0) exit
            enddo
           endif
           if (iaph==3 .or.iaph==4) then
            do
             write(iy,'(''   "     "   consider without printing [%]'', &
              & ''  : '')',advance='no')
             call pcheck(2,dwi2,1,dm,imod,ire)
             if (ire==0) exit
            enddo
            do
             write(iy,'(''   "     "   print beyond aphelion/per.[%]'', &
              & ''  : '')',advance='no')
             call pcheck(2,dwi3,1,dm,imod,ire)
             if (ire==0) exit
            enddo
           endif
          endif
          if (ipla==3 .and.ilin>=3) then
```

```
        if (ikomb==0) then
         do
          write(iy,'(''  Ang. range of eclipt. longitude   (real)'', &
            & ''  : '')',advance='no')
          call pcheck(2,dwi,1,dm,imod,ire)
          if (ire==0) exit
         enddo
        else
         do
          write(iy,'(''  Ecl. angular range, VSOP short v. (real)'', &
            & ''  : '')',advance='no')
          call pcheck(2,dwi,1,dm,imod,ire)
          if (ire==0) exit
         enddo
         do
          write(iy,'(''   "       "      "  , VSOP  full v. (real)'', &
            & ''  : '')',advance='no')
          call pcheck(2,dwikomb,1,dm,imod,ire)
          if (ire==0) exit
         enddo
        endif
       endif

 ! . . Dreier- oder Viererkonjunktion nur mit Transit (nurtr)
       nurtr = 1
       if (ipla==3 .and.ilin>=3 .and.ison==5 .and.imod/=3 &
           .and.itran==1) then
         do
          write(iy,'(''  All conjunctions (1),  only transits (2)'', &
            & ''  : '')',advance='no')
          read(*,*,iostat=iox) nurtr
          if ((nurtr==1 .or.nurtr==2).and.iox==0) exit
          call emes(ire,com,dm)
         enddo
       endif

 ! . . Blickrichtung auf die Planetenbahnen (iek)
 !     (nur bei 2D-Berechnungen)
       iek = 1
       if (ipla<=2) then
        do
         if (ison<=2 .and.(ijd==15 .or.ijd==0)) then
          if ((imod==2 .and.iaph<=2).or.ijd==0) then
           write(iy,'(''  View from ecliptic  North (1), South (2)'', &
             & ''  : '')',advance='no')
           read(*,*,iostat=iox) iek
           if (iek>=1 .and.iek<=2 .and.iox==0) exit
          else
           write(iy,'(''  View from eclipt.  N (1), S (2), N/S (3)'', &
             & ''  : '')',advance='no')
           read(*,*,iostat=iox) iek
           if (iek>=1 .and.iek<=3 .and.iox==0) exit
          endif
          call emes(ire,com,dm)
         else
          iek = 1
          if ((ijd>=6 .and.ijd<=11).or.ijd==13 .or.ijd==14) iek=2; exit
         endif
        enddo
       endif
```



```
 !-----Input Teotihuacan-----------------------------

 ! . . Kind of distance measurement (ilin)
       if (ipla==4) then
        do
         write(iy,'(''  Distances   GPS (1), meters (2), Map (3)  : ''&
           & )',advance='no')
         read(*,*,iostat=iox) ilin
         if (ilin>=1 .and.ilin<=3 .and.iox==0) exit
         call emes(ire,com,dm)
        enddo

 ! . . .Time interval (zmin, zmax, step)
        do
         write(iy,'(''     from  the year      (real): ''&
           & )',advance='no')
         call pcheck(1,zmin,1,dm,imod,ire)
         if (ire==0) exit
        enddo
        do
         write(iy,'(''     until the year      (real): ''&
           & )',advance='no')
         call pcheck(1,zmax,1,dm,imod,ire)
         if (zmin>zmax.and.ire==0) then
           call emes(ire,com,dm); ire = 1
         endif
         if (ire==0) exit
        enddo
        step = 0.d0
        if (zmin<zmax) then
         do
          write(iy,'(''     Step width in years (real): ''&
            & )',advance='no')
          read(*,*,iostat=iox) step
          if (step>z0.and.step<=zmax-zmin.and.iox==0) exit
          call emes(ire,com,dm)
         enddo
        endif

 ! . . .Special length unit, Teotihuacan (isep)
        do
         write(iy,'(''  Teotih. unit    as given (1), "luna" (2)  : ''&
           & )',advance='no')
         read(*,*,iostat=iox) isep
         if ((isep==1 .or.isep==2).and.iox==0) exit
         call emes(ire,com,dm)
        enddo

 ! . . .Special length unit for planetary distances (iuniv)
        do
         write(iy,'(''  Planetary unit, kilometer (1), R-Sun (2)  : ''&
           & )',advance='no')
         read(*,*,iostat=iox) iuniv
         if ((iuniv==1 .or.iuniv==2).and.iox==0) exit
         call emes(ire,com,dm)
        enddo

 ! . . . Logarithmic base (ical, dwi)
        do
         write(iy,'(''  Logar. base    10 (1), 3 (3), custom (4)  : ''&
           & )',advance='no')
```

```
        read(*,*,iostat=iox) ical
        if (ical==1 .or.ical==3 .or.ical==4 .and.iox==0) exit
        call emes(ire,com,dm)
       enddo
       if (ical==4) then
        do
         write(iy,'(''  Logarithmic base                  (real)'', &
           & ''  : '')',advance='no')
         read(*,*,iostat=iox) dwi
         if (dwi>1.d0 .and.dwi<=1000.d0 .and.iox==0) exit
         call emes(ire,com,dm)
        enddo
       endif
      endif
!-----End of input Teotihuacan---------------------

! . . Ausgabe (io)
      if (io==0) then
        io = 2; if (iaph==5) io = 1
        if (imo4==0 .and.iaph/=5) then
          do
           write(iy,'(''  Output         normal (1),  extended (2)'',&
             & ''  : '')',advance='no')
           read(*,*,iostat=iox) io
           if ((io==1 .or.io==2).and.iox==0) exit
           call emes(ire,com,dm)
          enddo
        endif
      endif

! . . Ausgabegeraet (iout)
      do
        if (imod<=2 .and.ipla<=2 .and.ison==5) then
          write(iy,'(''  Mon.(1), file (2), special (3), exit (4)'', &
            & ''  : '')',advance='no')
          read(*,*,iostat=iox) iout
          if (iout>=1 .and.iout<=4 .and.iox==0) exit
        else
          write(iy,'(''  Monitor (1),  mon. + file (2),  exit (4)'', &
            & ''  : '')',advance='no')
          read(*,*,iostat=iox) iout
          if ((iout==1 .or.iout==2 .or.iout==4).and.iox==0) exit
        endif; call emes(ire,com,dm)
      enddo
      end subroutine

      subroutine inputfile(ipla,ilin,imod,imo4,ikomb,io,lv,ivers, &
       itran,isep,iuniv,ical,ika,iaph,iamax,step,ison,ihi,irb,ijd, &
      zmin,zmax,ak,zjde1,dwi,dwikomb,dwi2,dwi3,nurtr,iek,iop,irw,iout)
!-----Einlesen der Inputdaten bei Schnellstart-----------------------
!     irw=1: lesen aus "inparm.t",  irw=2: schreiben in "inedit.t"
!     Mit Hilfe von inedit.t kann inparm.t manuell editiert werden.
      implicit double precision (a-h,o-z)
      if (irw==1) then
        if (iop/=999) then
          open(unit=10,file='inparm.t')
          do i=1,10*iop+1; read(10,*); enddo
        else
          open(unit=10,file='inedit.t')
          do i=1,26; read(10,*); enddo
        endif
```



```
        read(10,*) ipla,ilin,imod,imo4,ikomb
        read(10,*) lv,itran,isep,iuniv,ical
        read(10,*) ika,iaph,iamax,step
        read(10,*) ison,ihi,irb,ijd
        read(10,*) zmin,zmax,ak,zjde1
        read(10,*) dwi,dwikomb,dwi2,dwi3
        read(10,*) nurtr,iek,io,iout
        ivers = 3; if (lv==2) ivers = 1
      elseif (irw==2) then
        open(unit=10,file='inedit.t')
        do i=1,36; read(10,*); enddo
        write(10,'(5i3)') ipla,ilin,imod,imo4,ikomb
        write(10,'(5i3)') lv,itran,isep,iuniv,ical
        write(10,'(2i3,i6,f12.5)') ika,iaph,iamax,step
        write(10,'(3i3,i4)') ison,ihi,irb,ijd
        write(10,'(3f13.5,f15.5)') zmin,zmax,ak,zjde1
        write(10,'(4f8.3)') dwi,dwikomb,dwi2,dwi3
        write(10,'(4i3)') nurtr,iek,io,iout
        write(10,*) ('-',i=1,59)
        write(10,*) ('*',i=1,27),' END ',('*',i=1,27)
      endif
      close(10)
      end subroutine

      subroutine chambers(ig,rx)
!-----Aenderung der Planeten-Kammer-Zuordnung------------------------
!     Reihenfolge Koenigs-, Koeniginnen- u. Felsenkammer mit Planeten:
!     ig: 1. E-V-M, 2. E-M-V, 3. V-E-M, 4. V-M-E, 5. M-E-V, 6. M-V-E
      implicit double precision (a-h,o-z)
      dimension :: rx(3,4),x(5),y(5)
      if (ig==3 .or.ig==5) call pchange(1,1,2,rx,x,y,indx)
      if (ig==2 .or.ig==4 .or.ig==5) call pchange(1,2,3,rx,x,y,indx)
      if (ig==4) call pchange(1,1,2,rx,x,y,indx)
      if (ig==6) call pchange(1,1,3,rx,x,y,indx)
      end subroutine

      subroutine pchange(imodus,iz,jz,rxx,x,y,indx)
!-----Vertauschen von Input-Zeilen oder Zahlen in "fitmin"------------
      implicit double precision (a-h,o-z)
      dimension :: rxx(3,4),x(5),y(5)
      if (imodus==1) then; do i=1,4
        rpc=rxx(iz,i); rxx(iz,i)=rxx(jz,i); rxx(jz,i)=rpc; enddo
      elseif (imodus==2) then
        z=x(iz); x(iz)=x(jz); x(jz)=z
        z=y(iz); y(iz)=y(jz); y(jz)=z
        if (indx==iz) then; indx = jz; return; endif
        if (indx==jz) indx = iz
      endif
      end subroutine

      subroutine pcheck(i,p,n,dm,imod,ire)
!-----Read and check of input parameter p----------------------------
!     modus i: read + check time (1), tolerance (2)
!     time  n: year (1), k-number (2), JDE (3)
!     p: input parameter, dm: maximum allowed value
!     error code ire (ire = 0 means "no error.")
      implicit double precision (a-h,o-z)
      character(36) :: com
      ire = 0
      read(*,*,iostat=iox) p
      if (iox/=0) ire = 1
```

```
      if (i==1 .and.ire==0) then
        ire = 2
        if (imod/=3) then
          if (n==1 .and.(p<-13000.00001d0 .or.p>17000.00001d0)) then
            com = ' (-13 000. <= year <= 17 000.)      '
          elseif (n==2 .and.(p<-63000.001d0 .or.p>63000.001d0)) then
            com = ' (-63 000. <=  k  <=  63 000.)      '
          elseif (n==3 .and.(p<-3030000.1d0 .or.p>7940000.1d0)) then
            com = ' (-3 030 000. <= JDE <= 7 940 000.) '
          else
            ire = 0
          endif
        else
          if (n==1 .and.(p<-30000.00001d0 .or.p>30000.00001d0)) then
            com = ' (-30 000. <= year <= 30 000.)      '
          elseif (n==2 .and.(p<-133000.01d0 .or.p>117000.01d0)) then
            com = ' (-133 000. <=  k  <=  117 000.)    '
          elseif (n==3 .and.(p<-9240000.1d0 .or.p>12680000.1d0)) then
            com = ' (-9 240 000. <= JDE <= 12 680 000.)'
          else
            ire = 0
          endif
        endif
      elseif (i==2 .and.ire==0) then
        if (p<=0.d0) ire = 1
        if (p>dm) ire = 3
      endif
      if (ire/=0) call emes(ire,com,dm)
      end subroutine

      subroutine emes(ire,com,dm)
!-----Error message--------------------------------------------------
      implicit double precision (a-h,o-z)
      character(36) :: com
      iy = 6
      if (ire<=1) write(iy,'(/''  --->  Insert a correct number.''/)')
      if (ire==2) write(iy,'(/''  --->  Insert a correct number. '', &
        & a36/)')com
      if (ire==3) write(iy,'(/''  ---->  number too large '', &
        & ''(max.'',f6.2,'').''/)') dm
      end subroutine

      subroutine konst(ik,kon)
!-----Automatische Erkennung der Planetenkonst. 1 bis 14 --> kon------
!     Suchtoleranz (+/-) fuer Konst.: 53 Tage, fuer "->": 880 Tage
      use base, only : akon
      implicit double precision (a-h,o-z)
      character(2) :: kon,tkon(14)
      data tkon/' 1',' 2',' 3',' 4',' 5',' 6',' 7', &
                ' 8',' 9','10','11','12','13','14'/
      ye  = 10.d0; kon = '  '
      ep  = 0.6d0
      ako = dfloat(ik)
      do i=1,14
        a1 = dabs(ako-akon(i))
        a2 = dabs(ako-(akon(i)-1.d0))
        if (a1<ye.or.a2<ye) kon = '->'
        if (a1<ep.or.a2<ep) kon = tkon(i)
      enddo
      end subroutine
```



```
      subroutine ephim(i,iaph,ipla,ical,ak,iak,day,year,delt)
!-----Julian Ephemeris Day and Year (Merkur im Aphel)-----------------
!     Input ist "ak" (Nummer des Apheldurchgangs), "day" oder "year".
!     i = 0:  ak   --> day, year, delt
!     i = 1:  day  --> ak, iak, year, delt
!     i = 2:  year --> day, ak, iak
      implicit double precision (a-h,o-z)
      if (i==0) call akday(0,iaph,ipla,ak,iak,day)

! . . Neue Werte (Buch 2)
!     Diese Zahlen verbessern nur die Genauigkeit der dezimalen
!     Jahreszahl auf +/- 0,5 Tage im Vergleich zum Datum, aendern
!     jedoch nichts an den bisherigen astronomischen Berechnungen
!     und Datumsberechnungen. Alle durch 400 teilbaren Jahreszahlen,
!     wie z.B. -1200.0 oder 2000.0, entsprechen jetzt exakt dem
!     1. Januar, 12 Uhr. Das heisst, das dezimale Jahr 2000.0 be-
!     deutet die Standard-Epoche J2000.0.
      if (ical==2 .and.((i<=1 .and.day>=0.d0 .and.day<2299160.5d0) &
       .or.(i==2 .and.year>=-4712.d0 .and.year<1582.7854097d0))) then
        A = 365.25d0;   B = 0.d0;       C =-4712.d0  ! (Julian. Kal.)
      else
        A = 365.2425d0; B = 2451545.d0; C = 2000.d0  ! (Gregor. Kal.)
      endif
! . . Vorherige Werte (Buch 1)
!c      A = 365.248d0;  B = 0.d0;  C = -4711.9986d0  ! (Programm P3)

! . . Umrechnung der Daten
      if (i<=1) year = (day - B)/A + C
      if (i==1) call akday(1,iaph,ipla,ak,iak,day)
      if (i<=1) then
        aik = dnint(ak); call akday(0,iaph,ipla,aik,iak,aiday)
        delt = day - aiday
      else
        day = A * (year - C) + B; call akday(1,iaph,ipla,ak,iak,day)
      endif
      end subroutine

      subroutine akday(j,iaph,ipla,ak,iak,day)
!-----Julian Ephemeris Day------------------------------------------
!     j = 0:  ak  --> day
!     j = 1:  day --> ak,iak
!     ymer = Umlaufzeit des Merkur in Tagen
      use base, only : pmer,ymer
      implicit double precision (a-h,o-z)
      if (j==0) then
        aak = ak
        if (iaph==1 .or.iaph==3 .or.(iaph==5 .and.ipla==1)) &
          aak = aak - 0.5d0
        day = pmer + ymer * aak
      endif
      if (j==1) then
        ak = (day - pmer)/ymer
        if (iaph==1 .or.iaph==3 .or.(iaph==5 .and.ipla==1)) &
          ak = ak + 0.5d0
        iak = idnint(ak)
      endif
! . . Apheldurchgang der Erde
!c    day = 2451547.507d0 + 365.2596358d0 * (ak + 0.5d0) &
!c          + 1.58d-8 * (ak + 0.5d0)**2
      end subroutine
```

```fortran
      subroutine delta_T(zjd)
!-----Umrechnung: Terrestrial Time --> Universal Time-----------------
!     Gleichungen von Fred Espenak und Jean Meeus, entwickelt auf Ba-
!     sis des "Five Millennium Canon of Solar Eclipses", nach Artikeln
!     von Morrison/Stephenson (2004) und Stephenson/Houlden (1986).
!     (NASA Eclipse Web Site, Polynom. expressions for DELTA-T, 2005)
!     DELTA-T (del) in Sekunden.
      implicit double precision (a-h,o-z)
      call ephim(1,1,1,1,ak,iak,zjd,y,delt)
      if (y>-500.d0 .and.y<=500.d0) then
         u = y/100.d0
         del = 10583.6d0 - 1014.41d0 * u + 33.78311d0  * u**2 &
               - 5.952053d0 * u**3 - 0.1798452d0 * u**4 &
               + 0.022174192d0 * u**5 + 0.0090316521d0 * u**6
      elseif (y>500.d0 .and.y<=1600.d0) then
         u = (y-1000.d0)/100.d0
         del = 1574.2d0 - 556.01d0 * u + 71.23472d0 * u**2 &
               + 0.319781d0 * u**3 - 0.8503463d0 * u**4 &
               - 0.005050998d0 * u**5 + 0.0083572073d0 * u**6
      elseif (y>1600.d0 .and.y<=1700.d0) then
         t = y - 1600.d0
         del = 120.d0 - 0.9808d0 * t - 0.01532d0 * t**2 &
               + t**3 / 7129.d0
      elseif (y>1700.d0 .and.y<=1800.d0) then
         t = y - 1700.d0
         del = 8.83d0 + 0.1603d0 * t - 0.0059285d0 * t**2 &
               + 0.00013336d0 * t**3 - t**4 / 1174000.d0
      elseif (y>1800.d0 .and.y<=1860.d0) then
         t = y - 1800.d0
         del = 13.72d0 - 0.332447d0 * t + 0.0068612d0 * t**2 &
               + 0.0041116d0 * t**3 - 0.00037436d0 * t**4 &
               + 0.0000121272d0 * t**5 - 0.0000001699d0 * t**6 &
               + 0.000000000875d0 * t**7
      elseif (y>1860.d0 .and.y<=1900.d0) then
         t = y - 1860.d0
         del = 7.62d0 + 0.5737d0 * t - 0.251754d0 * t**2 &
               + 0.01680668d0 * t**3 - 0.0004473624d0 * t**4 &
               + t**5/233174.d0
      elseif (y>1900.d0 .and.y<=1920.d0) then
         t = y - 1900.d0
         del = -2.79d0 + 1.494119d0 * t - 0.0598939d0 * t**2 &
               + 0.0061966d0 * t**3 - 0.000197d0 * t**4
      elseif (y>1920.d0 .and.y<=1941.d0) then
         t = y - 1920.d0
         del = 21.20d0 + 0.84493d0 * t - 0.076100d0 * t**2 &
               + 0.0020936d0 * t**3
      elseif (y>1941.d0 .and.y<=1961.d0) then
         t = y - 1950.d0
         del = 29.07d0 + 0.407d0 * t - t**2/233.d0 + t**3/2547.d0
      elseif (y>1961.d0 .and.y<=1986.d0) then
         t = y - 1975.d0
         del = 45.45d0 + 1.067d0 * t - t**2/260.d0 - t**3/718.d0
      elseif (y>1986.d0 .and.y<=2005.d0) then
         t = y - 2000.d0
         del = 63.86d0 + 0.3345d0 * t - 0.060374d0 * t**2 &
               + 0.0017275d0 * t**3 + 0.000651814d0 * t**4 &
               + 0.00002373599d0 * t**5
      elseif (y>2005.d0 .and.y<=2050.d0) then
         t = y - 2000.d0
         del = 62.92d0 + 0.32217d0 * t + 0.005589d0 * t**2
      elseif (y>2050.d0 .and.y<=2150.d0) then
```



```fortran
         del = -20.d0 + 32.d0 * ((y-1820.d0)/100.d0)**2 &
               - 0.5628d0 * (2150.d0 - y)
      else
         u = (y - 1820.d0)/100.d0; del = -20.d0 + 32.d0 * u**2
      endif
!     Spaetere Korrektur (NASA Eclipse Web Site):
      if (y<1955.d0 .or.y>2005.d0) del = del-1.2932d-5*(y-1955.d0)**2
      zjd = zjd - del/86400.d0

! . . Alternativ: Jean Meeus, "Transits", S. 73, der wiederum fol-
!     gende Referenz zitiert: L.V. Morrison, F.R. Stephenson, Sun
!     and Planetary System, Vol. 96, Reidel, Dordrecht, 1982, S. 73
!c    zjd = zjd - ((zjd-2382148.d0)**2/41048480.d0 - 15.d0)/86400.d0
      end subroutine

      subroutine jdedate(zjd,ical,ida,da,dmo)
!-----Umrechnung Julian Day --> Kalenderdatum + Uhrzeit (TT)----------
!     Basierend auf einem Algorithmus aus "Astronomical Algorithms"
!     von Jean Meeus (S. 63). Copyright: 1991, Willmann-Bell,
!     Anmerkung: Der Algorithmus wurde geringfuegig modifiziert
!     (Ersetzung der Integer- durch die Floor-Funktion), so dass
!     er jetzt fuer beide Kalender auch fuer JDE < 0 gilt.
!     Indizes:
!     1: dez.Tag, 2: Mon., 3: Jahr, 4: Std, 5: Min, 6: Sek, 7: int.Tag
      implicit double precision (A-H,O-Z)
      dimension :: ida(7),da(7)
      character(5) :: monat(12),dmo
      data monat/' Jan.',' Feb.',' Mar.',' Apr.',' May ',' June', &
                 ' July',' Aug.',' Sep.',' Oct.',' Nov.',' Dec.'/
      Z = sdint(zjd + 0.5d0); F = zjd + 0.5d0 - Z
      if (z>=0.d0 .and.Z<2299161.d0 .and.ical==2) then
        A = Z
      else
        alpha = sdint((Z - 1867216.25d0)/36524.25)
        A = Z + 1.d0 + alpha - sdint(alpha*0.25d0)
      endif
      B = A + 1524.d0
      C = sdint((B - 122.1d0)/365.25d0)
      D = sdint(365.25d0 * C)
      E = sdint((B - D)/30.6001d0)
      da(1) = B - D - sdint(30.6001d0*E) + F + 5.d-9
      if (E<14.d0) then
        da(2) = E - 1.d0
      else
        if (E==14.d0 .or.E==15.d0) then
          da(2) = E - 13.d0
        else
          da(2) = 999.d0
        endif
      endif
      M = idnint(da(2))
      if (M>2) then
         da(3) = C - 4716.d0
      else
        if (M==1 .or.M==2) then
          da(3) = C - 4715.d0
        else
          da(3) = 9999999999999.d0
        endif
      endif
      st = da(1) - sdint(da(1)); dst = st*24.D0
```

```
      da(4) = sdint(dst)
      da(5) = (dst - sdint(dst))*60.D0
      da(6) = (da(5) - sdint(da(5)))*60.D0
      da(7) = sdint(da(1))                     ! day
      ida(3) = idnint(da(3))                   ! year
      ida(4) = idnint(da(4))                   ! hours
      ida(5) = idnint(da(5)-0.5d0+1.d-10)      ! minutes
      ida(6) = idnint(da(6))                   ! seconds
      imo    = idnint(da(2))                   ! month

!     Geringfuegige Korrektur der Darstellung
!     (Beispiel: Uhrzeit 13:44:60 wird zu 13:45:00)
      do i=6,5,-1
        if (ida(i)>=60) then
          ida(i) = ida(i) - 60
          ida(i-1) = ida(i-1) + 1
        endif
      enddo
      if (ida(4)>=24) then
        ida(4) = ida(4) - 24
        da(1) = da(1) + 1.d0
        da(7) = sdint(da(1))
      endif
!     (Beispiel: 31. Mai, 23:59:60 wird zu 1. Juni, 0:0:0.)
      if ((dabs(da(7)-32.d0)<=1.d-8.and.(imo==1.or.imo==3 &
         .or.imo==5.or.imo==7.or.imo==8.or.imo==10.or.imo==12)).or. &
          (dabs(da(7)-31.d0)<=1.d-8.and.(imo==4.or.imo==6.or.imo==9 &
         .or.imo==11)).or.(dabs(da(7)-30.d0)<=1.d-8.and.imo==2)) then
        do k=30,32
          q = dfloat(k); if (dabs(da(7)-q)<=1.d-8) da(1)=da(1)+1.d0-q
        enddo
        da(7) = sdint(da(1)); imo = imo + 1
        if (imo==13) then
          imo = 1
          da(3) = da(3) + 1.d0
          ida(3) = idnint(da(3))
        endif
      endif
      dmo = monat(imo)
      end subroutine

      double precision function sdint(x)
!-----Floor function----------------------------------------------
!     replacing some integer-functions in the subroutine "jdedate"
!     in order to expand the domain of definition for JDE < 0
      real(8) :: x
      sdint = dint(x)
      if (x<0.d0 .and.dmod(x,1.d0)/=0.d0) sdint = sdint - 1.d0
      end function

      subroutine weekday(ZJD,wd)
!-----Berechnung des Wochentages--------------------------------------
      real(8) :: ZJD,ZJS
      character(10) :: wday(0:6),wd
      data wday/'    Sunday','    Monday','   Tuesday',' Wednesday', &
                '  Thursday','    Friday','  Saturday'/
      ZJS = ZJD + 700000001.5d0
      if (ZJS<0.d0 .and.dmod(ZJS,1.d0)/=0.d0) ZJS = ZJS - 1.d0
      wd = wday(idnint(dmod(dint(ZJS),7.d0)))
      end subroutine
```



```
      subroutine vsop1(l,tau,resu)
!-----Berechnung der ekliptikalen Koordinaten (VSOP87D-Kurzversion)---
      use base, only : gdpi,z0,lmax,jp
      use astro, only : par1
      implicit double precision (a-h,o-z)
      resu = z0
      do j=1,lmax(l)
        sum0 = z0
        do i=1,jp(l,j)
          sum0 = sum0 + par1(i,j,1,l) * &
                 dcos(par1(i,j,2,l) + par1(i,j,3,l)*tau)
        enddo
        resu = resu + sum0*tau**(j-1)
      enddo
      resu = resu * 1.d-8
      if (l==1 .or.l==4 .or.l==7 .or.l==10) call reduz(resu,1,1)
      if (l/=3 .and.l/=6 .and.l/=9 .and.l/=12) resu = resu*gdpi
      end subroutine

      subroutine vsop2(zjde,ivers,ibody,md,ix,prec,lu,r,ierr,rku)
!-----Aufruf der VSOP-Subroutine (VSOP87A/C-Vollversionen)------------
!     (Index von rku  1: L,  2: B,  3: r)
      implicit double precision (a-h,o-z)
      dimension :: r(6),rku(3),md(0:9)
      character(11) :: afile(9),cfile(8)
      data afile/'VSOP87A.mer','VSOP87A.ven','VSOP87A.ear', &
                 'VSOP87A.mar','VSOP87A.jup','VSOP87A.sat', &
                 'VSOP87A.ura','VSOP87A.nep','VSOP87A.emb'/
      data cfile/'VSOP87C.mer','VSOP87C.ven','VSOP87C.ear', &
                 'VSOP87C.mar','VSOP87C.jup','VSOP87C.sat', &
                 'VSOP87C.ura','VSOP87C.nep'/
      if (md(ibody)==1) then
        if (ivers==1) open(unit=10,file=afile(ibody))
        if (ivers==3) open(unit=10,file=cfile(ibody))
      endif
      call VSOP87Z(zjde,ivers,ibody,prec,lu,r,ierr,md)
      if (md(ibody)==1) close(10)
      call kugelko(r(1),r(2),r(3),rku)
!c    write(6,'(/''  x, y, z  = '',3f14.10)') (r(i),i=1,3)
!c    write(6,'( ''  vx,vy,vz = '',3f14.10)') (r(i),i=4,6)
!c    write(6,'( ''  L, B, r  = '',3f14.10)') (rku(i),i=1,3)
      do iu=ix,6,5
        if (ierr/=0) write(iu,'(''  In VSOP87Z: ierr = '',i2)')ierr
      enddo
      end subroutine

      subroutine vsop3(l,k,ix,ke,time,res)
!-----Bahn-Elemente, abgeleitet aus VSOP82 (nach Meeus)--------------
!     fuer J2000.0 und Ekliptik der Epoche; Berechnung der wahren
!     Anomalie (ekliptikale Laenge) mit der Keplerschen Gleichung.
!     (Index von res  1: L, 2: a, 3: e, 4: i, 5: Omega, 6: pi, 7: M,
!                     8: omega, 9: E, 10: nue, 11: eklipt. Laenge)
      use base, only : pidg,gdpi
      use astro, only : par3
      implicit double precision (a-h,o-z)
      dimension :: res(12),t(0:3)
      u360 = 360.d0; ke = 0; eps = 1.d-13; t(0) = 1.d0
      do i=1,3; t(i) = t(i-1)*time; enddo
      do j=1,6; resu = 0.d0
        do i=1,4
          resu = resu + par3(i,j,k,l)*time**(i-1)
```

```
          if (j==1 .or.j>=5) call reduz(resu,0,1)
          res(j) = resu
        enddo
      enddo
      res(7) = res(1) - res(6)
      if (res(7)<0.d0) res(7) = res(7) + u360
      res(8) = res(6) - res(5)
      if (res(8)<0.d0) res(8) = res(8) + u360

! . . Loesung der Keplerschen Gleichung (Resultat: zen)
      ii = 0; E = res(3); zm = res(7)*pidg; ze = zm
      itmax = 100  ! Maximalzahl der Iterationen

      meth = 1  ! Drei iterative Methoden zur Auswahl (meth = 1..3)
      if (meth<3) then
        do
         if (meth==1) then
!     1. Verfahren von Newton-Raphson (schnellste Methode)
           zen = ze + (zm + E*dsin(ze) - ze)/(1.d0 - E*dcos(ze))
         else
!     2. Fixpunktverfahren (Keplersche Gleichung)
           zen = zm + E*dsin(ze)
         endif
         if (dabs(zen-ze)<eps) exit
         if (ii>itmax) then; ke = 2; go to 20; endif
         ii = ii+1; ze = zen
        enddo
      else
!     3. Sekantenverfahren (verwendet Sekantensteigung)
         ke = 1; ze2 = zm
   10    fze2 = zm + E*dsin(ze2) - ze2
         call sekante(ze1,ze2,fze1,fze2,eps,0.1d0,ii,itmax,ix,ke)
         if (ke==1) go to 10
         if (ke==2) go to 20 ! ("Ringfit" hat hier keinen Zeitvorteil
         zen = ze2           ! gegenueber "sekante", da die Keplersche
      endif                  ! Gleichung weniger Rechenzeit benoetigt
      go to 30               ! als "Ringfit" selbst.)

! . . zu viele Iterationen
   20 do iu=ix,6,5
        write(iu,'(/''  ---->  error in "vsop3" '', &
          & ''(Keplers equation), ke ='',I2/)') ke
      enddo; return
   30 res(9) = zen*gdpi; if (res(9)<0.d0) res(9) = res(9) + u360

! . . Berechnung der wahren Anomalie
      res(10) = 2.d0 * datan(dsqrt((1.d0 + E)/(1.d0 - E)) &
                * dtan(zen*0.5d0))*gdpi
      if (res(10)<0.d0) res(10) = res(10) + u360
      res(11) = res(10) + res(6)
      if (res(11)>u360) res(11) = res(11) - u360
      end subroutine

      subroutine transit(ip,ikomb,imod,ipla,ilin,iap,ivers,isep, &
        ical,iuniv,tr,sepmin,itt,sep,zjde,id5,da5,dmo5,zjahr, &
        rk,md,ddx1,ddx2,dfd,test,itin,is,ires,ix,pan,sd,sl,iop0,inum)
!-----Ueberpruefung der Transite von Merkur bzw. Venus----------------
!     Die berechneten Zeitpunkte sind optional dieselbe ekliptikale
!     Laenge bei Erde und Merkur bzw. Venus, die minimale Separation
!     oder die genauen Phasen. "M" bedeutet "normaler", "C" (geozen-
!     trischer) zentr. Transit des Merkurs und "m"/"c", dass irgend-
```



```
!     wo auf der Erde der Transit partiell/zentral erscheint. Analog
!     stehen "V" und "v" fuer die Venus. Das Minuszeichen "-" bedeu-
!     tet, dass der Planet die Sonne knapp verfehlt und dass der
!     dichteste Abstand der "sichtbaren" Scheiben (Sonnen- und Plane-
!     tenrand) nicht mehr als etwa 1 Prozent des scheinbaren Sonnen-
!     radius' betraegt (verwendet nur bei Syzygy-Berechnungen). Die
!     Planetenscheibe ist in diesem Fall natuerlich nicht sichtbar.
!     Index (ip): 1 = Merkur, 2 = Venus
      use base
      implicit double precision (a-h,o-z)
      dimension :: zi(2),sd(2),tcorr(2),rem(78)
      dimension :: ida(7),da(7),id5(5,7),da5(5,7),pan(5)
      dimension :: r(6),rku(3),rk(12),md(0:9),inum(0:4)
      dimension :: xx(5),yy(5),xk(2),yk(2),test(10)
      character(5) :: dmo,dmo5(5)
      character(1) :: tr,tp(8),sl
      data tp/'M','m','V','v','-',' ','C','c'/
      data blim/0.d0/,shift/0.d0/,xj3/0.d0/,yy3/0.d0/ ! pre-init.
      data ba/0.d0/,del/0.d0/ ! pre-init.

! . . Einige Konstanten
      T = (zjde-zjd0)/tcen
!     Axel D. Wittmann: we = Schiefe der Ekliptik der Epoche
      we = (23.4458042d0 - 0.856033d0 * &
           dsin(0.015306d0 * (T + 0.50747d0))) * pidg
      zi(1) = re(35); zi(2) = re(41)
      wfact = 3600.d0*gdpi; eps = 2.d-7
!     (Der folgende Korrekturfaktor "tcorr" zur Berechnung
!     der minimalen Separation ist nur eine Abschaetzung.)
      do j=1,2; tcorr(j) = tsyn(j)/tsid(j); enddo
      ee = dsqrt(R3a*R3a-R3p*R3p)/R3a
      R3 = R3p/(AE*dsqrt(1.d0-(ee*dsin(we))**2))
      a  = dasin(R0/(AE*re(9)))
      b3 = dasin(R3*re(3*ip)/(re(9)*(re(9)-re(3*ip))))
      bp = dasin(Ra(ip)/(AE*(re(9)-re(3*ip))))
      bmin1 = a-bp; bmin2 = a-bp-b3
      bmax1 = a+bp; bmax2 = a+bp+b3

!.....OPTIONEN 1/ 2: gleiche eklipt. Laenge u. minimale Separation
      if (isep==1) then
        din = dcos(zi(ip)*pidg*tcorr(ip))
        dre = (re(3*ip-1)-re(8))*pidg
        ba  = din*datan(re(3*ip)*dsin(dre)/(re(9)-re(3*ip)*dcos(dre)))
        bap = dabs(ba)
      else
        bap = sepmin
      endif
      if (ikomb==1 .and.imod==1) bmax2 = bmax2*1.8d0
      bout = bmax2*1.01d0; tr = tp(6)
      if (bap<=bmin2) tr = tp(2*ip-1)
      if (bap>bmin2.and.bap<=bmax2) tr = tp(2*ip)
      if (bap>bmax2.and.bap<=bout.and.ilin>=3) tr = tp(5)
      if (isep<=2 .and.ilin<=2) then
        if (bap<=bp+b3) tr = tp(8)
        if (bap<=bp) tr = tp(7)
      endif
!c    do iu=ix,6,5; write(iu,'(a15,a18,i3,5f8.5)')'ip,bmin2,bmin1,', &
!c      'bmax1,bmax2,bap = ',ip,bmin2,bmin1,bmax1,bmax2,bap; enddo

! . . Min. Separation (sep) zw. Sonne und Planet in Bogensekunden.
!     "Plus/minus" bedeutet noerdlich/suedlich des Sonnenzentrums.
```

```
      if (isep==1) then
        sep = ba*wfact
      else
        sep = bap*wfact; if (re(3*ip-1)<0.d0) sep = -sep
      endif
      if (isep<=2) then
        if (tr==' '.or.ilin>=3) return; go to 60
      endif

!.....OPTIONEN 3/ 4: Transitphasen ohne/mit Positionswinkeln
!     (Beginn, Ende und minimale Separation des geozentrischen Tran-
!     sits => Ein, drei oder fuenf Zeitpunkte werden berechnet.)
      if (bap>bmax2*1.005d0 .or.(ikomb==1 .and.imod==1)) then
        itt = 0; return
      endif

! . . Weitere Parameter festlegen
      prec = z0; lu = 10; itr = 1
      do j=1,78; rem(j) = re(j); enddo
      do j=1,5
        do k=1,7; id5(j,k) = 0; da5(j,k) = z0; enddo
      enddo
      xj2 = zjde

! . . Mitte des Transits, minimale Separation mit Lichtlaufzeit
      if (itr==1) then
        idr = 3; ke = 1; indx = 1
        step = 5.d-2; iflag = 0
        ddx1 = dfd + 1.d0; nu = 0
        if (ilin<=2) ddx1 = 1; ddx2 = ddx1
        xx(1) = xj2; itin = 0; iex = 0
        do j=1,10; test(j) = z0; enddo
!       Mittlere Laufzeit des Lichtes, optimierter Startwert [Tage]
        if (ip==1) del = 320.d0/86400.d0  ! Merkur
        if (ip==2) del = 150.d0/86400.d0  ! Venus
        if (imod==1) then; ept=3.d-14; else; ept=2.d-9; endif

!       VSOP87-Berechnung mit Beruecksichtigung der Lichtlaufzeit
   10   if (imod==1) then
         call vsop1tr(ip,rk,(xj2-zjd0-del)/tmil,del,r3i,ept,inum,resu)
        else
         call vsop2tr(xj2-del,ivers,ip,md,ix,prec,lu,r,rk, &
           ierr,del,r3i,ept,inum,rku)
        endif
        if (iex==1) go to 20
!       Bestimmung: auf- bzw. absteigender Knoten
        if (nu==1 .or.nu==2) then
          xk(nu) = xj2; yk(nu) = re(3*ip-1)
        endif
        if (nu==2) then
          sl = '/'; if ((yk(2)-yk(1))/(xk(2)-xk(1))<0.d0) sl = ' '
        endif
!       Ende Knotenbestimmung
        call sepa(ip,2,rk,sep0i); yy(indx) = sep0i
        epv = 1.d-6; if (sep0i<30.d0) epv = 1.d-7
        call fitmin(imod,2,iap,ke,xx,yy,epv,step,nu,iflag, &
              ddx1,ddx2,test,itin,indx,ix)
        xj2 = xx(indx)
        if (ke==0 .and.isep==4 .and.iex==0) then
          iex = 1; go to 10
        endif
```



```
        if (ke==1) go to 10

!       Art des (streifenden) Transits
   20   if (sep0i<=bmin2) then; tr=tp(2*ip-1); itt=3; endif
        if (sep0i>bmin2.and.sep0i<=bmin1) itt=3
        if (sep0i>bmin1.and.sep0i<=bmax1) itt=2
        if (sep0i>bmax1.and.sep0i<=bmax2) itt=1
        if (sep0i>bmax2) then; itt = 0; return; endif
        if (sep0i>bmin2.and.sep0i<=bmax2) then
          inum(3) = inum(3) + 1
          tr=tp(2*ip)
        endif
        sep = sep0i*wfact
        if (re(3*ip-1)<0.d0) sep = -sep
        xjdt = xj2
        zjde = xj2
        if (iuniv==2) call delta_T(xjdt)
        call jdedate(xjdt,ical,ida,da,dmo)
        call ephim(1,iaph,ipla,ical,ak,iak,zjde,zjahr,delt)

!       Berechnung des Positionswinkels (minimale Separation)
        if (isep==4) call pos_angle(ip,zjde,rk,ang)

!       Radien (semidiameter) von Sonne und Merkur/Venus
        if (isep>=3 .and.ilin<=2) then
          sd(1) = dasin(R0/(AE*re(9))) * wfact
          sd(2) = dasin(Ra(ip)/(AE*r3i)) * wfact
!         Kennzeichnung des zentralen Transits
          csep = (r3*re(3*ip)/re(9)+Ra(ip)/AE)*wfact/(re(9)-re(3*ip))
          if (dabs(sep)<csep) then
            tr = tp(8)
            if (dabs(sep)<sd(2)) tr = tp(7)
            inum(4) = inum(4) + 1
          endif
!         Mit der zeitlichen Verschiebung "shift" (in julian. Tagen)
!         wird der spaeter folgende Startpunkt fuer "ringfit" bzw.
!         "sekante" moeglichst nahe an die Nullstelle verlegt.
          wu = 1.d0-(sep/sd(1))**2
          if (wu<1.d-2) wu = 1.d-2
          if (ip==1) shift = 0.115d0 * dsqrt(wu)
          if (ip==2) shift = 0.17d0  * dsqrt(wu)
        endif
      endif

      if (itr==1) then
        if (itt==1) itr = 6
        go to 50
      endif

! . . Vorbereitung zur naechsten Berechnung im selben Transit
   30 iis = 0; ke = 1
      itr = itr + 1
!     Kontaktpunkt I
      if (itr==2) then
        idr = 1; blim = bmax1
        xj2 = zjde - shift
      endif
!     Kontaktpunkt II
      if (itr==3) then
        if (itt==2) itr = 5
        idr = 2; blim = bmin1
```

```
        xj2 = zjde - shift
      endif
!     Kontaktpunkt III
      if (itr==4) then
        idr = 4; blim = bmin1
        xj2 = zjde + shift
      endif
!     Kontaktpunkt IV
      if (itr==5) then
        idr = 5; blim = bmax1
        xj2 = zjde + shift
      endif

! . . Berechnung der Kontaktzeiten I bis IV
      if (imod==1) then; ept=1.d-12; else; ept=2.d-7; endif
   40 tau = (xj2 - zjd0)/tmil
!     VSOP87D Kurzversion (imod=1), VSOP87C Vollversion (imod=2)
      if (imod==1) then
        call vsop1tr(ip,rk,tau,del,r3i,ept,inum,resu)
      else
        call vsop2tr(xj2,ivers,ip,md,ix,prec, &
          lu,r,rk,ierr,del,r3i,ept,inum,rku)
      endif
!     "Sekante" wurde durch das etwas schnellere "ringfit" ersetzt.
      call sepa(ip,2,rk,sep0i)
      yy2 = sep0i-blim
      call ringfit(xj1,xj2,xj3,yy1,yy2,yy3,eps,1.d-3,iis,25,ix,ke)
      if (ke==1 .or.ke==5) go to 40
      if (ke==2) go to 60
      xjdt = xj2 + del
      if (iuniv==2) call delta_T(xjdt)
      call jdedate(xjdt,ical,ida,da,dmo)

! . . Berechnung des Positionswinkels (Planet am Sonnenrand)
      if (isep==4 .and.itr/=1) call pos_angle(ip,xj2,rk,ang)

! . . Ruecksprung
   50 do k=1,7; id5(idr,k) = ida(k); da5(idr,k) = da(k); enddo
      dmo5(idr) = dmo; pan(idr) = ang
      if (itr<=4) go to 30
      do j=1,78; re(j) = rem(j); enddo

!.....Berechnung der Transitserie
   60 if (ikomb==0 .or.(ikomb==1 .and.imod==2)) &
        call tserie(ip,zjde,is,iop0,ires)
      end subroutine

      subroutine sepa(ip,iv,rk,sep0i)
!-----Berechnung der Separation Sonne-Merkur bzw. Sonne-Venus---------
!     Index ip:  1 = Merkur, 2 = Venus
      use base, only : pidg,re
      implicit double precision (a-h,o-z)
      dimension :: rk(12),rd(3)
      if (iv==1) then
! . . . 1. Variante - raeumliche Geometrie (Testvariante)
        cos0i = dsin(re(3*ip-1)*pidg) * dsin(re(8)*pidg) + &
                dcos(re(3*ip-1)*pidg) * dcos(re(8)*pidg) * &
                dcos((re(3*ip-2)-re(7))*pidg)
        sep0i = datan(re(3*ip)*dsqrt(1.d0-cos0i*cos0i)/ &
                      (re(9)-re(3*ip)*cos0i))
      else
```



```
! . . . 2. Variante - Vektoranalysis
        do j=1,3; rd(j) = rk(3*(ip-1)+j) - rk(6+j); enddo
        ab = -rk(7)*rd(1)-rk(8)*rd(2)-rk(9)*rd(3)
        a  = dsqrt(rk(7)**2 + rk(8)**2 + rk(9)**2)
        b  = dsqrt(rd(1)**2 + rd(2)**2 + rd(3)**2)
        sep0i = dacos(ab/(a*b))
      endif
      end subroutine

      subroutine pos_angle(ip,xjd,rk,ang)
!-----Positionswinkel des Planeten fuer beliebigen Transit in Bezug
!     auf die Richtung zum Himmelsnordpol (y-Achse auf Sonnenscheibe),
!     vergleiche scheinbare Bewegungsrichtung der Sonne.
!     ip        : 1 fuer Merkur, 2 fuer Venus
!     xjd       : Zeitpunkt der Ankunft des Lichtes auf der Erde
!     rk(1..9)  : rechtwinklige heliozentrische Koordinaten
!                 von Merkur, Venus und Erde (VSOP87C)
!     eeps      : Stellung Erdachse gegen Ekliptik in jener Epoche
!     rgeo(1..9): transformierte geozentrische Koordinaten von Sonne,
!                 Merkur und Venus (rechtwinklig, dann sphaerisch)
!     ang       : Positionswinkel des Planeten vor der Sonne
      use base, only : pidg,gdpi,zjd0,tcen
      implicit double precision (a-h,o-z)
      dimension :: rk(12),rgeo(9),rku(3),xx(3)
      do i=1,9; rgeo(i) = rk(i); enddo

!.....Die Berechnung des Positionswinkels erfolgt in 4 Schritten.
!     Schritte 1-3: Koordinatentransformation helio- zu geozentrisch.

!  1. Rotation um x-Achse um Winkel der Schiefe der Ekliptik (Epoche);
!     Axel D. Wittmann: "On the variation of the obliquity of the
!     ecliptic", Univ.-Sternwarte Goettingen, 1984, MitAG 62, S.203
      T = (xjd-zjd0)/tcen
      eeps = (23.4458042d0 - 0.856033d0 * &
             dsin(0.015306d0 * (T + 0.50747d0))) * pidg
      call rotmat(1,-eeps,0.d0,0.d0,rgeo)

!  2. Translation des heliozentrischen Koordinatenursprungs von der
!     Sonne zur Erde. Das ergibt neue Koordinaten fuer Sonne und
!     Merkur bzw. Venus.
      do i=1,3
        xx(i) = -rgeo(6+i); rgeo(6+i) = rgeo(3+i)
        rgeo(3+i) = rgeo(i); rgeo(i) = 0.d0
      enddo
      call translat(xx(1),xx(2),xx(3),rgeo)

!  3. Umrechnung in sphaerische Koordinaten
!     (Positionen von Sonne, Merkur und Venus)
      do i=0,6,3
        call kugelko(rgeo(i+1),rgeo(i+2),rgeo(i+3),rku)
        do j=1,3; rgeo(i+j) = rku(j); enddo
      enddo

!  4. Berechnung des Positionswinkels nach Andre Danjon: "Astronomie
!     Generale", S.36, Gl."3 bis". Siehe auch Jean Meeus: "Transits",
!     S.15 ("kartesische" Koordinaten x und y in Bogensekunden).
      sdec = rgeo(2) * pidg
      dra  = (rgeo(3*ip+1)-rgeo(1)) * pidg
      ddec = (rgeo(3*ip+2)-rgeo(2)) * pidg
      tdra = dsin(sdec) * dtan(dra) * dtan(dra*0.5d0)
      zk = 206264.8062d0/(1.d0 + dsin(sdec) * tdra)
```

```
      x = -zk * (1.d0 - dtan(sdec)*dsin(ddec)) * dcos(sdec)*dtan(dra)
      y =  zk * (dsin(ddec) + dcos(sdec) * tdra)
      ang = datan(-x/y)*gdpi
      if (y*dcos(ang*pidg)<0.d0) ang = ang + 180.d0
      call reduz(ang,0,1)
      end subroutine

      subroutine tserie(ip,zjde,is,iop0,ires)
!-----Bestimmung der Transit-Serie-----------------------------------
!     Die Seriennummern entsprechen denen der "NASA Eclipse Web Site".
!     (Die Liste der Seriennummern "inserie.t" wird nur einmal verwen-
!     det, um die Startnummern, d.h. die Nummern zu bestimmen, die den
!     ersten gefundenen Transiten zugeordnet werden. Danach werden al-
!     le weiteren Seriennummern unabhaengig von der Liste berechnet.)
!     Index (ip):  1 = Merkur
!                  2 = Venus
      use astro, only : ser,ase,cc,t13BC,t17AD, &
        zstart,ise,ji,jj,isflag,ismax
      implicit double precision (a-h,o-z)
      if (dabs(zstart-99.99d0)<1.d-10) zstart = zjde
      if (iop0/=-804) then
        if (zjde<t13BC-365.d0 .or.zjde>t17AD+365.d0) then
          ires = 999
          return
        endif

! . . . Seriennummer (is) fuer Startzeitpunkt suchen
        if (isflag==0) then
          do j=jj(2*ip-1),jj(2*ip)
            if (ser(j,ip)>zjde) then
              is = j
              isflag = 1
              exit
            endif
          enddo
        endif
      endif

! . . Aktuelle Seriennummer bestimmen
      kflag = 0
      do j=is-ji(ip),is
        zlim = dmax1(t13BC,zstart)
        if (zjde-zlim>cc(ip)+100.d0) then
          do k=jj(2*ip-1),is
            ise(k) = 1
          enddo
        endif
        a = (zjde-ser(j,ip))/cc(ip)
        x = dabs((a-dnint(a))*cc(ip))
        b = dabs(zjde-ase(j)-cc(ip))
!c      write(6,'(''a,x,b,ise(j),j,is,ismax ='',f9.3,f10.3,f16.6, &
!c       & i3,3i5)')a,x,b,ise(j),j,is,ismax
        if (x<10.d0 .and.(b<2.d0 .or.ise(j)==0)) then
          ires = j
          kflag = 1
          if (j>ismax) ismax = j
        endif
        if (j==is.and.kflag==1) go to 20
      enddo
      if (ismax==-10000 .or.is>ismax) ismax = is - 1
      is = ismax + 1
```



```
      ismax = is
      ser(is,ip) = zjde
      ires = is
   20 ase(ires) = zjde
      ise(ires) = 1
      end subroutine

      subroutine VSOP87Z(tdj,ivers,ibody,prec,lu,r,ierr,md)
!-------------------------------------------------------------------
! >>
! >>  UPGRADE (by H. Jelitto): As proposed by Bretagnon and Francou
! >>  for rapidity of computation, the parameters in the VSOP87-files
! >>  are read only once at the first call for each planet. The main
! >>  data are copied into the 5-dimensional array "par2" for random
! >>  access, covering all planets of one VSOP87-version. For the
! >>  calculation of the transit phases (TYMT test), this reduces the
! >>  computing time by a factor 20 to 30. Thus, the original subrou-
! >>  tine "VSOP87" is extended and renamed as "VSOP87Z."
! >>
! >>  The new VSOP87Z-routine has been checked only for the use of the
! >>  theory versions VSOP87A and VSOP87C. Furthermore, the code is
! >>  converted to the Fortran 95 standard and the free source form.
! >>  The version VSOP87D is applied only in a short form, taken from
! >>  the book "Astronomical Algorithms" of Jean Meeus --> vsop1.
! >>
! >>  PARALLEL PROCESSING: To realize parallel processing, the VSOP87-
! >>  subroutine is further modified with the application programming
! >>  interface (API) "OpenMP." For compilation of P5, we use the com-
! >>  mand: "gfortran -fopenmp -static-libgfortran -O3 -Wall p5.f95."
! >>  For single-thread application, use: "gfortran -static -O3 -Wall
! >>  p5.f95." VSOP87Z is adapted to any number of threads (including
! >>  one). Notice: For the parallelization, the if-statement for com-
! >>  parison with the parameter p in the inner do-loop had to be de-
! >>  activated. This statement probably had an advantage in former
! >>  times, when the data were read from magnetic tape. However, this
! >>  branching is not allowed from an OpenMP structured block.
! >>
! >>  The following text belongs to the original VSOP87-subroutine.
! >>  (The quantity "ua" indicates the astronomical unit.)
! >>
!-------------------------------------------------------------------
!
!     Reference : Bureau des Longitudes - PBGF9502
!
!     Object :
!
!     Substitution of time in VSOP87 solution written on a file. The
!     file corresponds to a version of VSOP87 theory and to a body.
!
!     Input :
!
!     tdj      julian date (real double precision).
!              time scale : dynamical time TDB.
!
!     ivers    version index (integer).
!              0: VSOP87 (initial solution).
!                 elliptic coordinates
!                 dynamical equinox and ecliptic J2000.
!              1: VSOP87A.
!                 rectangular coordinates
!                 heliocentric positions and velocities
```

```
!              dynamical equinox and ecliptic J2000.
!           2: VSOP87B.
!              spherical coordinates
!              heliocentric positions and velocities
!              dynamical equinox and ecliptic J2000.
!           3: VSOP87C.
!              rectangular coordinates
!              heliocentric positions and velocities
!              dynamical equinox and ecliptic of the date.
!           4: VSOP87D.
!              spherical coordinates
!              heliocentric positions and velocities
!              dynamical equinox and ecliptic of the date.
!           5: VSOP87E.
!              rectangular coordinates
!              barycentric positions and velocities
!              dynamical equinox and ecliptic J2000.
!
!     ibody    body index (integer).
!           0: Sun (not used here in VSOP87Z)
!           1: Mercury
!           2: Venus
!           3: Earth
!           4: Mars
!           5: Jupiter
!           6: Saturn
!           7: Uranus
!           8: Neptune
!           9: Earth-Moon barycenter
!
!     prec     relative precision (real double precision).
!
!           if prec is = 0 then the precision is the precision
!              p0 of the complete solution VSOP87.
!              Mercury    p0 =  0.6 10**-8
!              Venus      p0 =  2.5 10**-8
!              Earth      p0 =  2.5 10**-8
!              Mars       p0 = 10.0 10**-8
!              Jupiter    p0 = 35.0 10**-8
!              Saturn     p0 = 70.0 10**-8
!              Uranus     p0 =  8.0 10**-8
!              Neptune    p0 = 42.0 10**-8
!
!           if prec is not equal to 0, let us say in between p0 and
!           10**-2, the precision is :
!              for the positions :
!              - prec*a0 ua for the distances.
!              - prec rd for the other variables.
!              for the velocities :
!              - prec*a0 ua/day for the distances.
!              - prec rd/day for the other variables.
!                a0 is semi-major axis of the body.
!              Mercury    a0 =  0.3871 ua
!              Venus      a0 =  0.7233 ua
!              Earth      a0 =  1.0000 ua
!              Mars       a0 =  1.5237 ua
!              Jupiter    a0 =  5.2026 ua
!              Saturn     a0 =  9.5547 ua
!              Uranus     a0 = 19.2181 ua
!              Neptune    a0 = 30.1096 ua
!
```



```
!     lu       logical unit index of the file (integer).
!              The file corresponds to a version of VSOP87 theory and
!              a body, and it must be defined and opened before the
!              first call to subroutine VSOP87.
!
!     Output :
!
!     r(6)     array of the results (real double precision).
!
!              for elliptic coordinates :
!                  1: semi-major axis (ua)
!                  2: mean longitude (rd)
!                  3: k = e*cos(pi) (rd)
!                  4: h = e*sin(pi) (rd)
!                  5: q = sin(i/2)*cos(omega) (rd)
!                  6: p = sin(i/2)*sin(omega) (rd)
!                     e:     eccentricity
!                     pi:    perihelion longitude
!                     i:     inclination
!                     omega: ascending node longitude
!
!              for rectangular coordinates :
!                  1: position x (ua)
!                  2: position y (ua)
!                  3: position z (ua)
!                  4: velocity x (ua/day)
!                  5: velocity y (ua/day)
!                  6: velocity z (ua/day)
!
!              for spherical coordinates :
!                  1: longitude (rd)
!                  2: latitude (rd)
!                  3: radius (ua)
!                  4: longitude velocity (rd/day)
!                  5: latitude velocity (rd/day)
!                  6: radius velocity (ua/day)
!
!     ierr     error index (integer).
!                  0: no error.
!                  1: file error (check up ivers index).
!                  2: file error (check up ibody index).
!                  3: precision error (check up prec parameter).
!                  4: reading file error.
!
!----------------------------------------------------------------------

!     -------------------------------
!     Declarations and initializations
!     -------------------------------
      use astro, only : par2,it2,in2,iv2
      implicit double precision (a-h,o-z)
      character(7) :: bo,body(0:9)
      dimension :: r(6),t(-1:5),a0(0:9),md(0:9)
      data body/'SUN','MERCURY','VENUS','EARTH','MARS','JUPITER', &
                'SATURN','URANUS','NEPTUNE','EMB'/
      data a0/0.01d0,0.3871d0,0.7233d0,1.d0,1.5237d0,5.2026d0, &
             9.5547d0,19.2181d0,30.1096d0,1.d0/
      data dpi/6.283185307179586476d0/
      data t/0.d0,1.d0,5*0.d0/
      data t2000/2451545.d0/
      data a1000/365250.d0/
```

```
      k=0; ierr=3
      if (md(ibody)==1) then
        ideb=0
        do i=1,3; do j=0,5; it2(j,i,ibody) = -1; enddo; enddo
      endif
      do i=1,6; r(i)=0.d0; enddo
      t(1)=(tdj-t2000)/a1000
      do i=2,5; t(i)=t(1)*t(i-1); enddo
      if (prec<0.d0 .or.prec>1.d-2) return
      if (md(ibody)/=1) ierr = 0
!v    q=dmax1(3.d0,-dlog10(prec+1.d-50))

!     -----------------------------------------------------------
!     File reading, for each planet only at first call to VSOP87Z
!     -----------------------------------------------------------
      if (md(ibody)==1) then
   10   read (lu,1001,end=20) iv,bo,ic,it,inn
        iv2(ibody) = iv
        it2(it,ic,ibody) = 1
        in2(it,ic,ibody) = inn
        if (ideb==0) then
          ideb=1; ierr=1
          if (iv/=ivers) return
          ierr=2
          if (bo/=body(ibody)) return
          ierr=0
        endif
        if (inn==0) go to 10
        do n=1,inn
          read (lu,1002) (par2(n,i,it,ic,ibody),i=1,3)
        enddo
        go to 10
   20   md(ibody) = 2
      endif

!     ------------------------------------
!     Computation of planetary coordinates
!     ------------------------------------
      ic = 1; it = 0
      iv = iv2(ibody)
      if (iv==0) k=2
      if (iv==2 .or.iv==4) k=1
   30 inn = in2(it,ic,ibody)
      if (inn==0) go to 50
!v    p=prec/10.d0/(q-2)/(dabs(t(it))+it*dabs(t(it-1))*1.d-4+1.d-50)
!v    if (k==0 .or.(k/=0 .and.ic==5-2*k)) p=p*a0(ibody)
!$omp parallel do reduction(+:r) shared(inn,par2,it,ic,ibody,t) &
!$omp private(n,a,b,c,cu)
      do 40 n=1,inn
!       a = par2(n,1,it,ic,ibody)   [a,b,c are replaced in cu and
!       b = par2(n,2,it,ic,ibody)   r(ic) because of speed increase.]
!       c = par2(n,3,it,ic,ibody)
!v      if (dabs(a)<p) go to 50
!v      u = b + c*t(1)
        cu = dcos(par2(n,2,it,ic,ibody) + par2(n,3,it,ic,ibody)*t(1))
        r(ic) = r(ic) + par2(n,1,it,ic,ibody)*cu*t(it)
!v      if (iv==0) go to 40
!v      su=dsin(u)   ! velocity of planet (not used)
!v      r(ic+3)=r(ic+3)+t(it-1)*it*a*cu-t(it)*a*c*su
   40 enddo
!$omp end parallel do
```



```
   50 if (it<=4 .and.it2(it+1,ic,ibody)/=-1) then
        it = it + 1
        go to 30
      else
        if (ic<3) then
          it = 0
          ic = ic + 1
          go to 30
        endif
      endif
      if (iv/=0) then
        do i=4,6
          r(i)=r(i)/a1000
        enddo
      endif
      if (k==0) return
      r(k)=dmod(r(k),dpi)
      if (r(k)<0.d0) r(k)=r(k)+dpi
      return

!     -------
!     Formats
!     -------
 1001 format (17x,i1,4x,a7,12x,i1,17x,i1,i7)
 1002 format (79x,f18.11,f14.11,f20.11)
      end subroutine

      subroutine kartko(ison)
!-----Umwandlung in kartesische Koordinaten, re(1..9) --> xyr(1..9)---
!     mit Merkur bei x-Achse
!     Indizes von "re" :  1: Lm'  2: Bm   3: rm   4: Lv'   5: Bv
!                         6: rv   7: Le'  8: Be   9: re
!     Indizes von "xyr":  1: xm   2: ym   3: zm   4: xv    5: yv
!                         6: zv   7: xe   8: ye   9: ze   10: leer
      use base
      implicit double precision (a-h,o-z)
      rr = re(1)
      if (ison==2) rr = re(4)
      if (ison==0) rr = 0.d0
      do i=3,9,3
        xyr(i-2) = re(i)*dcos(re(i-1)*pidg)*dcos((re(i-2)-rr)*pidg)
        xyr(i-1) = re(i)*dcos(re(i-1)*pidg)*dsin((re(i-2)-rr)*pidg)
        xyr(i)   = re(i)*dsin(re(i-1)*pidg)
      enddo
      end subroutine

      subroutine relpos(ipla,ison,ijd,iek,iekk,ika)
!-----Vergleich der Positionen Pyramiden/Kammern mit Planeten,--------
!     daraus Bestimmung der Genauigkeit Fpos bzw. xyr(36) in Prozent
!     und der Polaritaet "iek" bzw. "iekk".
!     Weitere Indizes von "xyr":
!        11: xv-xm   12: xe-xm   13: xe-xv   14: yv-ym   15: ye-ym
!        16: ye-yv   17: zv-zm   18: ze-zm   19: ze-zv   20: leer
!        21: v - m   22: e - m   23: e - v   24: q1      25: q2
!        26: q3      27: alpha'  28: beta'   29: gamma'  30: leer
!        31: x-Son   32: y-Son   33: z-Son   34: delta-s 35: M
!        36: Fpos, F'pos, F"pos
!     Indizes 11-19 und 21-29 bei "pyr" und "xyr" entsprechen sich.
      use base
      implicit double precision (a-h,o-z)
```

```
!  . . Pyramidenabstaende
       xyr(11) = xyr(4)-xyr(1);  xyr(12) = xyr(7)-xyr(1)
       xyr(13) = xyr(7)-xyr(4);  xyr(14) = xyr(5)-xyr(2)
       xyr(15) = xyr(8)-xyr(2);  xyr(16) = xyr(8)-xyr(5)
       xyr(17) = xyr(6)-xyr(3);  xyr(18) = xyr(9)-xyr(3)
       xyr(19) = xyr(9)-xyr(6)
       ax = xyr(11);  ay = xyr(14)
       bx = xyr(12);  by = xyr(15)
       cx = xyr(13);  cy = xyr(16)
       if (ison==3) then
         az = z0; bz = z0
         cz = z0
       else
         az = xyr(17); bz = xyr(18)
         cz = xyr(19)
       endif

!  . . Feststellen der Polaritaet (Blickrichtung auf die Ekliptik)
!      gemaess Vorzeichen der z-Komponente des Vektorproduktes a x c.
       if (ijd==15 .or.ijd==0) then
         if (iek/=3) iek  = 1
         if (iek==3) iekk = 1
         ez = ax*cy-ay*cx
         if ((ipla==1 .and.ez>=z0).or.(ipla==2 .and. &
           ((ez<z0.and.(ika==1 .or.ika==4 .or.ika==5)).or. &
           (ez>=z0.and.(ika==2 .or.ika==3 .or.ika==6))))) then
           if (iek/=3) iek  = 2
           if (iek==3) iekk = 2
         endif
       endif

!  . . Berechnung der rel. Abweichung [%] --> xyr(36)
!      Sonnenposition auf Nordsuedachse
       if (ison<=2) then
         xyr(24) = bx/ax; xyr(25) = by/ay; xyr(26) = by/bx
         s = 1.d0
         if (iek==3 .and.iekk==2) s = -1.d0
         dx1 = (xyr(24) - pyr(24))/pyr(24)
         dx2 = (xyr(25) - pyr(25))/pyr(25)
         dx3 = (xyr(26)-s*pyr(26))/pyr(26)
         xyr(36) = 100.d0 * dsqrt((dx1*dx1 + dx2*dx2 + dx3*dx3)/3.d0)
         return
       endif

!.....Relative Abweichung, Sonnenposition frei (2- und 3-dimensional)
!      Anmerkung: Bei Berechnung von F"pos (Sonnenpos. frei) laesst
!      sich statt der Strecken Mykerinos-/Chefren-Pyramide u. Myker.-/
!      Cheops-Pyramide auch ein anderes Streckenpaar verwenden, wie
!      z.B. Mykerinos-/Chefren-Pyramide und Chefren-/Cheops-Pyramide.
!      F"pos hat dann eventuell etwas andere Werte, aber die Minimie-
!      rung von F"pos liefert dieselben Zeitpunkte. Das heisst, die
!      wesentlichen Ergebnisse bleiben identisch.
       xyr(21) = dsqrt(ax*ax + ay*ay + az*az)
       xyr(22) = dsqrt(bx*bx + by*by + bz*bz)
       xyr(23) = dsqrt(cx*cx + cy*cy + cz*cz)
       xyr(24) = xyr(22)/xyr(21)
!c     xyr(25) = xyr(23)/xyr(21)
!c     xyr(26) = xyr(23)/xyr(22)
       xyr(27) = dacos((ax*bx + ay*by + az*bz)/(xyr(21) * xyr(22)))
!c     xyr(28) = dacos((ax*cx + ay*cy + az*cz)/(xyr(21) * xyr(23)))
!c     xyr(29) = dacos((bx*cx + by*cy + bz*cz)/(xyr(22) * xyr(23)))
```



```
       dx1 = (xyr(24)-pyr(24))/pyr(24)
       dx2 = xyr(27)-pyr(27)
       xyr(36) = 100.d0 * dsqrt((dx1*dx1 + dx2*dx2)*0.5d0)
       end subroutine

       subroutine sonpos(ison,iek,ix,xp3,yp3,zp3, &
         rcm,dmi,iter,iw,ke,m,n,f,x,e,w,y,z)
!-----Bestimmung von Sonnenposition und Massstab --> xyr(31 - 35)-----
!      Indizes von xyr wie in relpos
       use base
       implicit double precision (a-h,o-z)
       dimension :: D(3,3),xsta(n),ysta(m),rcm(3)
       dimension :: x(n),e(n),iw(100),f(m),y(m),z(m),w(1000)

!.....Zweidimensionale Berechnung der Sonnenpos. (x- und y-Koord.)
!      Projektion der Planetenpositionen in die Ekliptikebene.
!      Zusammengehoerige Pyramiden- und Planetenabstaende werden paral-
!      lel ausgerichtet und in der Mitte zur Deckung gebracht. (Wegen
!      des gemeinsamen Massstabsfaktors "zmas" haben die entsprechenden
!      Strecken leicht unterschiedliche Laengen.)
       em = 1.d0
       if (iek==2) em = -1.d0
       if (ison<=3) then
        sax = (xyr(4)+xyr(1)) * 0.5d0
        say = (xyr(5)+xyr(2)) * 0.5d0
        sbx = (xyr(7)+xyr(1)) * 0.5d0
        sby = (xyr(8)+xyr(2)) * 0.5d0
        scx = (xyr(7)+xyr(4)) * 0.5d0
        scy = (xyr(8)+xyr(5)) * 0.5d0
        al1 = - em * pyr(31) - datan(ay/ax) + datan(say/sax)
        al2 = - em * pyr(32) - datan(by/bx) + datan(sby/sbx)
        al3 = - em * pyr(33) - datan(cy/cx) + datan(scy/scx)
        r1 = dsqrt(sax*sax + say*say)
        r2 = dsqrt(sbx*sbx + sby*sby)
        r3 = dsqrt(scx*scx + scy*scy)
        zmas = (pyr(21)/xyr(21) + pyr(22)/xyr(22) + &
                pyr(23)/xyr(23))/3.d0
        xso1 = - r1 * zmas * dcos(al1) + pyr(34)
        xso2 = - r2 * zmas * dcos(al2) + pyr(36)
        xso3 = - r3 * zmas * dcos(al3) + pyr(38)
        yso1 = - r1 * zmas * dsin(al1) + pyr(35) * em
        yso2 = - r2 * zmas * dsin(al2) + pyr(37) * em
        yso3 = - r3 * zmas * dsin(al3) + pyr(39) * em
        xyr(31) = (xso1 + xso2 + xso3)/3.d0
        xyr(32) = (yso1 + yso2 + yso3)/3.d0
        if (iek==2) xyr(32) = - xyr(32)
        xyr(33) = z0

!  . .  Fehlerabschaetzung fuer die Sonnenposition
        xyr(34) = dsqrt((xyr(31)-rcm(1))**2 + (xyr(32)-rcm(2))**2) &
          * xyr(36) * 1.d-2
!  . .  Massstabsfaktor (nur fuer "Sonne" suedlich der
!       dritten Pyramide, zweidimensional gerechnet.)
        xyr(35)=AE*0.25d0*(dabs(xyr(11)/pyr(11))+dabs(xyr(12)/pyr(12))&
                         + dabs(xyr(14)/pyr(14))+dabs(xyr(15)/pyr(15)))
       endif

!.....Dreidimensionale Berechnung (x-, y- und z-Koordinate)
!      Loesung eines linearen inhomogenen Gleichungssystems bzgl. der
!      Planetenpositionen und Uebertragung des Ergebnisses auf die
!      Pyramidenpositionen.
```

```
! . . Erzeugung eines (schiefwinkligen) Vektordreibeins fuer die Pla-
!     neten (mit Hilfe des Vektorproduktes). Die 3 Vektoren bilden
!     dann die Spalten der Koeffizienten-Matrix.
      if (ison==4) then
        D(1,1) = ax; D(2,1) = ay; D(3,1) = az
        D(1,2) = bx; D(2,2) = by; D(3,2) = bz
        dx = by*az - ay*bz
        dy = ax*bz - bx*az
        dz = bx*ay - ax*by
        aba = dsqrt(ax*ax + ay*ay + az*az)
        abb = dsqrt(bx*bx + by*by + bz*bz)
        abd = dsqrt(dx*dx + dy*dy + dz*dz)
        dfakt = (aba + abb) * 0.5d0/abd
        D(1,3) = dx * dfakt
        D(2,3) = dy * dfakt
        D(3,3) = dz * dfakt
! . . . Inversion der Matrix D
        call invert(D)
! . . . Berechnung der Loesung mit x = Inv.(D) * (- Merkur-Koord.)
        x1 = - D(1,1) * xyr(1) - D(1,2) * xyr(2) - D(1,3) * xyr(3)
        x2 = - D(2,1) * xyr(1) - D(2,2) * xyr(2) - D(2,3) * xyr(3)
        x3 = - D(3,1) * xyr(1) - D(3,2) * xyr(2) - D(3,3) * xyr(3)
! . . . Koordinaten der Sonnenposition in Giza
        xyr(31) = x1 * pyr(11) + x2 * pyr(12) + x3 * pyr(7)
        xyr(32) = x1 * pyr(14) + x2 * pyr(15) + x3 * pyr(8)
        xyr(33) = x1 * pyr(17) + x2 * pyr(18) + x3 * pyr(9)
! . . . Massstabsfaktor
        xyr(35) = AE * dsqrt((xyr(12)**2 + xyr(15)**2 + xyr(18)**2)/ &
                             (pyr(12)**2 + pyr(15)**2 + pyr(18)**2))
      endif

!.....Dreidimensionale Berechnung (x-, y- und z-Koordinate)
!     mit Hilfe des Fit-Programms FITEX. Die Konstellation der Plane-
!     ten wird durch Translation, Rotation und Groessenaenderung mit
!     der Anordnung der Pyramiden bzw. der Kammern in der Cheops-Pyra-
!     mide zur Deckung gebracht. Anschliessend wird die resultierende
!     Transformation auf die Sonnenposition (Koordinatenursprung)
!     angewendet.
      if (ison==5) then
        istart = 0
        ke = 0
        if (iter/=0) then
          do iu=ix,6,5; write(iu,*); enddo
        endif

! . . . Koordinatentransformation --> y(i)
        do
          do i=1,m; y(i) = xyr(i); enddo
          call translat(x(1),x(2),x(3),y)
          call rotmat(5,x(4),x(5),x(6),y)
          call mastab(x(7),y)
          if (istart==0) then
            do i=1,n; xsta(i) = x(i); enddo
            do i=1,m; ysta(i) = y(i); enddo
          endif

! . . . . Die Fehlerquadrate dabs(F)**2
          w(4) = z0
          do i=1,m; f(i) = y(i) - z(i); w(4) = w(4) + f(i)*f(i); enddo
          istart = istart + 1
```



```
! . . . . Ausgabe der Iterationen (Aufruf von FITEX)
          do iu=ix,6,5
           if (iter/=0)write(iu,152)iw(3),iw(4),w(3),w(4),(x(i),i=1,n)
          enddo
          call fitex(ke,m,n,f,x,e,w,iw); if (ke/=1) exit
        enddo

! . . . Ausgabe der Ergebnisse
        if (iter/=0) then
          do iu=ix,6,5
            write(iu,153) ke,iw(3),iw(4),w(3),w(4)
            j2 = n+n
            write(iu,154) x,(w(4+j),j=1,j2)
            if (w(5)==z0) go to 10
            j2=4+j2
!c          do i=1,n
!c            j1=j2+1; j2=j1+i-1
!c            write(iu,154) (w(j),j=j1,j2)
!c          enddo
   10       write(iu,*)
            write(iu,'(''  start   x(1..'',i1,''):'',7F13.3)') &
              n,(xsta(i),i=1,3),(xsta(i)*gdpi,i=4,6),xsta(7)
            write(iu,'(''      "    y(1..'',i1,''):'',9f13.3)') &
              m,(ysta(i),i=1,m)
            write(iu,'(''  results x(1..'',i1,''):'',7f13.3)') &
              n,(x(i),i=1,3),(x(i)*gdpi,i=4,6),x(7)
            write(iu,'(''      "    y(1..'',i1,''):'',9f13.3/)') &
              m,(y(i),i=1,m)
          enddo
        endif

! . . . Berechnung der Sonnenposition im Pyramidengelaende mit Hilfe
!       der gerade bestimmten Parameter x(1)..x(7) durch Transforma-
!       tion des Koordinatenursprungs (Sonne)
        do i=1,m; y(i) = z0; enddo
        call translat(x(1),x(2),x(3),y)
        call rotmat(5,x(4),x(5),x(6),y)
        call mastab(x(7),y)
        xyr(31) = y(1)
        xyr(32) = y(2)
        xyr(33) = y(3)
        xyr(35) = AE/x(7)
      endif

      if (ison>=4) then
!.......Korrektur der Koordinaten (1/4 Hoehe oder ganze Hoehe der
!       3. Pyramide bzw. Positionskoordinaten der Felsenkammer)
        xyr(31) = xyr(31) + xp3
        xyr(32) = xyr(32) + yp3
        xyr(33) = xyr(33) + zp3

! . . . Fehlerabschaetzung fuer die Sonnenposition
!c      if (ison==4) then
          dcm = dsqrt((xyr(31)-rcm(1))**2 + (xyr(32)-rcm(2))**2 &
                    + (xyr(33)-rcm(3))**2)
          qu = dcm
          if (dcm<dmi) qu = dmi * ((dcm/dmi)**2 + 1.d0)*0.5d0
          xyr(34) = qu * xyr(36) * 1.d-2
!c      else
!c        xyr(34) = dsqrt(w(4))
!c      endif
```

```
      endif

      return
  152 format(5x,2i5,1p,9e13.5)
  153 format(3i5,1p,8e23.15)
  154 format('  ',1p,6e13.5)
      end subroutine

      subroutine invert(a)
!-----Inversion der 3x3-Matrix a, d.h. a -> inv(a)--------------------
      real(8) :: a(3,3),b(3,3),dei
      integer(2) :: i,j

! . . Die Kofaktoren
      b(1,1) = a(2,2)*a(3,3) - a(2,3)*a(3,2)
      b(1,2) = a(2,3)*a(3,1) - a(2,1)*a(3,3)
      b(1,3) = a(2,1)*a(3,2) - a(2,2)*a(3,1)
      b(2,1) = a(3,2)*a(1,3) - a(3,3)*a(1,2)
      b(2,2) = a(3,3)*a(1,1) - a(3,1)*a(1,3)
      b(2,3) = a(3,1)*a(1,2) - a(3,2)*a(1,1)
      b(3,1) = a(1,2)*a(2,3) - a(1,3)*a(2,2)
      b(3,2) = a(1,3)*a(2,1) - a(1,1)*a(2,3)
      b(3,3) = a(1,1)*a(2,2) - a(1,2)*a(2,1)

! . . Kehrwert der Determinante und Transponieren
      dei = 1.d0/(a(1,1)*b(1,1) + a(1,2)*b(1,2) + a(1,3)*b(1,3))
      do i=1,3; do j=1,3; a(i,j) = b(j,i)*dei; enddo; enddo
      end subroutine

      subroutine rotmat(iachse,w1,w2,w3,a)
!-----Erstellung der Dreh-Matrix und Multiplikation------------------
!     3 Vektoren fuer Merkur bis Erde: a(1..9) --> a(1..9)
!     iachse = 1-3:  Drehung um x-, y- oder z-Achse (Winkel w1)
!
!                                (  cos w1  sin w1   0  )
!                   z.B.   Dz(w1) = ( -sin w1  cos w1   0  )
!                                (    0       0      1  )
!
!     iachse = 4:    Drehung um Knotenlinie (Winkel w1, w2)
!     iachse = 5:    Drehung um beliebige Achse (Winkel w1, w2
!                    und w3: die Eulerschen Winkel)
      implicit double precision (a-h,o-z)
      dimension :: a(9),b(9),D(3,3)
      z0  = 0.d0
      one = 1.d0
      s1 = dsin(w1)
      c1 = dcos(w1)
      if (iachse<=3) then
        do j=1,3; do i=1,3; D(i,j) = z0; enddo; enddo
        if (iachse==1) then
          D(1,1) = one           ! axis 1
          D(2,2) = c1
          D(2,3) = s1
          D(3,2) = - s1
          D(3,3) = c1
        else
          D(1,1) = c1
          if (iachse==2) then
            D(1,3) = s1          ! axis 2
            D(2,2) = one
            D(3,1) = - s1
```



```
            D(3,3) = c1
          else
            D(1,2) = s1          ! axis 3
            D(2,1) = - s1
            D(2,2) = c1
            D(3,3) = one
          endif
        endif
      else
        s2 = dsin(w2)
        c2 = dcos(w2)
        if (iachse==4) then
          D(1,1) = - s1 * s1 * (one - c2) + one    ! axis 4
          D(1,2) =   s1 * c1 * (one - c2)
          D(1,3) = - s1 * s2
          D(2,1) =   s1 * c1 * (one - c2)
          D(2,2) = - c1 * c1 * (one - c2) + one
          D(2,3) =   c1 * s2
        else
          s3 = dsin(w3)
          c3 = dcos(w3)
          D(1,1) =   c1 * c3 - s1 * c2 * s3        ! axis 5
          D(1,2) =   s1 * c3 + c1 * c2 * s3
          D(1,3) =   s2 * s3
          D(2,1) = - c1 * s3 - s1 * c2 * c3
          D(2,2) = - s1 * s3 + c1 * c2 * c3
          D(2,3) =   s2 * c3
        endif
        D(3,1) =   s1 * s2
        D(3,2) = - c1 * s2
        D(3,3) =   c2
      endif

! . . Ausfuehrung der Transformation (Merkur-, Venus- und Erdposition)
!c    do i = 1,3; write(6,'(3f13.8)')(D(i,j),j=1,3); enddo
      do i=1,9; b(i) = z0; enddo
      do k=0,6,3
        do i=1,3
          do j=1,3
            b(k+i) = b(k+i) + D(i,j)*a(j+k)
          enddo
        enddo
      enddo
      do i=1,9; a(i) = b(i); enddo
!c    write(6,'(a12,3f13.8)') '  Mercury : ',(a(j),j=1,3)
!c    write(6,'(a12,3f13.8)') '  Venus   : ',(a(j),j=4,6)
!c    write(6,'(a12,3f13.8)') '  Earth   : ',(a(j),j=7,9)
      end subroutine

      subroutine translat(a1,a2,a3,a)
!-----Translation der Positionen der 3 Planeten----------------------
!     3 Vektoren a(1..9) --> a(1..9)
      real(8) :: a1,a2,a3,a(9)
      integer(2) :: i
      do i=1,7,3
        a(i)   = a(i)   + a1
        a(i+1) = a(i+1) + a2
        a(i+2) = a(i+2) + a3
      enddo
      end subroutine
```

```fortran
      subroutine mastab(zmas,a)
!-----Massstabsaenderung---------------------------------------------
!     3 Vektoren a(1..9) --> a(1..9)
      real(8) :: zmas,a(9)
      integer(2) :: i
      do i=1,9; a(i) = zmas * a(i); enddo
      end subroutine

      subroutine transfo(irb,rku)
!-----Transformation ins Merkurbahn-System (Venusbahn-System)---------
!     re(1..9) --> re(1..9),  xyr(1..9) --> xyr(1..9)
!     Die Transformationen A, B und C liefern dasselbe Ergebnis.
!     Die Eingabewinkel ao, ai, at sind im Modul "base" gespeichert.
      use base
      implicit double precision (a-h,o-z)
      dimension :: xyt(9),rku(3)
      pi2 = pi * 2.d0
      if (irb>=2 .and.irb<=4) then
        ao = (re(34) - re(1))*pidg
      else
        ao = (re(40) - re(1))*pidg
      endif
      if (ao<z0)  ao = ao + pi2
      if (ao>pi2) ao = ao - pi2
!c    write(6,'(a10,f23.8)') '  re(4)    ',re(4)
!c    write(6,'(a10,f23.8)') '  re(40)   ',re(40)
      if (irb>=2 .and.irb<=4) then
        ai = dabs(datan(xyr(3)/(xyr(1)*dsin(ao))))
      else
        rxy = dsqrt(xyr(4)*xyr(4) + xyr(5)*xyr(5))
        aov = (re(40) - re(4))*pidg
        ai  = dabs(datan(xyr(6)/(rxy*dsin(aov))))
      endif
      at = dasin(dsin(ao)/dsqrt(1.d0-(dsin(ai)*dcos(ao))**2))+ao-pi
      a1 = ao; a2 = ai; a3 = at
!c    write(6,'(a12,3f13.8)') '  Mercury : ',(xyr(j),j=1,3)
!c    write(6,'(a12,3f13.8)') '  Venus   : ',(xyr(j+3),j=1,3)
!c    write(6,'(a12,3f13.8)') '  Earth   : ',(xyr(j+6),j=1,3)
      do i=1,9; xyt(i) = xyr(i); enddo

!.....Transformation A  -->  Dz(at) * K(ao,ai)
!     (Reihenfolge der Matrizen von rechts nach links!)
      if (irb==2 .or.irb==5) then
! . . . Matrix K(ao,ai)
        call rotmat(4,a1,a2,z0,xyt)
! . . . Matrix Dz(at)
        if (irb==5) then
          at = datan(xyt(2)/xyt(1))
          a3 = at
        endif
        call rotmat(3,a3,z0,z0,xyt)
      endif

!.....Transformation B  -->  Dz(at-ao) * Dx(ai) * Dz(ao)
      if (irb==3) then
! . . . Matrix Dz(ao)
        call rotmat(3,a1,z0,z0,xyt)
! . . . Matrix Dx(ai)
        call rotmat(1,a2,z0,z0,xyt)
! . . . Matrix Dz(at-ao)
        call rotmat(3,a3-a1,z0,z0,xyt)
```



```fortran
      endif

!.....Transformation C  -->  R(ao,ai,at-ao)
      if (irb==4) then
! . . . Matrix R(ao,ai,at-ao)
        call rotmat(5,a1,a2,a3-a1,xyt)
      endif

! . . Ruecktransformation in Kugelkoordinaten
      do i=1,9; xyr(i) = xyt(i); enddo
      do i=0,6,3
        call kugelko(xyr(i+1),xyr(i+2),xyr(i+3),rku)
        do j=1,3; re(i+j) = rku(j); enddo
      enddo
      end subroutine

      subroutine kugelko(r1,r2,r3,rku)
!-----Umrechnung in Kugelkoordinaten rku(1)..rku(3)-------------------
!     (Index von rku  1: phi, 2: theta, 3: r)
      use base, only : gdpi
      implicit double precision (a-h,o-z)
      dimension :: rku(3)
      ra = dsqrt(r1*r1 + r2*r2)
      rku(1) = datan(r2/r1)*gdpi
      rku(2) = datan(r3/ra)*gdpi
      rku(3) = dsqrt(ra*ra + r3*r3)
      if (r1<0.d0) rku(1) = rku(1) + 180.d0
      if (rku(1)<0.d0) rku(1) = rku(1) + 360.d0
      end subroutine

      subroutine aphelko(imod,ivers,iaph,ipla, &
        ison,ijd,io,iop0,ix,dh3,x,y,rcm,dmi)
!-----Berechnung der "Merkur-Aphelposition" in Giza-------------------
!     fuer Konstell. 13, 14, sowie "quick start option" 322 und 323.
!     Die Berechnung kann mit VSOP87A (ivers=1) und VSOP87C (ivers=3)
!     durchgefuehrt werden. Die Ortsabweichungen im Pyramidengelaende
!     zwischen beiden Versionen liegen fuer Konst. 13 bzw. 14 bei ca.
!     10 cm und 5 mm, bei der Schatten-Konstellation 12 bei ca. 4 mm.
!     Sollte sich an den Zeitpunkten dieser Konstellationen etwas aen-
!     dern, sind die astron. Aphelkoordinaten in "aphelm" anzupassen.
      use base
      implicit double precision (a-h,o-z)
      dimension :: aphelm(18),x(7),y(9),rcm(3)

!.....Sphaerische ekliptikale Koordinaten L, B und r des Merkur-Aphels
!     fuer Konst. 13 und 14 jeweils fuer J2000.0 und Ekl. der Epoche
!     und fuer "Schatten-Konstellation 12" mit J2000.0 (Option 323)
!     und Ekliptik der Epoche (Option 322).
!
! . . A. Berechnung mit Gl. (7.1) --> Konst. 13:  JDE = 5909973.28368
!                                     Konst. 14:  JDE =  671046.63581
!                          Optionen 322 und 323:  JDE = 2849071.14940
!     data aphelm/
!       272.2596751d0, -5.4263369d0, 0.4672908784d0,   (K.13, VSOP87A)
!        46.8137077d0, -6.4048699d0, 0.4670482474d0,   (K.13, VSOP87C)
!       249.5729904d0, -1.9354192d0, 0.4662991040d0,   (K.14, VSOP87A)
!       182.1787524d0, -1.3530604d0, 0.4662950222d0,.. (K.14, VSOP87C)
!
! . . B. r(Mer.) optimiert --> Konst. 13 (VSOP87A): JDE = 5909973.264
!        (r maximal fuer Aphel)          (VSOP87C): JDE = 5909973.255
!                           Konst. 14 (VSOP87A/C): JDE =  671046.632
```

```
      data aphelm/272.2054713d0, -5.4229877d0, 0.4672909313d0, &
                   46.7345218d0, -6.4007584d0, 0.4670483641d0, &
                  249.5625348d0, -1.9341303d0, 0.4662991059d0, &
                  182.1682931d0, -1.3518259d0, 0.4662950244d0, &
                  258.9945271d0, -3.6947988d0, 0.4667842406d0, &
                  274.2350325d0, -3.8355115d0, 0.4667842399d0/

      if ((ijd==13 .or.ijd==14 .or.iop0==322 .or.iop0==323).and. &
       imod<=2 .and.ison==5 .and.iaph==1 .and.ipla==1 .and.io==2) then
        if (ijd==13 .and.ivers==1) j = 1
        if (ijd==13 .and.ivers/=1) j = 4
        if (ijd==14 .and.ivers==1) j = 7
        if (ijd==14 .and.ivers/=1) j = 10
        if (iop0==322) j = 16
        if (iop0==323) j = 13
        do i=4,6; re(i) = aphelm(j+i-4); enddo
!       Umrechnung in kartesische Koordinaten
        call kartko(ison)
!       Koordinatentransformation: Weltraum --> Pyramidengelaende
        do i=4,6; y(i) = xyr(i); enddo
        call translat(x(1),x(2),x(3),y)
        call rotmat(5,x(4),x(5),x(6),y)
        call mastab(x(7),y)
        y(6) = y(6) + dh3
!       Fehler in Metern (dr)
        dcm = dsqrt((y(4)-rcm(1))**2 + (y(5)-rcm(2))**2 &
                  + (y(6)-rcm(3))**2)
        qu = dcm
        if (dcm<dmi) qu = dmi * ((dcm/dmi)**2 + 1.d0)*0.5d0
        dr = qu * xyr(36) * 1.d-2
!       Ausgabe des Ergebnisses
        do iu=ix,6,5
          write(iu,'(''  Mercury aphelion coordinates [m]:'', &
            & f13.2,2f10.2,f9.2)') y(4),y(5),y(6),dr
          call linie(iu,1)
        enddo
      endif
      end subroutine

      subroutine plako(diff,ipla,ijd,ik,ison,ipos, &
        rcm,x,y,ort,rp,dd,dn,dss,pla,plan,emp,text,tt,titab, &
        is12,dmi,zjda,zjde,ivers,md,ix,prec,lu,r,ierr,rku)
!-----Koordinaten fuer Merkur bis Neptun------------------------------
!     und Berechnung der "Planetenpositionen" im Giza-Gelaende fuer
!     Konst. 1-14 mit ison = 5 (FITEX) und imod = 2 (VSOP87-Vollv.).
!     Zusaetzlich:
!     Spezialausgabe fuer Konst. 12 mit iuniv = 1 (TT) und iout = 3
!     (spezial). In diesem Fall sind nur noch folgende Parameter
!     variierbar: ipla (Pyr.- oder Kammerpositionen), imod (VSOP87
!     Voll- oder Kurzv.), lv (VSOP87A oder VSOP87C, bei Vollv.) und
!     ihi (z-Koordinate)
      use base
      implicit double precision (a-h,o-z)
      dimension :: diff(9),r(6),rku(3),md(0:9),x(7),y(9),rcm(3)
      dimension :: ort(0:9,4),rp(3,4),zjda(4)
      character(2) :: dd,dn,dss
      character(3) :: pla(0:9),line
      character(7) :: emp
      character(10) :: plan(0:9)
      character(18) :: date(4)
      character(23) :: text(0:9),tt(2)
```



```
      character(49) :: titab
      data date/'date of chambers: ','date of syzygy:   ', &
                'date of transit:  ','date of pyramids: '/
      data line/'---'/

! . . Tabellenkopf
      do iu=ix,6,5
        if (is12==0) then
          write(iu,*); call linie(iu,1)
          write(iu,*)'pla.   x[AU]     y[AU]     z[AU]      L', &
            '       B       r[AU]     Lm-L     dev.'
          call linie(iu,2)
        else
          write(iu,'(/27x,''Celestial positions in Giza'')')
          call linie(iu,1)
          write(iu,*)' body           x[m]       y[m]      z[m]', &
            '   dr[m]      latitude N     longitude E'
        endif
      enddo

!.....Positionen von Merkur bis Neptun und Sonne im Pyramiden-
!     gelaende und im System innerhalb der Cheops-Pyramide (nur
!     VSOP87-Vollversion)
      icm = 1
      imax = 8
      if (ivers==1) imax = 9
      if (is12/=0)  imax = 8  ! (urspruenglich imax = 4, Aug. 2022)
      icmax = 1; if (is12/=0) icmax = 4
   10 if (is12/=0) then       ! (Spezial-Output, Konst. 14)
        if (ijd==14) then; icm = 4; zjda(icm) = 671034.65042d0; endif
        zjde = zjda(icm)
        do iu=ix,6,5
          call linie(iu,2)
          write(iu,'(4x,a18,''JDE ='',f14.5)') date(icm),zjda(icm)
          call linie(iu,2)
        enddo
      endif
      if (is12/=0 .and.(icm==1 .or.ijd==14)) then ! "Sonnenposition"
        if (ipla==1) then
          call geoko(ort(0,1),-ort(0,2),ipla,iB1,zB2,iL1,zL2)
        else
          call geoko(ort(0,1),ort(0,3),ipla,iB1,zB2,iL1,zL2)
        endif
        do iu=ix,6,5
          write(iu,102) plan(0),(ort(0,j),j=1,4),iB1,zB2,iL1,zL2
        enddo
      endif
      do 20 id=1,imax
        call vsop2(zjde,ivers,id,md,ix,prec,lu,r,ierr,rku)
        dif = re(1) - rku(1); call reduz(dif,0,0)
        err = dif-diff(id); call reduz(err,0,0)
        if (is12==0) then
          do iu=ix,6,5
            if (id/=4 .and.(id<=6 .or.id==9)) then
             write(iu,100)pla(id),(r(i),i=1,3),(rku(i),i=1,3),dif,err
            else
             write(iu,101)pla(id),(r(i),i=1,3),(rku(i),i=1,3),dif,emp
            endif
          enddo
        endif
```

```
     !....."Planetenpositionen" im Giza-Gelaende (kartesische Koord.)
             if (((ijd>=1 .and.ijd<=14).or.(ik==4519 .and.ipla==1).or. &
               ((ik==4518 .or.ik==5349).and.ipla==2)).and.ison==5) ipos = 1
             if (ipos==1) then
               if (id==1) then
                 do j=1,3; y(j) = rku(j); enddo
               endif
               do j=1,3; re(j+3) = rku(j); enddo
               call kartko(ison)
               do j=4,6; y(j) = xyr(j); enddo
               call translat(x(1),x(2),x(3),y)
               call rotmat(5,x(4),x(5),x(6),y)
               call mastab(x(7),y)
               do j=1,3
                 ort(id,j) = y(3+j) + rp(3,j)
               enddo
     !         Genauigkeit der "Planetenpositionen"
               if (id<=3 .and.is12==0) then
                 ort(id,4) = dsqrt((ort(id,1)-rp(4-id,1))**2 &
                                 + (ort(id,2)-rp(4-id,2))**2 &
                                 + (ort(id,3)-rp(4-id,3))**2)
               elseif (id==9 .and.is12==0) then
                 ort(id,4) = dsqrt((ort(id,1)-rp(1,1))**2 &
                                 + (ort(id,2)-rp(1,2))**2 &
                                 + (ort(id,3)-rp(1,3))**2)
               else
                 dcm = dsqrt((ort(id,1)-rcm(1))**2 &
                           + (ort(id,2)-rcm(2))**2 &
                           + (ort(id,3)-rcm(3))**2)
                 qu = dcm
                 if (dcm<dmi) qu = dmi * ((dcm/dmi)**2 + 1.d0)*0.5d0
                 ort(id,4) = qu * xyr(36) * 1.d-2
               endif
     !         Geographische Koordinaten (Laenge und Breite) der
     !         transformierten Sonnen- und Planetenpositionen
               if (is12/=0) then
                 if (ipla==1) then
                   call geoko(ort(id,1),-ort(id,2),ipla,iB1,zB2,iL1,zL2)
                 else
                   call geoko(ort(id,1),ort(id,3),ipla,iB1,zB2,iL1,zL2)
                 endif
                 do iu=ix,6,5
                   write(iu,102) plan(id),(ort(id,j),j=1,4),iB1,zB2,iL1,zL2
                 enddo
               endif
             endif
        20 enddo

     ! . . Ruecksprung zum naechsten Planeten
           icm = icm + 1; if (icm<=icmax) go to 10

     ! . . Weitere Ergebnis-Ausgabe
           if (ipos==1 .and.is12==0) then
             text(2) = tt(ipla)
             do iu=ix,6,5
               call linie(iu,1)
               write(iu,'(''  Celestial pos. in Giza'',4x,a49)')titab
               call linie(iu,2)
               write(iu,'(''  Local coordinates'',9x,''Sun        '', &
                 & f10.2,2f10.2,f9.2)') (ort(0,j),j=1,4)
             enddo
```



```
             do i=1,imax
               dd = dn
               if ((i>=1 .and.i<=3).or.i==9) dd = dss
               do iu=ix,6,5
                 write(iu,'(a23,5x,a10,3f10.2,f9.2,a2)') &
                 text(i),plan(i),(ort(i,j),j=1,4),dd
               enddo
             enddo
           endif
           do iu=ix,6,5; call linie(iu,1); enddo
           return
       100 format(1x,a3,3f10.6,f9.4,f8.4,f10.6,2f9.4)
       101 format(1x,a3,3f10.6,f9.4,f8.4,f10.6,f9.4,1x,a7)
       102 format(2x,a10,f9.2,f10.2,f9.2,f7.2,i7,f10.5,i6,f9.5)

     ! . . Groessere Stellenanzahl fuer Schnellstart-Optionen 3 und 8
     !f100 format(2x,a3,f11.6,2f10.6/28x,f13.7,f11.7,f14.10/58x,f13.7,f8.3)
     !f101 format(2x,a3,f11.6,2f10.6/28x,f13.7,f11.7,f14.10/58x,f13.7,a8)!f
           end subroutine

           subroutine geoko(x,y,ipla,iB1,zB2,iL1,zL2)
     !-----Berechnung der geographischen Koordinaten----------------------
     !     (iB1,zB2 und iL1,zL2, jeweils in Grad und Minuten)
           use base, only : pi,pidg,R3a,R3p
           implicit double precision (a-h,o-z)

     ! . . Erdumfang ueber Pole. Anstelle von Ue = 40008 km folgt
     !     Ellipsenumfang nach Srinivasa Ramanujan.
           zl = 3.d0*((R3a-R3p)/(R3a+R3p))**2
           Ue = pi*(R3a+R3p) * (1.d0 + zl/(10.d0 + dsqrt(4.d0-zl)))

     !     Geographische Position des Koordinatenursprungs fuer jeweils
     !     die Pyramiden und Kammern (Genauigkeit ca. +/- 0,000010°)
           if (ipla==1) then
             zB0 = 29.972529d0  !  Zentrum der Mykerinos-Pyramide
             zL0 = 31.128243d0  !  (Pyramiden-System)
           else
             zB0 = 29.979197d0  !  Senkrechte Mittelachse der Ostwand der
             zL0 = 31.134275d0  !  Koeniginnenkammer (Kammer-System)
           endif

     ! . . Geographische Breite (zB)
           dBa = 360.d0 * x/Ue
           zBa = zB0 + dBa
           call geokar(zBa,ua,va)
           call geokar(zB0,u0,v0)
           xa  = dsqrt((ua-u0)**2 + (va-v0)**2)
           dB  = dBa * dabs(x/xa)
           zB  = zB0 + dB
           iB1 = idint(zB)
           zB2 = dmod(zB,1.d0)*60.d0

     ! . . Geographische Laenge (zL)
           zBm = 0.5d0*(zB + zB0)
           call geokar(zBm,um,vm)
           dL  = y/(pidg*um)
           zL  = zL0 + dL
           iL1 = idint(zL)
           zL2 = dmod(zL,1.d0)*60.d0
           end subroutine
```

```
      subroutine geokar(B,u,v)
!-----Abstand eines Punktes der geographischen Breite B---------------
!     zur Erdachse (u) und zur Aequatorebene (v) (kartesische Koord.)
      use base, only : pidg,R3a,R3p
      implicit double precision (a-h,o-z)
      u = R3a/dsqrt(1.d0 + (dtan(B*pidg)*R3p/R3a)**2)
      v = R3p*dsqrt(1.d0 - (u/R3a)**2)
      end subroutine

      subroutine reduz(a,i,j)
!-----Winkelreduzierung  a --> a  (z.B. 387 Grad --> 27 Grad)---------
!     i = 0:  dezimale Grad
!     i = 1:  Bogenmass
!     j = 0:  a --> -180...180 Grad
!     j = 1:  a -->    0...360 Grad
      use base, only : pidg,gdpi
      implicit double precision (a-h,o-z)
      u360 = 360.d0; z1 = 1.d0
      if (a<0.d0) z1 = -1.d0
      if (i/=0) a = a*gdpi
      ab = dabs(a); if (ab>u360) ab = dmod(ab,u360)
      if ((j==0 .and.ab>180.d0).or. &
          (j==1 .and.a<0.d0)) ab = ab - u360
      a = z1 * ab; if (i/=0) a = a * pidg
      end subroutine

      subroutine distance(i1,i2,dis)
!-----Entfernung zweier Punkte in Teotihuacan-------------------------
!     (linear bestimmt in Metern anhand der GPS-Koordinaten)
      use base, only : pidg
      use astro, only : teot
      integer(4) :: i1,i2 ! Nummern bzw. Kennzeichnung beider Punkte
      real(8) :: u1,v1,u2,v2,x,y,dis
      call geokar(teot(i1,1),u1,v1)
      call geokar(teot(i2,1),u2,v2)
      x = dsqrt((u2-u1)**2 + (v2-v1)**2)
      y = dabs((teot(i1,2)-teot(i2,2))*pidg) * (u1+u2)*0.5d0
      dis = dsqrt(x*x+y*y)
      end subroutine

      subroutine rcoef2(k,n,bmas)
!-----Bestimmtheitsmass-----------------------------------------------
!     Zusammenhang zw. Wallpositionen in Teotih. und Planetenbahnen
!     k=1: Periheldistanzen
!     k=2: grosse Halbachsen
!     k=3: Apheldistanzen
!     n  : Anzahl der Datenpunkte
      use astro, only : comp
      integer(4) :: i,k,n
      real(8) :: v(5),bmas(2,3),xn
      xn = dfloat(n)
      do i=1,5; v(i) = 0.d0; enddo
      do i=0,n-1
        v(1) = v(1) + comp(i,1)*comp(i,k+1)/xn
        v(2) = v(2) + comp(i,1)/xn
        v(3) = v(3) + comp(i,k+1)/xn
      enddo
      do i=0,n-1
        v(4) = v(4) + ((comp(i,1)-v(2))**2)/xn
        v(5) = v(5) + ((comp(i,k+1)-v(3))**2)/xn
      enddo
```



```
      bmas(1,k) = ((v(1) - v(2)*v(3))/(dsqrt(v(4)*v(5))))**2     ! R^2
      bmas(2,k) = 1.d0-(1.d0-bmas(1,k))*(xn-1.d0)/(xn-2.d0) ! adj. R^2
      end subroutine

      subroutine memo(zz1,zz2,zz3,zz4,zz5,zz6,zz7,zmem,ik,imem)
!-----Ergebnis-Parameter merken---------------------------------------
      use base, only : re
      implicit double precision (a-h,o-z)
      dimension :: zmem(78)
      zmem(1) = zz1; zmem(2) = zz2
      zmem(3) = zz3; zmem(4) = zz4
      zmem(5) = zz5; zmem(6) = zz6
      zmem(7) = zz7
      do i=1,12; zmem(10+i) = re(i); enddo
      do i=31,78; zmem(i) = re(i); enddo
      imem = ik
      end subroutine

      subroutine info
!-----Information zu den Copyrights (aus der Datei "ingiza.t")--------
      integer(2) :: i
      character(70) :: itext(38)
      open(unit=10,file='ingiza.t')
      do i=1,105; read(10,*); enddo
      do i=1,38; read(10,*) itext(i); enddo
      close(10); write(6,'(///38(5x,a70/))') (itext(i),i=1,38)
      end subroutine

      subroutine titel1(iaph,ijd,ia,ison,ipla, &
        ilin,isep,nurtr,iuniv,is12,iop0)
!-----Haupttitel und Untertitel---------------------------------------
      implicit double precision (a-h,o-z)
      character(3)  :: xt
      character(10) :: pc,pd
      pc = '(PYRAMIDS)'; if (iop0==321) pc = '(CHAMBERS)'
      pd = ' pyramids)'; if (ipla==2)   pd = ' chambers)'
      xt = 'TT)'; if (iuniv==2) xt = 'UT)'; write(ia,*)
      if (iop0==300) then
        write(ia,'(20x,A20,A22)')'4 PLANETS IN A LINE ', &
             '(SYZYGY), MAY 17, 3088'
        go to 20
      elseif (iop0==301) then
        write(ia,'(17x,A16,A31)')'MERCURY TRANSIT ', &
             '(MIN. SEPARATION), MAY 18, 3088'
        go to 20
      elseif (iop0==310) then
        write(ia,'(18x,A14,A32)')'VENUS TRANSIT ', &
             '(MIN. SEPARATION), DEC. 18, 3089'
        go to 20
      elseif (iop0==311) then
        write(ia,'(19x,A20,A23)')'3 PLANETS IN A LINE ', &
             '(SYZYGY), DEC. 23, 3089'
        go to 20
      elseif (iop0==320 .or.iop0==321) then
        write(ia,'(18x,A34,1x,A10)') &
             'SEARCH FOR "SHADOW-CONSTELLATIONS"',pc
        go to 10
      elseif (iop0==322 .or.iop0==323) then
        write(ia,'(11x,A20,A29,1x,A10)')'PRECEDING "SHADOW-CO', &
             'NSTELLATION" 12, MAY 22, 3088',pc
        go to 20
```

```
        elseif (iop0==338) then
          write(ia,'(22x,A37)')'ORBITAL ELEMENTS OF OUR EIGHT PLANETS'
          go to 20
        endif
        if (ipla==1) write(ia,*)'                  PLANETS IN ', &
            'ALIGNMENT WITH THE PYRAMIDS OF GIZA'
        if (ipla==2) write(ia,*)'           PLANETS IN ALIGNME', &
            'NT WITH THE CHAMBERS OF THE CHEOPS PYRAMID'
        if (ipla==3) then
          if (ilin>=3) write(ia,'(28x,a11,a15)')'PLANETS IN ', &
            'A LINE (SYZYGY)'
          if (ilin==1) write(ia,'(31x,a19)')'TRANSITS OF MERCURY'
          if (ilin==2) write(ia,'(32x,a17)')'TRANSITS OF VENUS'
        endif

   ! . . Untertitel
     10 if (ipla<=2 .and.is12==0) then
          if (iaph==1 .and.ijd/=13 .and.ijd/=14) &
            write(ia,'(30x,a21)')'(Mercury at aphelion)'
          if (iaph==2 .and.ijd/=13 .and.ijd/=14) &
            write(ia,'(29x,a23)')'(Mercury at perihelion)'
          if (iaph==3 .or.(iaph==1 .and.(ijd==13 .or.ijd==14))) &
            write(ia,'(29x,a23)')'(Mercury near aphelion)'
          if (iaph==4 .or.(iaph==2 .and.(ijd==13 .or.ijd==14))) &
            write(ia,'(28x,a25)')'(Mercury near perihelion)'
          if (iaph==5) write(ia,'(24x,a34)') &
            '(time not restricted, F minimized)'
        elseif (ipla<=2 .and.is12/=0) then
          write(ia,'(17x,a38,a10)') &
            '(more positions - coordinate system of',pd
        elseif (ipla==3) then
          if (isep==1) then
            if (ison/=5) then
              write(ia,'(14x,a21,a33)')'(eclipt. longitudes, ', &
                'all within an angular range, JDE)'
            else
              if (ilin>=3) then
                if (nurtr==1) then
                  write(ia,'(13x,a18,a37)')'(angular range of ', &
                  'eclipt. longitudes dL minimized, JDE)'
                else
                  write(ia,'(5x,a18,a52)')'(angular range of ', &
                  'eclipt. longitudes dL minimized, only transits, JDE)'
                endif
              else
                write(ia,'(11x,a18,a38,a3)')'(equal eclipt. lon', &
                  'gitudes for Earth und transit planet, ',xt
              endif
            endif
          elseif (isep==2) then
            write(ia,'(14x,a51,a3)') &
              '(minimum separation, without travel time of light, ',xt
          else
            if (iuniv==1) then
              write(ia,'(17x,a48)') &
                '(geocentric transit phases, terrestrial time TT)'
            else
              write(ia,'(18x,a46)') &
                '(geocentric transit phases, universal time UT)'
            endif
          endif
```



```
        endif
     20 if (isep/=4) then
          write(ia,'(32x,a11,i4,a2)')'< P5-option',iop0,' >'
        else
          write(ia,'(10x,a11,i4,a47)')'< P5-option',iop0, &
          ' >  (monitor line width minimal 148 characters)'
        endif
        end subroutine

        subroutine titel2(ia,imod,ivers,irb,ipla, &
          ison,ihi,iek,ijd,ika,iaph,ilin,ical,ak,zjde1,zjahr,delt, &
          dwi,dwikomb,dwi0,dwi2,dwi3,iamax,step,ikomb,zmin,zmax)
  !-----Zwei weitere Titelzeilen---------------------------------------
        implicit double precision (a-h,o-z)
        dimension :: ida(7),da(7)
        character(5)  :: ca(2),dmo
        character(7)  :: cal(2)
        character(10) :: wd
        character(15) :: text0
        character(27) :: text1
        character(19) :: text2
        character(8)  :: text3(0:6)
        character(25) :: text4
        character(22) :: text5(2)
        data ca/' (c1)',' (c2)'/,cal/'Gregor.','Julian.'/
        data text3/ '         ',' E-V-M, ',' E-M-V, ', &
         ' V-E-M, ',' V-M-E, ',' M-E-V, ',' M-V-E, '/
        data text5/',  only Greg. calendar',',  Jul./Greg. calendar'/
        if (imod==1)  text1 = '  VSOP87D short ver.(Meeus)'
        if (imod==2 .and.ivers==1) text1='  VSOP87A (2005) full ver.,'
        if (imod==2 .and.ivers==3) text1='  VSOP87C (2005) full ver.,'
        if (imod==3)  text1 = '  "Keplers equation",       '
        if (ikomb==1 .and.ivers==1)text1='  VSOP87A, comb. search,   '
        if (ikomb==1 .and.ivers==3)text1='  VSOP87C, comb. search,   '
        if (ivers==1) text2 = '  standard J2000.0,'
        if (ivers==3) text2 = '  ecliptic of date,'
        if (ipla<=2) then
         if (irb==1) then
          if (ison==1) text4 = ' "Sun" south of Myker. P.'
          if (imod==3 .and.ipla==2) text4 ='"Sun" south of sub. cham.'
          if (ison==2) text4 = '"Sun" south of Chefren P.'
          if (ison==3) text4 = '"Sun position" free, 2D  '
          if (ipla==1) then
           if (ison==4 .and.ihi==1) text4 ='"Sun" free, 3D, base, SLE'
           if (ison==4 .and.ihi==2) text4 =' "Sun" free, 3D, C-M, SLE'
           if (ison==4 .and.ihi==3) text4 =' "Sun" free, 3D, top, SLE'
           if (ison==5 .and.ihi==1) text4 ='"Sun" free 3D base, FITEX'
           if (ison==5 .and.ihi==2) text4 ='"Sun" free 3D, C-M, FITEX'
           if (ison==5 .and.ihi==3) text4 ='"Sun" free 3D, top, FITEX'
          endif
          if (ipla==2) then
           if (ison==4 .and.ihi==1) text4 ='"Sun" free, 3D, east, SLE'
           if (ison==4 .and.ihi==2) text4 ='"Sun" free, 3D, mid., SLE'
           if (ison==4 .and.ihi==3) text4 ='"Sun" free, 3D, west, SLE'
           if (ison==5 .and.ihi==1) text4 ='"Sun" free 3D east, FITEX'
           if (ison==5 .and.ihi==2) text4 ='"Sun" free 3D mid., FITEX'
           if (ison==5 .and.ihi==3) text4 ='"Sun" free 3D west, FITEX'
          endif
         endif
         if (irb==2) text4 = '   ref. Mercury orbit (A)'
         if (irb==3) text4 = '   ref. Mercury orbit (B)'
```

```fortran
          if (irb==4) text4 = '   ref. Mercury orbit (C)'
          if (irb==5) text4 = '    reference Venus orbit'
         elseif (ipla==3) then
          if (ilin==1) text4 = '     all Mercury transits'
          if (ilin==2) text4 = '       all Venus transits'
          if (ilin==3) text4 = 'linear c., Merc. to Earth'
          if (ilin==4) text4 = 'linear c. Mercury to Mars'
         endif
         write(ia,'(/a27,a19,a8,a25)') text1,text2,text3(ika),text4
         if (ipla<=2) then
          if (iek==1)  text0 ='  Ecl. north p/'
          if (iek==2)  text0 ='  Ecl. south p/'
          if (ison>=3 .or.iek==3) text0 ='  Ecl. N and S,'
         elseif (ipla==3) then
          text0 ='  Period   (yea'
         endif
         if (ijd==15 .and.(imod/=2 .or.(imod==2 .and. &
            (iaph==3 .or.iaph==4)))) then
          if (ipla<=2) then
           if (ison<=2) then
            if (ikomb/=1) write(ia,'(a15,'' years'',f10.2, &
             & '' to'',f10.2,a5,''    angular range:'',f8.4,'' deg'')') &
             text0,zmin,zmax,ca(ical),dwi0
            if (ikomb==1) write(ia,'(a15,'' years'',f10.2, &
             & '' to'',f10.2,a5,'', angular r.:'',f6.2,''/'',f6.2, &
             & '' deg'')') text0,zmin,zmax,ca(ical),dwi,dwikomb
           else
            if (ikomb/=1 .and.iaph/=5) then
             write(ia,'(a15,'' years'',f10.2,'' to'',f10.2,a5, &
              &'',       tolerance F <'',f8.4,'' %'')') &
              text0,zmin,zmax,ca(ical),dwi0
            else
             write(ia,'(a15,'' years'',f10.2,'' to'',f10.2,a5, &
              & '', tolerance F <'',f6.2,''/'',f6.2,'' %'')') &
              text0,zmin,zmax,ca(ical),dwi,dwikomb
            endif
           endif
          elseif (ipla==3) then
           if (ilin>=3) then
            if (ikomb==1) write(ia,'(a15,''rs)'',f10.2, &
             & ''  to'',f10.2,a5,'',    angular r.:'',f6.2,''/'',f6.2, &
             & '' deg'')') text0,zmin,zmax,ca(ical),dwi,dwikomb
            if (ikomb/=1) write(ia,'(a15,''rs)'',f10.2,''  to'', &
             & f10.2,a5,3x,''   angular range:'',f8.4,'' deg'')') &
             text0,zmin,zmax,ca(ical),dwi0
           else
            write(ia,'(5x,a15,''rs) from'',f10.2,''  to'',f10.2,a22)') &
             text0,zmin,zmax,text5(ical)
            return
           endif
          endif
         else
          call ephim(1,iaph,ipla,ical,ak,iak,zjde1,zjahr,delt)
          if (ijd>=1 .and.ijd<=14) then
           write(ia,'(a15,''  constellation'',i3,'',    JDE ='', &
            & f15.5,'', year ='',f9.2,a5)')text0,ijd,zjde1,zjahr,ca(ical)
          else
           write(ia,'(a15,20x,''  JDE ='',f15.5,'', year ='',f9.2,a5)') &
             text0,zjde1,zjahr,ca(ical)
          endif
          if (iaph<=2) then
```



```fortran
           call jdedate(zjde1,ical,ida,da,dmo)
           call weekday(zjde1,wd)
           k=1; if (zjde1>=0.d0 .and.zjde1<2299161.d0 .and.ical==2) k=2
           if (zjde1>=1356183.d0 .and.zjde1<=5373484.d0) then
            write(ia,'(25x,''date ('',a7,'',TT) ='', &
              & f4.0,a5,i5,'','',i3,2('':'',i2),'','',A10)') &
              cal(k),da(7),dmo,(ida(i),i=3,6),wd
            return
           else
            write(ia,'(24x,''date ('',a7,'',TT) ='', &
              & f4.0,a5,i6,'','',i3,2('':'',i2),'','',A10)') &
              cal(k),da(7),dmo,(ida(i),i=3,6),wd
            return
           endif
          endif
         endif
         if (iaph==3 .or.iaph==4) then
          write(ia,'(''  Special search (interval), step number ='',i6,&
           & '',  step width ='',f8.3,'' hour(s)'')')iamax,24.d0*step
         endif
         if ((iaph==3 .or.iaph==4).and.ijd==15) then
          write(ia,'(''  Consider without printing by tolerance ='', &
           & f8.4)') dwi2
          write(ia,'(''  Print beyond aphelion (per.) by toler. ='', &
           & f8.4)') dwi3
         endif
         end subroutine

         subroutine tabe(iaph,imod,iek,ia,io, &
          ison,ipla,ilin,itran,is12,iop0,iout)
!-----Tabellenkopf---------------------------------------------------
!        Bei Datumsberechnungen uebernimmt das Unterprogramm
!        "zwischenzeile" die Tabellenueberschrift.
         implicit double precision (a-h,o-z)
         character(2) :: trs
         if (ilin>=3.) then
          write(ia,*)
          if (io==2 .and.imod/=3) call linie(ia,1)
         endif
         if (ipla==3) then
          trs = 'tr'
          if (itran==2 .or.ison/=5 .or.imod==3) trs = '  '
          if (ilin>=3) then
           if (ison==5) then
            write(ia,'('' co '',a2,''     k          JDE           year'',&
              & ''    dt[days]  Lm-Lv    Lm-Le  Lm-Lma    dLmin'')')trs
           else
            write(ia,'('' co '',a2,''     k          JDE           year'',&
              & ''    dt[days]  Lm-Lv    Lm-Le  Lm-Lma      dL'')')trs
           endif
          endif
         elseif (ipla<=2) then
          if (ison<=2) then
           if (imod/=3 .and.iek/=3) then
            write(ia,'('' con   k         JDE          year  '', &
              & ''    Lm     Lm-Lv    Lm-Le     del1    del2    F[%]'')')
           else
            write(ia,'('' con   k          JDE           year'', &
              & ''       Lm     Lm-Lv    Lm-Le     del1    del2  P'')')
           endif
          else
```

```
      if (ison==3 .or.ison==4) then
       write(ia,'('' con    k       year      Lm      Lm'', &
         & ''-Lv   Lm-Le  x-Sun   y-Sun  z-Sun  dr  P   F[%]'')')
       if (iaph==3 .or.iaph==4) then
        write(ia,'(''       (  ~k        JDE            M'', &
         & ''       no.     "      "      "    "   "    "   )'')')
       endif
      endif
      if (ison==5) then
       if (iaph==3 .or.iaph==4 .or.iout/=3) then
        if (iaph/=5) then
         write(ia,'('' con    k       year    Lm-Lv   Lm'', &
           & ''-Le    e  it  x-Sun  y-Sun  z-Sun   dr  P   F[%]'')')
        else
         write(ia,'('' con    k          JDE           ye'', &
           & ''ar    e  it   x-Sun  y-Sun  z-Sun   dr  P  F[%]'')')
        endif
       else
        if (ipla==1) then
         if (iaph/=5) then
          write(ia,'('' con    k       year      X5    M/1'', &
           & ''0^7  h-Sun  x-Sun  y-Sun  z-Sun    dr  P   F[%]'')')
         else
          write(ia,'('' con    k        year   dt[days]  '', &
           & ''  X5     M/10^7   x-Sun  y-Sun  z-Sun  P   F[%]'')')
         endif
        elseif (ipla==2) then
         if (iaph/=5) then
          write(ia,'('' con    k       year      X5    M/1'', &
           & ''0^9   h-Sun  x-Sun  y-Sun  z-Sun   dr  P   F[%]'')')
         else
          write(ia,'('' con    k        year   dt[days]  '', &
           & ''  X5     M/10^9   x-Sun  y-Sun  z-Sun  P   F[%]'')')
         endif
        endif
       endif
       if (iaph==3 .or.iaph==4) then
        if (iout==3) then
         if (ipla==1) then
          write(ia,'(''    (  JDE        dt[h]   X5   M/'', &
           & ''10^7  h-Sun    "      "      "     "   "    "   )'')')
         elseif (ipla==2) then
          write(ia,'(''    (  JDE        dt[h]   X5   M/'', &
           & ''10^9  h-Sun    "      "      "     "   "    "   )'')')
         endif
        else
         write(ia,'(''       (  ~k        JDE           M'', &
          & ''     "  "     "      "      "     "   "    "   )'')')
        endif
       endif
      endif
     endif
!    (Output zum Vergleich mit den Pyramidenabstaenden)
     if (ilin>=3) then
      if (imod==3) then
        call linie(ia,1)
      else
        call linie(ia,io)
      endif
      if (io==2 .and.imod/=3 .and.is12==0) then
```



```
         write(ia,'(''   Lm        Bm        Rm        Lv        Bv  '', &
           & ''      Rv        Le        Be        Re   '')')
         if (ipla==3) write(ia,'(''   Lma       Bma       Rma'')')
         if (ipla<=2) then
         write(ia,'(''    xm      ym      zm          xv      yv  '', &
           & ''    zv        xe        ye       ze   '')')
         write(ia,'(''  xv-xm    xe-xm    yv-ym    ye-ym    zv-zm'', &
           & ''    ze-zm             rel. deviation'')')
         endif
         call linie(ia,1)
        endif
       endif
       if (iop0==-804) write(ia,'(/24x,a33/31x,a19)') &
        'calculation of the file inser-2.t','--- please wait ---'
       end subroutine

       subroutine elements(ia,ivers,pla)
!-----Ausgabe der Bahnelemente aller Planeten------------------------
!      im Rahmen der erweiterten Ergebnisausgabe
       use base, only : re
       implicit double precision (a-h,o-z)
       character(3) :: pla(0:9)
       write(ia,'('' pla.   mean long.   a [AU]   '', &
         & ''eccentr.  asc.node    incl.   per.[°]   per.[AU]'')')
       call linie(ia,2)
       do i=1,8
         pd = re(26+6*i) * (1.d0-re(27+6*i))
         if (ivers==3 .and.i==3) then
          write(ia,'(1x,a3,f13.5,2f10.5,a11,f9.5,f11.5,f10.5)')pla(i),&
            (re(24+6*i+j),j=1,3),'     ---    ',(re(24+6*i+j),j=5,6),pd
         else
          write(ia,'(1x,a3,f13.5,2f10.5,f11.5,f9.5,f11.5,f10.5)') &
            pla(i),(re(24+6*i+j),j=1,6),pd
         endif
       enddo
       end subroutine

       subroutine linie(ia,ib)
!-----Linie, waagerecht----------------------------------------------
       implicit double precision (a-h,o-z)
       if (ib==1) write(ia,'(1x,79a1)') ('=',i=1,79)
       if (ib==2) write(ia,'(1x,79a1)') ('-',i=1,79)
       if (ib==3) write(ia,'(1x,147a1)') ('=',i=1,147)
       if (ib==4) write(ia,'(1x,147a1)') ('-',i=1,147)
       end subroutine

       subroutine zwizeile(ia,io,zjde,ilin,imod,isep,ical,izp)
!-----Tabellenueberschrift und Zwischenzeile bei Datumsausgaben-------
!      Bei Transitbestimmungen werden abhaengig von der Wahl der
!      Kalender-Option Zwischenzeilen eingefuegt, die den Uebergang
!      von einem zum anderen Kalender kennzeichnen.
       implicit double precision (a-h,o-z)
       ipar = 0; if (isep==4) ipar = 2
       is = isep; if (is==2) is = 1
       if (izp==1) then
         if (isep/=4) then
           write(ia,*)
         else
           write(ia,'(93x,''position angles [deg]'',12x, &
             & ''semidiameters ["]'')')
         endif
```

```
      endif
      if (izp==1) then
        if (ilin<=2 .and.io==2) call linie(ia,1+ipar)
        if (isep<=2) then
         write(ia,'('' co/p   k         date         time'', &
           & ''    dt[days]  Lm-Lv   Lm-Le  Lm-Lma  sep["]  S'')')
        elseif (isep==3) then
         write(ia,'('' co/p    date, phase:   I         I'', &
           & ''I      nearest     III       IV    sep["]a  S'')')
        else
         write(ia,'('' co/p    date, phase:    I         II     '', &
         &'' nearest     III       IV     sep["]  a      P1      '',&
         &''P2     near.    P3      P4        s-Sun    s-pl.    S'')')
        endif
        if (imod/=3 .and.io/=2) then
          call linie(ia,1+ipar)
        else
          call linie(ia,io+ipar)
        endif
        if (io==2 .and.imod/=3) then
          write(ia,'(''   Lm       Bm       Rm       Lv       Bv '', &
           & ''      Rv       Le       Be       Re  '')')
          write(ia,'(''   Lma      Bma      Rma'')')
          call linie(ia,1+ipar)
        endif
        if (ia==6) then
          izp=2; if (zjde>=0) izp=3
          if (zjde>=2299161.d0) izp=4
        endif
      elseif (zjde>=0.d0 .and.izp==2 .and.ical==2) then
       select case (is)
        case(1);write(ia,'(1x,13(''-''),'' (Jul. cal.) '',53(''-''))')
        case(3);write(ia,'(1x,''----- (Jul. cal.) '',61(''-''))')
        case(4);write(ia,'(1x,''----- (Jul. cal.) '',129(''-''))')
       end select
       if (ia==6) izp = 3
      elseif (zjde>=2299161.d0 .and.izp==3 .and.ical==2) then
       select case (is)
       case(1);write(ia,'(1x,12(''-''),'' (Greg. cal.) '',53(''-''))')
       case(3);write(ia,'(1x,''---- (Greg. cal.) '',61(''-''))')
       case(4);write(ia,'(1x,''---- (Greg. cal.) '',129(''-''))')
       end select
       if (ia==6) izp = 4
      endif
      end subroutine

      subroutine comtime(i,za,zb,iw1,iw2,ihour,imin,sec)
!-----Bestimmung der Rechenzeit -------------------------------------
!     i = 1: CPU time, i = 2: run time
!     Stopzeit zb - Startzeit za = Rechenzeit [hhh:mm:ss.sss]
      implicit double precision (a-h,o-z)
      dimension :: iw1(8),iw2(8)
      if (i==1) then
        t1 = za; t2 = zb
      else
        t1 = dfloat(iw1(5)*3600+iw1(6)*60+iw1(7))+dfloat(iw1(8))*1.d-3
        t2 = dfloat(iw2(5)*3600+iw2(6)*60+iw2(7))+dfloat(iw2(8))*1.d-3
      endif
      zt = t2-t1; if (zt<0.d0) zt = zt + 86400.d0
      zih = dint(zt/3600.d0); ihour = idnint(zih)
      zm  = (zt-zih*3600.d0)/60.d0; zim = dint(zm)
```



```
      imin = idnint(zim); sec = (zm-zim)*60.d0
      end subroutine

      subroutine endzeile(ipla,imod,ilin,iaph,isep,ison,ijd,ipos, &
       io,ia,inum,ihour,imin,sec,ihour2,imin2,sec2,is12,iop0)
!-----Endzeilen des Outputs------------------------------------------
!     Zusammenfassung: Anzahl gefundener Ereignisse, Rechenzeit
      implicit double precision (a-h,o-z)
      dimension :: inum(0:4)
      character(37) :: te1
      character(8)  :: te2,te22
      character(1)  :: te3
      character(29) :: te4
      character(15) :: te5
      te1 = '                                     '
      te2 =  'CPU time'; te3 = ':'; te5 = ' -- end of run.'
      te22 = 'run time'; te4 = '("<" exact deviation dr)      '
      ipar = 0; if (isep==4) ipar = 2
      if (io==2 .and.inum(2)==0) call linie(ia,1+ipar)
      if ((imod/=3 .and.ison>=3).or.imod==3) then
        if (ipla==1) te1 = '(P: polarity, * view from ecl. south)'
        if (ipla==2) te1 = '(P: polarity, resp. view on ecliptic)'
      endif
      if (ilin<=2 .and.isep>=3) &
        te1 = '           ("/" means ascending node)'
      if (ipla<=3 .and.ijd==15 .and.iop0/=-804 .and.(imod/=2 .or. &
         & (imod==2 .and.(iaph==3 .or.iaph==4 .or.ilin<=2)))) then
        write(ia,500)' Computed constellations:',inum(1),te1
        if (ilin<=2) then
          write(ia,501)' Tested planet. passages:',inum(0)
          write(ia,501)' Detected transits      :',inum(2)
          write(ia,502)' Centr./grazing transits:',inum(4),' /', &
            inum(3),te2,ihour,te3,imin,te3,sec
        else
          if (ipla<=2) then
            write(ia,503)' Detected constellations:',inum(2),te2, &
              ihour,te3,imin,te3,sec
          elseif (ipla==3) then
            if (ison==5) then
              inumber = inum(2)
            else
              write(ia,501)' Detected constellations:',inum(2)
              inumber = inum(3)
            endif
            write(ia,503)' Number of syzygies     :',inumber,te2, &
              ihour,te3,imin,te3,sec
          endif
        endif
      else
        if (ipos==1 .and.is12==0 .and.iop0/=-804) then
          write(ia,504)te4,te2,ihour,te3,imin,te3,sec
        else
          if (iop0==-804) write(ia,'(43x,a36)') &
            'The file inser-2.t has been created.'
          write(ia,505)te2,ihour,te3,imin,te3,sec
        endif
      endif
      write(ia,506)te22,ihour2,te3,imin2,te3,sec2,te5
  500 format(1x,a25,i10,6x,a37)
  501 format(1x,a25,i10)
  502 format(1x,a25,i5,a2,i3,7x,a8,i3,a1,i2,a1,f6.3)
```

```
  503 format(1x,a25,i10,7x,a8,i3,a1,i2,a1,f6.3)
  504 format(14x,a29,a8,i3,a1,i2,a1,f6.3)
  505 format(43x,a8,i3,a1,i2,a1,f6.3)
  506 format(43x,a8,i3,a1,i2,a1,f6.3,a15/)
      end subroutine

!h    subroutine histogramm(zz,ihis)  !h
!-----Einsortieren der Genauigkeiten Fpos (zz) in ein Array-----------
!     fuer Pyramiden oder Kammern (ipla <= 2, imod <= 2, ison >= 3).
!     Zur Nutzung muessen alle !h-Kommentarzeilen aktiviert werden.
!h    implicit double precision (a-h,o-z)
!h    dimension :: ihis(100)
!h    i = idnint(zz*20.d0 + 0.5d0); if (i<=100) ihis(i) = ihis(i) + 1
!h    end subroutine

      subroutine save_ser
!-----Speicherung von Daten in die Datei "inser-2.t"------------------
!     Wenn die Datei "inserie.t" mit den julianischen Tagen (JDE)
!     und den Nummern der Transit-Serien neu berechnet werden soll,
!     erfolgt dies mit der Schnellstart-Option -804. Hiermit wird
!     die neue Datei "inser-2.t" erzeugt. Falls gewuenscht kann
!     diese - durch Umbenennung in "inserie.t" - die vorherige bzw.
!     fehlende Datei "inserie.t" ersetzen. Die Verwendung dieser
!     Option ist normalerweise nicht erforderlich.
      use astro, only : ser
      implicit double precision (a-h,o-z)
      open(unit=10,file='inser-2.t')
      write(10,'(9x,a21,a42/6x,a10,a58)')'Julian Ephemeris Day ', &
       'of each first transit in a series (S-No.),','to be used', &
       ' for the years -13000 BC to 17000 AD, VSOP87C full version'
      write(10,'(34x,a9)')'(Mercury)'
      write(10,'(a14,4(12x,a3))')'S-No.      JDE',('JDE',i=1,4)
      write(10,'(79a1)')('-',i=1,79)
!     Serien, Merkur
      do i=-150,150,5
        write(10,'(I4,5f15.5)')i,(ser(i+j,1),j=0,4)
      enddo
      write(10,'(79a1)')('-',i=1,79)
      write(10,'(35x,a7)')'(Venus)'
      write(10,'(a14,4(12x,a3))')'S-No.      JDE',('JDE',i=1,4)
      write(10,'(79a1)')('-',i=1,79)
!     Serien, Venus
      do i=-10,10,5
        write(10,'(I4,5f15.5)')i,(ser(i+j,2),j=0,4)
      enddo
      ser(19,2) = 1.d12
      write(10,'(I4,4f15.5,e15.1)')i,(ser(15+j,2),j=0,4) ! "     "
      write(10,'(79a1/)')('-',i=1,79)
      close(10)
      end subroutine

      subroutine lintrend(k,n,u,v)
!-----Lineare Regression, f(x) = ux+v --> u, v (Teotihuacan)----------
!     k = 1: Periheldistanzen,  n = Anzahl der Punkte
!     k = 2: grosse Halbachsen
!     k = 3: Apheldistanzen
      use astro, only : comp
      integer(4) :: i,k,n
      real(8) :: sumx,sumy,sumx2,sumxy,sig2,u,v,xn
      xn = dfloat(n)
      sumx = 0.d0; sumy = 0.d0; sumx2 = 0.d0; sumxy = 0.d0
```



```
      do i=0,n-1
        sumx  = sumx  + comp(i,1)
        sumy  = sumy  + comp(i,k+1)
        sumx2 = sumx2 + comp(i,1)**2
        sumxy = sumxy + comp(i,1)*comp(i,k+1)
      enddo
      sig2 = xn*sumx2 - sumx**2
      u = (xn*sumxy - sumx*sumy)/sig2
      v = (sumx2*sumy - sumx*sumxy)/sig2
      end subroutine

      subroutine vsop1tr(ip,rk,tau,del,r3i,eps,inum,resu)
!-----Berechnung der ekliptikalen Koordinaten (Kurzversion VSOP87)----
!     Beruecksichtigung der Laufzeit des Lichtes, die bei Berechnung
!     der Transitphasen eine Rolle spielt (siehe "vsop2tr")
!     Index ip: 1 = Merkur, 2 = Venus
      use base
      implicit double precision (a-h,o-z)
      dimension :: rk(12),rd(3),inum(0:4)
      del = del/tmil  ! Laufzeit des Lichtes: Merkur/Venus --> Erde
      ist = 3*ip-2
      ii = 3*(ip-1)
      do j=ist,ist+2
        call vsop1(j,tau,resu)
        re(j) = resu
      enddo
      call kartko(0)
      do j=ist,ist+2; rk(j) = xyr(j); enddo
      do
        tau1 = tau + del; inum(1) = inum(1) + 1
        do j=7,9
          call vsop1(j,tau1,resu)
          re(j) = resu
        enddo
        call kartko(0)
        do j=7,9
          rk(j) = xyr(j)
        enddo
        do j=1,3
          rd(j) = rk(ii+j) - rk(6+j)
        enddo
        r3i  = dsqrt(rd(1)**2 + rd(2)**2 + rd(3)**2)
        del  = r3i*AE/(c*86400.d0*tmil)
        tau2 = tau + del
        if (dabs(tau2-tau1)<eps) exit
      enddo
      del = del*tmil
      end subroutine

      subroutine vsop2tr(xj2,ivers,ip,md, &
        ix,prec,lu,r,rk,ierr,del,r3i,eps,inum,rku)
!-----Aufruf der VSOP87-Subroutine (Vollversion)----------------------
!     Beruecksichtigung der Laufzeit des Lichtes
!     Index von rku: 1 = L, 2 = B, 3 = r;  ip: 1 = Merkur, 2 = Venus
!     Input: Zeitpunkt "xj2", Output: Koordinaten der Planeten und
!     Laufzeit des Lichtes "del" vom Planet "ip" zur Erde
      use base, only : re,c,AE
      implicit double precision (a-h,o-z)
      dimension :: rk(12),rd(3),r(6),rku(3),md(0:9),inum(0:4)
      ii = 3*(ip-1)
      call vsop2(xj2,ivers,ip,md,ix,prec,lu,r,ierr,rku)
```

```
      do k=1,3
        re(ii+k) = rku(k)
        rk(ii+k) = r(k)
      enddo
      do
        xj3 = xj2 + del
        inum(1) = inum(1) + 1
        call vsop2(xj3,ivers,3,md,ix,prec,lu,r,ierr,rku)
        do k=1,3
          re(6+k) = rku(k)
          rk(6+k) = r(k)
        enddo
        do j=1,3
          rd(j) = rk(ii+j) - rk(6+j)
        enddo
        r3i = dsqrt(rd(1)**2 + rd(2)**2 + rd(3)**2)
        del = r3i*AE/(c*86400.d0)
        xj4 = xj2 + del
        if (dabs(xj4-xj3)<eps) exit
      enddo
      end subroutine

      subroutine fitmin(imod,imodus,iap,ke,x,y,ee1, &
        step,nu,iflag,ddx1,ddx2,test,itin,indx,ix)
!-----Minimum stetiger aber nicht ueberall diff.-barer Funktionen-----
!     -->  Resultat = x(indx), indx = 1, 2 oder 3.
!
!     imodus = 1
!     Das Unterprogramm basiert auf einer Art ternaerem Suchen. Es
!     verwendet 3 Stuetzpunkte, um einen neuen Punkt zu finden und
!     einen alten durch diesen zu ersetzen. Dabei ruecken die Punkte
!     immer naeher zusammen, bis die Suchgenauigkeit (ee1) unter-
!     schritten wird. Das Minimum wird durch wiederholten Aufruf
!     von fitmin gefunden. Dieser Such-Algorithmus ist nicht beson-
!     ders schnell, konvergiert aber zuverlaessig und wird u.a. zur
!     Minimierung von "dL" bei Syzygien verwendet.
!
!     imodus = 2 (Spezialsuche)
!     Das Unterprogramm findet den Scheitelpunkt (Minimum) hyper-
!     bolischer Funktionen der Form: y = a * sqrt((x-b)**2 + c**2).
!     Dieser Algorithmus konvergiert deutlich schneller, findet
!     jedoch im konkreten Fall der Planetenbewegung die Loesung nur
!     dann, wenn sie zeitlich nicht zu weit entfernt liegt. Er dient
!     zur schnellen Berechnung der minimalen Separation des Transits.

      implicit double precision (a-h,o-z)
      dimension :: rx(3,4),x(5),y(5),test(10),d(3)
      ie = 0; ze = 0.d0; ee2 = 1.d-30
      zpa = 5.d0  ! zpa >= 2.d0
   10 iconv = 0
!c    do iu=ix,6,5; write(iu,'(''  nu,imod,imodus,indx,ddx1,ddx2 ='',&
!c       & i4,3i3,2f13.8)')nu,imod,imodus,indx,ddx1,ddx2
!c      write(iu,'(a12,3f18.8)') '  x(1..3) = ',(x(i),i=1,3)
!c      write(iu,'(a12,3f18.12/)')'  y(1..3) = ',(y(i),i=1,3); enddo
      nulim = 1

!.....Bestimmung der ersten drei x- und y-Werte
      if (iap==5 .and.imod==2) then
        nulim = 2
        if (nu==0) then; indx = 1; go to 99; endif
      endif
```



```
      if (nu<=nulim) then
        do i=1,2
          x(4-i) = x(3-i)
          y(4-i) = y(3-i)
        enddo
        x(1) = x(1) + step
        indx = 1
        go to 99
      endif
      dy1 = y(2)-y(1)
      dy2 = y(3)-y(2)

! . . Pruefen auf numerisches Rauschen (im Minimum) und Konvergenz-
!     problem. Letzteres Problem entsteht eventuell beim Umschalten
!     von der VSOP87-Kurzversion zur -Vollversion.
      if (dy1>=ze.and.dy2<ze) then
        i1 = 0; if (ddx1+ddx2>1.d-3) i1 = 1
        i2 = 0; if (dabs(dy1)+dabs(dy2)>1.d-3) i2 = 1
!c      if (i1==0.and.i2==0) write(6,*)' -->  num. noise, nu =',nu
!c      if (i2==1) write(6,'(a23,i3)') ' -->  switch-pr.(dy), ',nu
!c      if (i1==1) write(6,'(a23,i3)') ' -->  switch-pr.(dx), ',nu
        if (i1==1 .or.i2==1) then
          iconv = 1; go to 20
        endif
        if (imodus==1) then
          ke = 0
          return
        endif
      endif
   20 if (imodus==1) then

!.......Quasiternaeres Suchen (imodus = 1)
        if (dy1>=ze.and.dy2>=ze.and.iflag==0) then
          do i=1,2
            x(4-i) = x(3-i)                       ! way 1
            y(4-i) = y(3-i)
          enddo
          x(1) = x(1)+x(2)-x(3)
          if (dabs(x(1)-x(4))<1.d-8) then
            y(1) = y(4)
            go to 10
          endif
          indx = 1
        elseif ((dy1<ze.and.dy2<ze.and.iflag==0).or.iconv==1) then
          do i=1,2
            x(i) = x(1+i)                         ! way 2
            y(i) = y(1+i)
          enddo
          x(3) = x(3)+x(2)-x(1)
          if (dabs(x(3)-x(5))<1.d-8) then
            y(3) = y(5)
            go to 10
          endif
          indx = 3
        elseif ((dy1<ze.and.dy2>=ze).or.iflag==1) then
          select case (iflag)
          case(0)                                 ! way 3
            do i=1,2
              x(3+i) = x(2*i-1)
              y(3+i) = y(2*i-1)
            enddo
```

```
              x(3) = (x(3)+(zpa-1.d0)*x(2))/zpa
              indx = 3; iflag = 1
            case(1)
              x(1) = (x(1)+(zpa-1.d0)*x(2))/zpa
              indx = 1; iflag = 0
            end select
          endif
        else

!.......Suche mit hyperbolischem Fit (imodus = 2)
          a1 = x(1)-x(2); a3 = x(3)-x(2)
          b1 = (y(2)**2-y(1)**2)*a3
          b2 = (y(3)**2-y(2)**2)*a1
          if (dabs(b1+b2)<ee2) then; ke = 0; return; endif
          b = 0.5d0*(b1*a3+b2*a1)/(b1+b2) + x(2)
          d(1) = dabs(x(1)-b)
          d(2) = dabs(x(2)-b)
          d(3) = dabs(x(3)-b); indx = 1
          if (d(2)>d(1).and.d(2)>d(3)) indx = 2
          if (d(3)>d(1).and.d(3)>d(2)) indx = 3
          x(indx) = b
          if (x(1)>x(2)) call pchange(2,1,2,rx,x,y,indx)
          if (x(2)>x(3)) call pchange(2,2,3,rx,x,y,indx)
          if (x(1)>x(2)) call pchange(2,1,2,rx,x,y,indx)
        endif
        ddx1 = dabs(x(2)-x(1))
        ddx2 = dabs(x(3)-x(2))
        ddx3 = dabs(x(3)-x(1))
        if (imodus==2) then
          do i=1,10
            if (dabs(ddx3-test(i))<1.d-7) ie = 1
          enddo
        endif

!.....Hauptbedingung pruefen und Check auf Endlosschleife (ie=1)
        if (ddx1<=ee1.or.ddx2<=ee1.or.ie==1) then
!c        do iu=ix,6,5; write(iu,'(''  nu,imod,imods,indx,dx1,dx2,ie'',&
!c         & '' ='',i4,3i3,2f13.8,i3)') nu,imod,imodus,indx,ddx1,ddx2,ie
!c         write(iu,'(a12,3f18.8/)') '  x(1..3) = ',(x(i),i=1,3); enddo
          ke = 0
          return
        endif
        if (imodus==2) then
          itin = itin + 1
          if (itin>10) itin = 1
          test(itin) = ddx3
        endif

   99   nu = nu + 1
!c      write(6,'(a11,2i2,3f18.7)')' m,n,x1-3 =',imodus,nu,(x(i),i=1,3)
        if (nu<=100) return
        ke = 2
        do iu=ix,6,5
          write(iu,'(/''  ---->  error in "fitmin", ke ='',I2/)') ke
        enddo
        end subroutine

        subroutine ringfit(x1,x2,x3,y1,y2,y3,ep,step,nu,itmax,ix,ke)
!-----Nullstellenbestimmung-----------------------------------------
!     Die Routine liefert fuer die Kreisfunktion, die durch (x1,y1),
!     (x2,y2) und (x3,y3) verlaeuft, die naechstgelegene Nullstelle
```



```
!     (neuer x2-Wert). Wie bei "sekante" ergibt wiederholtes Aufrufen
!     von "ringfit" die Nullstelle einer stetig differenzierbaren
!     Funktion. Die Rechenzeit (TYMT Test) verkuerzt sich um ca. 3%,
!     was wenig ist. Da die Grundidee und die Gleichungen jedoch auch
!     eine gewisse Aesthetik besitzen, wurde diese Routine beibehal-
!     ten. Der Einsatz von "ringfit" ist nur sinnvoll, wenn die Be-
!     rechnung der Ausgangsfunktion deutlich mehr Zeit erfordert als
!     "ringfit" selbst.
      implicit double precision (a-h,o-z)
      if (ke/=5) ke = 1; ep0 = 1.d-15
      if (nu<=0 .or.ke==5) then
        call sekante(x1,x2,y1,y2,ep,step,nu,itmax,ix,ke); return
      endif
      if (nu==1) then  ! Erzeugung des 3. Startpunktes
        x31 = x1; y31 = y1; x32 = x2; y32 = y2
        call sekante(x1,x2,y1,y2,ep,step,nu,itmax,ix,ke)
        if (x1==x31) then
              x3 = x32; y3 = y32
        else; x3 = x31; y3 = y31
        endif; return
      endif
      sh = x2  ! Verschiebung (x2) zum Ursprung
      x1 = x1-sh; x2 = 0.d0; x3 = x3-sh
!c    do iu=ix,6,5; write(iu,'(a16,i3,6f10.6)') &
!c      'nu, x123, y123 =',nu,x1,x2,x3,y1,y2,y3; enddo
      z1 = x1*x1 + y1*y1;  ya = y2-y1;  xa = -x1
      z2 =         y2*y2;  yb = y3-y2;  xb = x3
      z3 = x3*x3 + y3*y3;  yc = y1-y3;  xc = x1-x3
      denom = x1*yb + x3*ya
      if (denom<ep0) go to 10
      xy = 0.5d0/denom
      if (dabs(xy)>=ep0) go to 20
   10 x1 = x1+sh; x2 = sh
      if (dabs(x1-x2)<ep0) x2 = x2 + 1.d0
      ke = 5; return ! switchover to "sekante"
   20 x0 =  (z1*yb + z2*yc + z3*ya)*xy
      y0 = -(z1*xb + z2*xc + z3*xa)*xy
      wu =  x0*x0 + (y2-y0)**2 - y0*y0
      if (wu<0.d0) then; ke = 4; go to 30; endif
      wu = dsqrt(wu); xx = x0+wu; xx2 = x0-wu ! (2 Loesungen)
      xmid = (x1+x2+x3)/3.d0
      if (dabs(xx-xmid)>dabs(xx2-xmid)) xx = xx2
      d1 = dabs(x1-xx); d2 = dabs(xx); d3 = dabs(x3-xx)
      if (d3>d1.and.d3>d2) then
        x3 = 0.d0; y3 = y2
      elseif (d1>d2.and.d1>d3) then
        x1 = 0.d0; y1 = y2
      endif
      x1 = x1+sh; x2 = xx+sh; x3 = x3+sh; nu = nu+1
      if (dabs(x2-x1)<ep.or.dabs(x3-x2)<ep) then
!c      do iu=ix,6,5; write(iu,'(a8,7x,a1,i3,3f14.10)') &
!c        'nu, x123','=',nu,x1-sh,x2-sh,x3-sh; enddo
        ke = 0; return
      endif
      if (nu<=itmax) return
      ke = 2
   30 do iu=ix,6,5
        write(iu,'(/''  ---->  error in "ringfit", ke ='',I2/)') ke
      enddo
      end subroutine
```

```
      subroutine sekante(x1,x2,y1,y2,ep,step,nu,itmax,ix,ke)
!-----Nullstellenbestimmung der Sekante-----------------------------
!     Das Programm liefert die Nullstelle der linearen Funktion, die
!     durch (x1,y1) und (x2,y2) verlaeuft. Das Ergebnis wird als
!     neuer x2-Wert ausgegeben. Wiederholtes Aufrufen dieser Routine
!     liefert die Nullstelle (erster Ordnung) einer stetig differen-
!     zierbaren, nicht notwendigerweise linearen Funktion.
      implicit double precision (a-h,o-z)
      if (ke/=5) ke = 1
!c    do iu=ix,6,5; write(iu,'(a16,i3,2f16.6,2f12.6)') &
!c       'nu,x1,x2,y1,y2 =',nu,x1,x2,y1,y2; enddo
      nu = nu + 1
      if (nu<=1) then
        x1 = x2                          !--------------------------------
        y1 = y2                          !  In sekante, ringfit,
        x2 = x1 + step                   !  fitmin, and vsop3 the
        return                           !  error code ke means:
      endif                              !
      if (y1==y2) then                   !  ke=0:  no error, result found
        ke = 3; go to 10                 !  ke=1:  routine runs (internal)
      endif                              !  ke=2:  too many iterations
      x0 = x2-y2*(x2-x1)/(y2-y1)         !  ke=3:  division by zero
      if (dabs(y2)<dabs(y1)) then        !  ke=4:  root of negative number
        x1 = x2                          !  ke=5:  switchover to "sekante"
        y1 = y2                          !--------------------------------
      endif
      x2 = x0
      if (dabs(x2-x1)<ep.and.nu>2) then
!c      do iu=ix,6,5; write(iu,'(a16,i3,2f16.6)') &
!c        'nu,x1,x2        =',nu,x1,x2; enddo
        ke = 0
        return
      endif
      if (nu<=itmax) return
      ke = 2
   10 do iu=ix,6,5
        write(iu,'(/''  ---->  error in "sekante", ke ='',I2/)') ke
      enddo
      end subroutine

! >>    Update: The 4 subroutines of FITEX have been updated        <<
! >>            for Fortran 95 standard, double precision,          <<
! >>            and free source form.                               <<

!-------------------------------------------------------------------
!     FITEX                                        M O D I N A 8 7
!-------------------------------------------------------------------
!
!  PROGRAMM BESCHREIBUNG NR. 320 VON G. W. SCHWEIMER    (VERSION 1985)
!
!  CHISQUARE MINIMISING SUBROUTINE
!  SOLVES THE NONLINEAR LEAST SQUARES PROBLEM
!  USING A LEAST SQUARES INTERPOLATION BETWEEN VARIABLES AND FUNCTIONS
!  OR THE EXACT GRADIENT OF THE FUNCTIONS
!  CALLED SUBROUTINES: LILESQ(LINEAR LEAST SQUARES PROBLEM)
!                      INVATA(INVERSION OF A(TRANSPOSED)*A)
!                      FIT1(ONE DIMENSIONAL MINIMUM SEARCH)
!  CALLING SEQUENCE
!     KE=0
!     M=NUMBER OF FUNCTIONS, M GE N
!     N=NUMBER OF VARIABLES, N GE 1
```



```
!     DO 1 I=1,N
!     X(I)=STARTING VALUES OF THE VARIABLES
!   1 E(I)=ABSOLUTE SEARCH ACCURACIES FOR THE VARIABLES, E(I) NE 0
!     W(1)=FIRST STEP SIZE IN UNITS OF E(I), IF LE 1 W(1) = 100 BY
!          FITEX THE MAXIMUM ALLOWED STEP SIZE IS 2*W(1)
!     W(2)=METHOD OF APPROXIMATION, 0 FOR LEAST SQUARES INTERPOLATION
!                             1 FOR EXACT GRADIENT OF THE FUNCTIONS
!     IW(1)=NUMBER OF POINTS TO BE REMEMBERED, IF LE N IW(1) = N+1
!     IW(2)=MAXIMUM NUMBER OF FUNCT. EVALUATIONS, IF EQ 0 IW(2)=2IW(1)
!           IF IW(2) LT 0 NO ACTION EXCEPT KE = 0
!     JA=4+MAX0(14,(N*(N+5))/2)+(M+N+1)*(IW(1)+1)
!   2 W(4)=0.
!     DO 3 I=1,M
!     F(I)=FUNCTION VALUES AT THE POINT X
!     IF(W(2)==0.) GO TO 3
!     W(JA+I+M*(J-1))= DF(I)/DX(J) FOR J=1,N
!   3 W(4)=W(4)+F(I)*F(I)
!     OPTIONAL WRITE(*,*) IW(3),IW(4),W(3),W(4),X,F
!     CALL FITEX(KE,M,N,F4,X4,E4,W4,IW)
!     IF(KE==1) GO TO 2
!     W(3)=ERROR RENORMALISATION FACTOR
!     W(4)=MINIMUM QUADRATIC SUM OF THE F(I)
!     X=MINIMUM POINT
!     F=FUNCTIONS AT THE MINIMUM POINT
!     KE=ERROR CODE  KE=0: WITHOUT ERRORS
!                    KE=2: USER INTERRUPT; RETURNS MINIMUM VALUES
!                          WITHOUT ERRORS. THE CURRENT POINT IS
!                          IGNORED. FOR NORMAL USER INTERRUPT SET
!                          IW(2)=IW(3).
!                    KE=3: MAXIMUM NUMBER OF FUNCTION EVALUATIONS
!                    KE=4: ROUNDING ERRORS
!                    KE=5: THE FUNCTIONS DO NOT DEPEND ON X(IW(4))
!                    KE=6: USELESS VARIABLES IN THE PREPARATORY CALLS,
!                          THE LABELS OF THE VARIABLES ARE IW(3),IW(4)
!                    KE=7: M LT N OR N LT 0 OR W(2)*(W(2)-1.) NE 0
!     W(4+I)=STANDARD ERRORS OF THE VARIABLES
!            THE ERROR CALCULATION ASSUMES LINEAR FUNCTIONS.
!            THE PROGRAM SHOWS THE LINEARITY BY THE KIND OF
!            PREDICTION IW(3)
!            IW(3)=0: LINEAR PREDICTION
!                 =1: STEP SIZE LIMITATION
!                 =2: ONE DIMENSIONAL SEARCH
!                 =3: RANDOM SEARCH
!            THE ERRORS ARE CORRECTLY CALCULATED IF THE LAST
!            N ITERATIONS WERE LINEAR, I.E. IW(3)=0.
!     W(4+N+I)=ERROR ENHANCEMENTS
!     W(4+N+N+I+(J*(J-1))/2)=ERROR CORRELATION BETW. X(I) AND X(J) I<J
!     IW(3): NUMBER OF FUNCTION EVALUATIONS
!     IW(4): NUMBER OF DEGREES OF FREEDOM
!  WORKING FIELD:  IW: LENGTH 4+K WITH K = IW(1)
!     W: LENGTH 4+MAX(14,(N*(N+5))/2)+(M+N+1)*(K+1)+M*N
!                ADRESSES IN IW
!                    4+L: LABELS OF THE QUADRATIC SUMS
!                ADRESSES IN W
!                    4+I: STANDARD ERROR OF X(I)
!                    4+N+I: ERROR ENHANCEMENT FOR X(I)
!                    FROM 4+N+N+1: MATRIX D AND ERROR CORRELATIONS
!                    FROM JS+1 MATRIX S; JS = 4+MAX0(14,(N*(N+5))/2)
!                    FROM JA+1: MATRIX A WITH JA = JS+(M+N+1)*(K+1)
!  THE WORKING FIELDS CONTAIN ALL INFORMATION FOR THE CONTINUATION OF
!  THE SEARCH. THIS ALLOWS A SEARCH WITHIN ANOTHER SEARCH JUST
```

```
!  CHANGING THE WORKING FIELDS.
!
!-------------------------------------------------------------------
      SUBROUTINE FITEX(KE,M,N,F,X,E,W,IW)
      IMPLICIT NONE
      INTEGER(4) :: KE,M,N,I,I1,I2,J,J1,J2,J3,JA,JD,JM,JS,K,KV
! >>  Sizes of IW and W are increased because of index overflow,
! >>  although FITEX ran correctly before. (The numbers 100 and 1000
! >>  are appropriate, if n = 7 and m = 9.)
      INTEGER(4) :: IW(100),L,LM,MF
      REAL(8) :: E(N),F(M),W(1000),X(N),EPS,S,T,U,V,BIG
      REAL(4) :: A
      INTEGER(2) :: IR
! >>  A and IR in the equivalence statement have still the original
! >>  single precision, since they are used to generate random numbers
! >>  and so the calculation is not changed.
      EQUIVALENCE (A,IR)
      DATA EPS/1.D-8/,BIG/7.D+75/
      DATA MF/0/,J/0/,LM/0/,JS/0/,JM/0/,JD/0/,JA/0/,J3/0/ ! pre-init.
      IF (IW(2)<0) GO TO 50
      JD = 4 + N + N
      JS = 4 + MAX0(14,(N*(N+5))/2)
      LM = M + N + 1
      IF (KE/=0) GO TO 2
      IF (IW(1)<=N) IW(1) = N + 1
      IF (IW(2)==0) IW(2) = 2*IW(1)
      IF (W(1)<=1.D0) W(1) = 100.D0
      IW(3) = 1
      K = IW(1)
      DO L = 1,K
        IW(L+4) = 1 + K - L
        W(JS+LM*L) = 7.D75
      ENDDO
      KE = 1
    2 K = IW(1)
      KV = K
      JA = JS + LM* (K+1)
      JM = JS + LM*IW(5) - LM
      J3 = JA - LM
      IF (KE==2) GO TO 52
      IF (M<N.OR.N<1 .OR.W(2)*(W(2)-1.D0)/=0.D0) GO TO 57
      IF (W(4)<=0.D0) GO TO 50
      L = IW(K+4)
      IF (W(JS+LM*L)==BIG) KV = L - 1
      DO I = 1,K
        J1 = JS + LM*IW(I+4)
        IF (W(4)<W(J1)) GO TO 4
      ENDDO
      GO TO 37
    4 IF ((W(2)==0.D0 .AND.I>MAX0(N+1,KV)).OR. &
         (W(2)==1.D0 .AND.I>1)) GO TO 37
      IF (KV<K) KV = KV + 1
      I1 = K + 4
      I2 = K - I
      IF (I2==0) GO TO 6
      DO J = 1,I2
        I1 = I1 - 1
        IW(I1+1) = IW(I1)
      ENDDO
      IW(I1) = L
      JM = JS + LM*IW(5) - LM
```



```
!  NEW ROW
    6 J1 = JS + LM* (L-1)
      DO I = 1,N
        J1 = J1 + 1
        W(J1) = X(I)
      ENDDO
      DO I = 1,M
        J1 = J1 + 1
        W(J1) = F(I)
      ENDDO
      W(J1+1) = W(4)
!  TEST MAXIMUM NUMBER OF FUNCTION EVALUATIONS
      IF (IW(3)>=IW(2)) GO TO 53
      IF (N==1) GO TO 42
!  EXACT GRADIENTS OR END OF PREPARATORY FUNCTION EVALUATIONS
      IF (W(2)==1.D0 .OR.IW(3)>N+1) GO TO 15
!  PREPARATORY FUNCTION EVALUATIONS
      MF = IW(3)
      IF (MF==1) GO TO 12
      X(MF-1) = W(3)
      J2 = JS + N
      S = 0.D0
      DO I = 1,M
        T = F(I) - W(J2+I)
        S = S + T*T
      ENDDO
      J = 2
      IF (S<EPS*EPS*W(JS+LM)) GO TO 55
      W(3) = S
      J1 = 2 + N + MF
      W(J1) = DSQRT(W(3))
      IF (MF<=2) GO TO 12
      I1 = N + 1
      DO J = 3,MF
        I2 = J2 + LM* (J-2)
        S = 0.D0
        DO I = 1,M
          S = S + (W(I2+I)-W(J2+I))* (F(I)-W(J2+I))
        ENDDO
        IF (DABS(W(J1)*W(I1+J)-DABS(S))<EPS*DABS(S)) GO TO 56
      ENDDO
   12 IF (MF==N+1) GO TO 15
      W(3) = X(MF)
      X(MF) = X(MF) + W(1)*E(MF)
      GO TO 100
!  END OF PREPARATORY FUNCTION EVALUATIONS
!  SUM OF INVERSES OF THE QUADRATIC SUMS
   15 S = 0.D0
      DO L = 1,KV
        T = W(JS+LM*L)
        S = S + 1.D0/ (T*T)
      ENDDO
      W(JA) = 1.D0/S
!  CENTRE OF THE VARIABLES AND FUNCTIONS
      I1 = M + N
      DO I = 1,I1
        J1 = JS
        S = 0.D0
        DO L = 1,KV
          T = W(J1+LM)
          S = S + W(J1+I)/ (T*T)
```

```
            J1 = J1 + LM
          ENDDO
          W(J3+I) = S*W(JA)
        ENDDO
        IF (KE/=1) GO TO 60
        IF (W(2)==0.D0) GO TO 20
        J1 = JA - M - 1
        DO I = 1,M; W(J1+I) = F(I); ENDDO
        GO TO 23
!     MATRIX A
      20 J1 = JA
        DO I = 1,N
          U = W(J3+I)
          DO J = 1,M
            J1 = J1 + 1
            J2 = JS
            S = 0.D0
            T = W(J3+N+J)
            DO L = 1,KV
              V = W(J2+LM)
              S = S + (W(J2+N+J)-T)* (W(J2+I)-U)/ (V*V)
              J2 = J2 + LM
            ENDDO
            W(J1) = S*W(JA)
          ENDDO
        ENDDO
        IF (KE/=1) GO TO 62
!     LINEAR LEAST SQUARES PROBLEM
      23 CALL LILESQ(M,N,IR,W(JA+1),W(JA-M),W(5),W(N+5))
        IF (IR<0) GO TO 54
        IF (IR==0) GO TO 24; GO TO 35
!     MATRIX D
      24 J1 = JD
        DO I = 1,N
          T = W(J3+I)
          DO J = 1,I
            J1 = J1 + 1
            J2 = JS
            S = 0.D0
            U = W(J3+J)
            DO L = 1,KV
              V = W(J2+LM)
              S = S + (W(J2+I)-T)* (W(J2+J)-U)/ (V*V)
              J2 = J2 + LM
            ENDDO
            W(J1) = S*W(JA)
          ENDDO
        ENDDO
!     NEW VARIABLES
        IF (W(2)==0.D0) GO TO 28
        DO I = 1,N
          X(I) = W(JM+I) - W(I+4)
        ENDDO
        GO TO 31
      28 DO I = 1,N
          I2 = 1; J1 = JD + (I*I-I)/2
          S = 0.D0
          DO J = 1,N
            J1 = J1 + I2
            IF (J>=I) I2 = J
            S = S + W(J1)*W(J+4)
```



```
          ENDDO
          X(I) = W(J3+I) - S
        ENDDO
!     TEST OF CONVERGENCE
      31 A = 0.E0
        DO I = 1,N
          W(I+4) = X(I) - W(JM+I)
          A = AMAX1(A,SNGL(DABS(W(I+4)/E(I))))
        ENDDO
        IF (A<1.E0) GO TO 50
        IW(4) = 0
        W(3) = 1.D0
        IF (A<2.E0*W(1)) GO TO 33
!     STEP SIZE LIMITATION
        IW(4) = 1
        W(3) = 2.D0*W(1)/A
      33 DO I = 1,N; X(I) = W(JM+I) + W(3)*W(I+4); ENDDO
        GO TO 100
!     RANDOM PREDICTION
      35 DO I = 1,N
          A = SNGL(W(J3+I))
          X(I) = W(JM+I) + W(1)*E(I)* &
                 (MOD(IABS(INT(IR,KIND=4)),200)-100)/100.D0
        ENDDO
        IW(4) = 3
        GO TO 100
!     ONE DIMENSIONAL SEARCH
      37 IF (N==1) GO TO 43
        IF (IW(3)>=IW(2)) GO TO 53
        IF (IW(4)==2) GO TO 39
        IW(4) = 2
        DO I = 1,N; W(J3+I) = X(I) - W(JM+I); ENDDO
        IR = 3
        W(5) = IR
        IR = 20
        W(6) = IR
        W(8) = 0.5D0
        W(11) = 0.D0
        W(12) = 0.D0
        W(13) = 0.D0
        W(14) = 1.D0
        W(16) = W(JM+LM)
        W(17) = W(4)
        GO TO 40
      39 W(9) = W(4)
        CALL FIT1(KE,W(5),W(8))
      40 DO I = 1,N; X(I) = W(JM+I) + W(8)*W(J3+I); ENDDO
        IF (KE==3) KE = 2
        IF (KE==2) GO TO 53
        KE = 1
        W(3) = W(8)
        GO TO 100
!     ONLY ONE VARIABLE X
      42 IF (IW(3)>1) GO TO 43
        KE = 0
        W(10) = W(1)*E(1)
        W(11) = E(1)
        W(12) = 0.D0
      43 IR = INT(IW(2),KIND=2)
        W(6) = A
        W(8) = X(1)
```

```
      W(9) = W(4)
      CALL FIT1(KE,W(5),W(8))
      IW(4) = 2
      X(1) = W(8)
      IF (KE==1) GO TO 100
      IF (KE>0) KE = KE + 1
      W(3) = 0.D0
      W(5) = 0.D0
      IF (W(6)/=0.D0) GO TO 74
      W(5) = DSQRT(DABS((W(13)-W(15))/ ((W(16)-W(17))/(W(13)-W(14))- &
      (W(17)-W(18))/ (W(14)-W(15)))))
      W(6) = 1.D0
      W(7) = 1.D0
      GO TO 71
!  END OF SEARCH
   50 KE = 0
      IF (W(4)==0.D0 .OR. IW(2)<0) GO TO 100
      GO TO 52
!  ERROR CODE DEFINITION
   57 KE = KE + 1
   56 KE = KE + 1
   55 KE = KE + 1
   54 KE = KE + 1
   53 KE = KE + 2
   52 DO I = 1,N; W(I+4) = 0.D0; ENDDO
      W(3) = 0.D0
      IF (KE*(KE-3)/=0 .OR.(KE==3 .AND.(W(2)==1.D0 .OR. &
         (W(3)==0.D0 .AND.IW(3)<=N)))) GO TO 74
!  COMPUTATION OF THE ERRORS OF THE VARIABLES
!  RESTORE MATRIX G
      IF (W(2)==0.D0) GO TO 15
      J1 = JA
      I1 = N + 1
      DO 45 I = 2,I1
        IF (I>M) GO TO 45
        DO J = I,M
          W(J1+J) = 0.D0
        ENDDO
        J1 = J1 + M
   45 ENDDO
      DO 49 I = 1,N
        DO I1 = I,N
          A = SNGL(W(4+N+I1))
          IF (IR==I) EXIT
        ENDDO
        IF (I1==I) GO TO 49
        J1 = JA + M* (I-1)
        J2 = JA + M* (I1-1)
        W(4+N+I1) = W(4+N+I)
        DO J = 1,N
          A = SNGL(W(J1+J))
          W(J1+J) = W(J2+J)
          W(J2+J) = A
        ENDDO
   49 ENDDO
      GO TO 66
!  INVERSE OF MATRIX D
   60 T = DSQRT(W(JA))
      J1 = JA
      DO I = 1,N
        S = W(J3+I)
```



```
          J2 = JS + I - LM
          DO L = 1,KV
            J1 = J1 + 1
            W(J1) = T*(W(J2+L*LM)-S)/W(JS+L*LM)
          ENDDO
        ENDDO
        CALL INVATA(KV,N,IR,W(JA+1),W(JD+1),X)
        IF (IR==0) GO TO 20
        GO TO 74
!  MATRIX G = A*INVERSE OF D
   62 DO L = 1,M
          J1 = L + JA - M
          DO I = 1,N
            I1 = JD + (I*I-I)/2
            I2 = 1
            S = 0.D0
            DO J = 1,N
              I1 = I1 + I2
              IF (J>=I) I2 = J
              S = S + W(I1)*W(J1+J*M)
            ENDDO
            X(I) = S
          ENDDO
          DO J = 1,N; W(J1+J*M) = X(J); ENDDO
        ENDDO
!  DIAGONAL ELEMENTS OF G(T)*G
   66 J1 = JA
        DO I = 1,N
          S = 0.D0
          DO L = 1,M
            J1 = J1 + 1
            S = S + W(J1)*W(J1)
          ENDDO
          W(4+N+I) = DSQRT(S)
        ENDDO
!  STANDARD ERRORS AND ERROR CORRELATIONS
        CALL INVATA(M,N,IR,W(JA+1),W(JD+1),X)
        IF (IR/=0) GO TO 74
        DO I = 1,N
          W(I+4) = DSQRT(W(JD+ (I*I+I)/2))
          W(4+N+I) = W(I+4)*W(4+N+I)
        ENDDO
        J1 = JD
        DO I = 1,N
          DO J = 1,I
            J1 = J1 + 1
            W(J1) = W(J1)/ (W(I+4)*W(J+4))
          ENDDO
        ENDDO
!  ERROR RENORMALISATION FACTOR
   71 S = 0.D0
        DO I = 1,M; S = S + W(JM+N+I); ENDDO
        W(3) = DSQRT(DABS(W(JM+LM)-S*S/M)/MAX0(M-N-1,1))
        DO I = 1,N; W(I+4) = W(I+4)*W(3); ENDDO
!  RESTORE OPTIMUM VALUES TO X AND F
   74 IW(4) = M - N - 1
        IF ((KE-5)* (KE-6)/=0) GO TO 75
        IW(3) = J - 2
        IW(4) = MF - 1
   75 DO I = 1,N; X(I) = W(JM+I); ENDDO
        DO I = 1,M; F(I) = W(JM+N+I); ENDDO
```

```fortran
      W(4) = W(JM+LM)
  100 IF (KE==1) IW(3) = IW(3) + 1
      END SUBROUTINE

!---------------------------------------------------------------------
!     FIT1                                          M O D I N A 8 7
!---------------------------------------------------------------------
!
!  PROGRAMM BESCHREIBUNG NR. 309 VON G. W. SCHWEIMER (VERSION 1985)
!
!  MINIMISATION OF A FUNCTION F(X) OF ONE VARIABLE X
!  CALLING SEQUENCE
!     KE=0
!     I(2)=MAXIMUM NUMBER OF FUNCTION EVALUATIONS
!     W(1)=START VALUE OF X
!     W(3)=FIRST STEP SIZE
!     W(4)=ABSOLUTE SEARCH ACCURACY
!     W(5)=RELATIVE SEARCH ACCURACY
!   1 W(2)=FUNCTION VALUE F(X) AT X=W(1)
!     OPTIONAL WRITE VI(1),X,F
!     CALL FIT1(KE,VI,W)
!     IF(KE==1) GO TO 1
!     XMIN=W(1)
!     FMIN=W(2)
!     NF=VI(1)
!  KE = ERROR CODE: KE=0 NO ERRORS, KE=
!   2 MAXIMUM NUMBER OF FUNCTION EVALUATIONS
!   3 ROUNDING ERRORS, PROB. BECAUSE BOTH W(4) AND W(5) ARE TOO SMALL
!  THE WORKING FIELDS I AND W HAVE THE LENGTH 3 AND 11 RESPECTIVELY
!  THEY CONTAIN ALL INFORMATION FOR THE CONTINUATION OF THE SEARCH
!  THEREFORE A SEARCH WITHIN ANOTHER SEARCH CAN BE DONE JUST CHANGING
!  THE WORKING FIELDS
!  IF 2 FUNCTION VALUES F1 AND F2 ARE KNOWN FOR X = X1 AND X2 RESPEC-
!  TIVELY WITH X1 NE X2 ENTER THE CALLING SEQUENCE AFTER DEFINING :
!  KE = 1; I(1) = 3; W(6) = X1; W(7) = X2; W(9) = F1; W(10) = F2 AND
!  W(1) = USERS CHOICE
!  WORKING FIELD VARIABLES:
!  I(1): CURRENT NUMBER OF FUNCTION EVALUATIONS
!  I(2): MAXIMUM NUMBER OF FUNCTION EVALUATIONS
!  I(3): MINIMUM POINTER, THE MINIMUM FUNCTION VALUE IS AT W(7+I(3))
!  W(1): CURRENT VALUE OF X
!  W(2): USER SUPPLIED FUNCTION VALUE
!  W(3): FIRST STEP SIZE
!  W(4 AND 5): SEARCH ACCURACIES
!  W(6, 7 AND 8): X1, X2 AND X3 WITH X1 < X2 < X3
!  W(9, 10 AND 11): FUNCTION VALUES AT X1, X2 AND X3 RESPECTIVELY
!
!---------------------------------------------------------------------
      SUBROUTINE FIT1(KE,V,W)
      IMPLICIT NONE
      INTEGER(4) :: KE,IV,J,K
      REAL(8) :: V(3),W(11)
      IF (KE==1) GO TO 2
      KE = 1
      V(1) = 1
      V(3) = -1
      W(6) = W(1)
      W(9) = W(2)
    1 W(1) = W(1) + W(3)
      GO TO 12
    2 IF (V(1)>2.D0) GO TO 3
```



```fortran
      V(3) = 0.D0
      W(7) = W(1)
      W(10) = W(2)
      IF (W(2)<=W(9)) GO TO 1
      V(3) = -1.D0
      W(1) = W(6) - W(3)
      GO TO 12
    3 IF (V(1)>3.D0) GO TO 5
      W(8) = W(1)
      W(11) = W(2)
      DO 4 J = 1,3
        K = 7 - MOD(J,2)
        IF (W(K)<=W(K+1)) GO TO 4
        W(1) = W(K)
        W(K) = W(K+1)
        W(K+1) = W(1)
        K = K + 3
        W(1) = W(K)
        W(K) = W(K+1)
        W(K+1) = W(1)
    4 ENDDO
      V(3) = 0.D0
      IF (W(9)<W(10).AND.W(9)<W(11)) V(3) = -1.D0
      IF (W(11)<W(10).AND.W(11)<W(9)) V(3) = 1.D0
      GO TO 9
!   SORT IN THE NEW VALUES OF X AND F
    5 IF (V(3)==0.D0) GO TO 6
      J = IDINT(V(3))
      W(7-J) = W(7)
      W(10-J) = W(10)
      IF ((W(7+J)-W(1))*(W(1)-W(7))>0.D0) GO TO 7
      W(7) = W(7+J)
      W(10) = W(10+J)
      W(7+J) = W(1)
      W(10+J) = W(2)
      IF (W(2)>=W(10)) V(3) = 0.D0
      GO TO 9
    6 J = -1
      IF (W(1)<W(7)) J = 1
      IF (W(2)>W(10)) GO TO 8
      W(7+J) = W(7)
      W(10+J) = W(10)
    7 W(7) = W(1)
      W(10) = W(2)
      IV = IDINT(V(3))
      IF (W(2)<=W(10+IV)) V(3) = 0.D0
      GO TO 9
    8 W(7-J) = W(1)
      W(10-J) = W(2)
    9 IV = IDINT(V(3))
      J = 7 + IV
!   ERROR TESTS
      IF (W(6)==W(7) .OR. W(7)==W(8) .OR. &
         (W(9)==W(10).AND.W(10)==W(11))) GO TO 15
      IF (V(1)>=V(2)) GO TO 16
      IF (V(3)==0.D0) GO TO 10
!   STEP SIZE LIMITATION
      W(1) = W(J) + 2.D0*V(3)* (W(8)-W(6))
      GO TO 12
   10 W(1) = DMIN1(W(8)-W(7),W(7)-W(6))/(W(8)-W(6))
      IF (W(1)>0.1D0) GO TO 11
```

```fortran
      W(1) = .5D0* (W(6)+W(8))
      GO TO 12
!  PREDICTION OF THE POSITION OF THE MINIMUM
   11 W(1) = ((W(9)-W(10))/ (W(6)-W(7))- (W(10)-W(11))/(W(7)-W(8)))/ &
       (W(6)-W(8))
      W(1) = .5D0* (W(6)+W(8)+ (W(11)-W(9))/ (W(1)* (W(6)-W(8))))
!  TEST OF CONVERGENCE
      W(2) = DABS(W(1)-W(J))
      IF (W(2)<DABS(W(4)) .OR. W(2)<DABS(W(5)*W(J))) GO TO 13
   12 V(1) = V(1) + 1.D0
      RETURN
   13 KE = 0
   14 IV = IDINT(V(3))
      W(1) = W(7+IV)
      W(2) = W(10+IV)
      RETURN
   15 KE = KE + 1
   16 KE = KE + 1
      GO TO 14
      END SUBROUTINE

!---------------------------------------------------------------------
!     INVATA                                          M O D I N A 8 7
!---------------------------------------------------------------------
!
!  PROGRAMM BESCHREIBUNG NR. 320 VON G. W. SCHWEIMER   (VERSION 1985)
!
!  INVERSION OF THE PRODUCT MATRIX A(TRANSPOSED)*A
!  THE MATRIX A IS REDUCED TO AN UPPER TRIANGULAR MATRIX R BY
!  HOUSEHOLDER TRANSFORMATIONS. THE REMAINING COMPUTATION IS STRAIGHT
!  FORWARD.
!  INPUT VARIABLES: N: NUMBER OF COLUMNS OF MATRIX A
!                   M: NUMBER OF ROWS OF MATRIX A, M >= N > 0
!                   A: INPUT MATRIX (DESTROYED)
!  OUTPUT VARIABLES:   IR: ERROR CODE
!                         IR=-2: M LT N OR N LT 1
!                         IR=-1 RANK OF MATRIX A IS ZERO
!                         IR=0 NO ERROR, RANK OF MATRIX A IS N
!                         IR>0 RANK OF MATRIX A IS IR, THE INVERSE
!                               OF A(T)*A IS COMPUTED CONSIDERING THE
!                               IR COLUMNS OF A INDICATED BY THE FIRST
!                               IR COMPONENTS OF IP
!                  A: TRIANGULAR MATRIX R, R=A(I,J) I<=J=1,N
!                  D: VECTOR OF LENGTH (N*(N+1))/2, IT CONTAINS THE
!                       UPPER TRIANGULAR PART OF THE INVERSE OF A(T)*A
!                 IP: PERMUTATION VECTOR OF LENGTH N, ITS FIRST IR
!                       COMPONENTS CONTAIN THE LABELS OF THE USEFULL
!                       COLUMNS OF A, THE LAST COMPONENTS CONTAIN
!                       THE LABELS OF THE COLUMNS WHICH ARE LINEAR
!                       COMBINATIONS OF THE FIRST.
!     THE RANK OF THE MATRIX A IS DETECTED COMPARING THE RESULT
!     OF A SUM WITH THE SUM OF ABSOLUTE VALUES.
!     IF SUM OVER I OF T(I) <= EPS * (SUM OF ABS(T(I))) THEN
!     SUM IS SET TO EXACTR ZERO.
!---------------------------------------------------------------------
      SUBROUTINE INVATA(M,N,IR,A,D,VP)
      IMPLICIT NONE
      INTEGER(2) :: IR
      INTEGER(4) :: M,N,I,I1,IJ,J,K,L
!     Size of D changed (see above, FITEX)
      REAL(8) :: A(M,N),D(15*N),VP(N)
```



```fortran
      REAL(8) :: EPS,P,Q,R,S,SIG,T,U,V,C
      DATA EPS/1.D-8/
      DATA I1/0/ ! pre-init.
      IR = INT(N,KIND=2)
      IF (M<N.OR.N<1) GO TO 19
      DO I = 1,IR; VP(I) = I; ENDDO
!  HOUSEHOLDER LOOP
      K = 0
    2 K = K + 1
!  PIVOT ELEMENT
    3 C = 0.D0
      DO 4 I = K,M
        IF (DABS(A(I,K))<=C) GO TO 4
        C = DABS(A(I,K))
        I1 = I
    4 ENDDO
      IF (C>0.D0) GO TO 8
      IR = IR - INT(1,KIND=2)
      IF (K>IR) GO TO 13
!  SET UP THE PERMUTATION VECTOR IP AND PERMUTE COLUMNS OF MATRIX A
      L = IDINT(VP(K))
      DO J = K,IR; VP(J) = VP(J+1); ENDDO
      VP(IR+1) = L
      DO I = 1,M
        C = A(I,K)
        DO J = K,IR; A(I,J) = A(I,J+1); ENDDO
        A(I,IR+1) = C
      ENDDO
      GO TO 3
!  ROTATION OF THE LOWER COLUMN FRAGMENTS OF A(K)
    8 DO J = K,IR
        C = A(K,J)
        A(K,J) = A(I1,J)
        A(I1,J) = C
      ENDDO
      S = A(K,K); V = 0.D0
      DO I = K,M
        U = A(I,K)/S
        V = V + U*U
      ENDDO
      V = 1.D0/DSQRT(V)
      SIG = S/V
      U = S + SIG
      A(K,K) = -SIG
      IF (K>=IR) GO TO 13
      L = K + 1
      DO J = L,IR
        S = V*A(K,J)
        P = DABS(S)
        DO I = L,M
          R = (A(I,K)/SIG)*A(I,J)
          S = S + R
          P = P + DABS(R)
        ENDDO
        IF (DABS(S)<=EPS*P) S = 0.D0
        T = (A(K,J)+S)/U
        IF (DABS(T)<=EPS*DABS(S/U)) T = 0.D0
        A(K,J) = -S
        DO I = L,M
          Q = A(I,J)
          P = T*A(I,K)
```

```
            R = Q - P
            IF (DABS(R)<=EPS*DABS(P)) R = 0.D0
            A(I,J) = R
          ENDDO
        ENDDO
        GO TO 2
!  END OF HOUSEHOLDER LOOP
   13 IF (IR==0) GO TO 20
!  INVERSE OF THE TRIANGULAR MATRIX R STORED IN D
        IJ = 0
        DO 16 K = 1,IR
          D(IJ+K) = 1.D0/A(K,K)
          IF (K==1) GO TO 16
          I = K
          DO L = 2,K
            I1 = I
            I = I - 1
            S = 0.D0
            DO J = I1,K; S = S + A(I,J)*D(IJ+J); ENDDO
            D(IJ+I) = -S/A(I,I)
          ENDDO
          IJ = IJ + K
   16 ENDDO
!  INVERSE OF THE PRODUCT MATRIX
        IJ = 0
        DO J = 1,IR
          DO I = 1,J
            IJ = IJ + 1
            I1 = IJ
            L = J - I
            S = 0.D0
            DO K = J,IR
              S = S + D(I1)*D(I1+L)
              I1 = I1 + K
            ENDDO
            D(IJ) = S
          ENDDO
        ENDDO
        GO TO 20
   19 IR = -2
   20 IF (IR==0) IR = -1
        IF (IR==N) IR = 0
        END SUBROUTINE

!---------------------------------------------------------------------
!     LILESQ                                          M O D I N A 8 7
!---------------------------------------------------------------------
!
!  PROGRAMM BESCHREIBUNG NR. 320 VON G. W. SCHWEIMER   (VERSION 1985)
!
!  LINEAR LEAST SQUARES PROBLEM !!B-A*X!!=MIN(X)
!  SOLVED BY HOUSEHOLDER TRANSFORMATIONS
!  REDUNDANT VARIABLES ARE DETECTED BY THE METHOD OF G.GOLUB,
!  NUMERISCHE MATHEMATIK, VOL. 7, PAGE 206-216, (1965)
!  INPUT VARIABLES:M: NUMBER OF ROWS OF A AND B
!                  N: NUMBER OF COLUMNS OF A AND ROWS OF X
!                  A: M*N MATRIX (DESTROYED)
!                  B: VECTOR OF M COMPONENTS (DESTROYED)
!  OUTPUT VARIABLES: X: VECTOR OF VARIABLES, THE REDUNDANT VARIABLES
!                      ARE SET TO ZERO. THE !!X!!=MIN IS NOT USED
!                      BECAUSE THE COMPONENTS OF X ARE ASSUMED TO BE
```



```
!                      NOT COMMENSURABLE
!                  IP: PERMUTATION VECTOR OF N COMPONENTS, IT CONTAINS
!                      THE COLUMN LABLES OF MATRIX A ORDERED ACCORDING
!                      THEIR IMPORTANCE IN REDUCING THE EUCLIDEAN NORM
!                  A: THE UPPER PART CONTAINS THE TRANSFORMED INPUT A
!                      A(2,1) CONTAINS THE SQUARE OF THE EUCLIDEAN
!                      NORM
!                  B: TRANSFORMED INPUT B
!                  IER: ERROR CODE
!                      IER=0 NO ERROR
!                      IER=-1 ALL COMPONENTS OF X ARE ZERO AND MAY BE
!                      REDUNDANT
!                      IER=-2 NO ACTION BECAUSE M < N OR N < 1
!                      IER>0 THE FIRST IER COMPONENTS OF IP CONTAIN
!                      THE LABELS OF THE NONZERO COMPONENTS OF X, THE
!                      REMAINING COMPONENTS OF X ARE ZERO AND MAY BE
!                      REDUNDANT
!  NOTE: ALL ARITHMETIC OPERATIONS ARE PERFORMED IN DOUBLE PRECISION,
!  AN ITERATIVE IMPROVEMENT IS IMPOSSIBLE WITHOUT SAVING A AND B.
!  THE ROUND OFF ERROR OF !!B-A*X!!**2 IS APPROXIMATLY GIVEN BY
!  !!B(INITIAL)!!**2 - !!B(TRANFORMED)!!**2
!---------------------------------------------------------------------
        SUBROUTINE LILESQ(M,N,IER,A,B,X,VP)
        IMPLICIT NONE
        INTEGER(2) :: IER
        INTEGER(4) :: M,N,I,IP,J,K,L,L1,L2
        REAL(8) :: C,DELTA,EPS,P,Q,R,S,SIG,T,U,V,W
        REAL(8) :: A(M,N),B(M),VP(N),X(N)
        DATA EPS/1.D-8/
        DATA W/0.d0/,SIG/0.d0/,L2/0/,L1/0/,L/0/ ! pre-init.
        IER = 0
        IF (M<N.OR.N<1) GO TO 19
        DO J = 1,N; VP(J) = J
        ENDDO
!  ROTATION LOOP
        DO 10 K = 1,N
!  PIVOT ELEMENT
          U = 0.D0
          DO 4 J = K,N
            C = 0.D0
            DO 2 I = K,M
              IF (DABS(A(I,J))<=DABS(C)) GO TO 2
              L2 = I
              C = A(I,J)
    2       ENDDO
            IF (C==0.D0) GO TO 4
            S = 0.D0
            T = 0.D0
            DO I = K,M
              V = A(I,J)/C
              S = S + V*V
              T = T + V*B(I)
            ENDDO
            IF (U>=T* (T/S)) GO TO 4
            U = T* (T/S)
            SIG = C*DSQRT(S)
            W = T
            L = J
            L1 = L2
    4     ENDDO
          IF (U==0.D0) GO TO 11
```

```
!  PERMUTE A(K) AND B(K)
        I = IDINT(VP(L))
        VP(L) = VP(K)
        VP(K) = I
        DO I = 1,M
          C = A(I,L)
          A(I,L) = A(I,K)
          A(I,K) = C
        ENDDO
        C = B(K)
        B(K) = B(L1)
        B(L1) = C
        DO J = K,N
          C = A(K,J)
          A(K,J) = A(L1,J)
          A(L1,J) = C
        ENDDO
!  ROTATION OF THE LOWER COLUMN FRAGMENT OF A(K) AND B(K)
        U = SIG + A(K,K)
        V = A(K,K)/SIG
        DELTA = (B(K)+V*W)/U
        A(K,K) = -SIG
        B(K) = -V*W
        L = K + 1
        IF (L>M) GO TO 10
        IF (K>=N) GO TO 8
        DO J = L,N
          S = V*A(K,J)
          P = DABS(S)
          DO I = L,M
            R = A(I,K)/SIG*A(I,J)
            S = S + R
            P = P + DABS(R)
          ENDDO
          IF (DABS(S)<=EPS*P) S = 0.D0
          T = (A(K,J)+S)/U
          IF (DABS(T)<=EPS*DABS(S/U)) T = 0.D0
          A(K,J) = -S
          DO I = L,M
            Q = A(I,J)
            P = T*A(I,K)
            R = Q - P
            IF (DABS(R)<=EPS*DABS(P)) R = 0.D0
            A(I,J) = R
          ENDDO
        ENDDO
    8   DO I = L,M
          B(I) = B(I) - DELTA*A(I,K)
        ENDDO
   10 ENDDO
!  END OF ROTATION LOOP
      K = N
      GO TO 12
   11 K = K - 1
      IER = int(K,KIND=2)
!  SQUARE OF THE EUCLIDEAN NORM
   12 S = 0.D0
      L = K + 1
      IF (K==M) GO TO 14
      DO I = L,M
        S = S + B(I)*B(I)
```



```
      ENDDO
   14 A(2,1) = S
      IF (K==N) GO TO 16
!  COMPONENTS OF X WHICH DO NOT REDUCE THE EUCLIDEAN NORM
      DO I = L,N
        DO J = L,N
          IP = IDINT(VP(J))
          X(IP) = 0.D0
        ENDDO
      ENDDO
      IF (K==0) GO TO 20
!  COMPUTATION OF X
   16 IP = IDINT(VP(K))
      X(IP) = B(K)/A(K,K)
      IF (K==1) GO TO 21
      DO J = 2,K
        L = K + 2 - J
        S = B(L-1)
        DO I = L,K
          IP = IDINT(VP(I))
          S = S - A(L-1,I)*X(IP)
        ENDDO
        IP = IDINT(VP(L-1))
        X(IP) = S/A(L-1,L-1)
      ENDDO
      GO TO 21
!  ERROR CODE
   19 IER = IER - INT(1,KIND=2)
   20 IER = IER - INT(1,KIND=2)
   21 RETURN
      END SUBROUTINE
!  Number of lines: 6681
```

Appendix A2 – TOPO Source Code

GFortran, free source form

Before displaying the TOPO source code, the data of the upper and lower lakes is provided (file: zlakes.txt). The rectangular borders are given by their geographical latitudes and longitudes. The reader is free to modify the numbers or extend the table by other lakes or continental seas. If the table is to be extended, the number 14 (Anzahl der Gebiete/ number of areas) must be accordingly adapted.

```
*                                                                     *
*                 --- HOCH LIEGENDE BINNENMEERE ---                   *
*          (zur Berechnung des tatsaechlichen Erdvolumens)            *
*                                                                     *
*   Diese Datei enthaelt geographische rechtwinklige Gebiete mit      *
*   den groessten Binnenseen, die ueber oder unter dem Meesesspiegel- *
*   niveau liegen. Die Begrenzungen werden durch die geographische    *
*   Breite und Laenge in dezimalen Grad gegeben. Vorzeichen: noerdl.  *
*   Breite pos./ suedl. Breite neg./ westl. Laenge neg./ oestl. Laen- *
*   ge pos. Die Hoehe des Wasserspiegels ueber dem Meeresniveau (NN,  *
*   Normal Null) ist in Metern anzugeben. Aus programmtechnischen     *
*   Gruenden darf ein Gebiet nicht ueber den Nullmeridian reichen.    *

  ======================---==
    Anzahl der Gebiete:  14
  ======================---==

       geograph. Breite        geograph. Laenge        Hoehe ueber NN
         von      bis            von      bis            [m]  (google)
=======================================================================
 1. Lake Superior -------------------------- North America ------------
         46.4     49.2          -92.7    -84.0          183.   (184)
 2. Lake Michigan
         41.3     46.3          -88.4    -84.3          176.   (176)
 3. Lake Huron
         42.9     46.5          -84.3    -79.7          176.   (176)
 4. Lake Erie
         41.2     43.0          -83.9    -78.7          174.   (174)
 5. Lake Ontario
         43.0     44.4          -80.0    -75.7           75.    (75)
 6. Great Bear Lake
         64.2     67.3         -126.0   -117.0          156.   (156 ?)
 7. Great Slave Lake
         60.7     63.2         -118.4   -108.8          156.   (156)
 8. Lake Winnipeg (*)
         50.3     54.0          -99.6    -96.0          217.   (217)
 9. Kaspijskoje More (*) ------------------------ Asia ----------------
         36.2     50.0           44.0     56.0          -28.   (-28)
10. Aralskoje More (*)
         43.1     47.2           57.7     62.5           53.   (35 ?)
11. Ozero Bajkal (*)
         51.4     55.9          103.4    110.0          455.   (455)
12. Lake Victoria (*) -------------------------- Africa ---------------
         -3.3      0.7           31.4     35.0         1134.  (1134 ?)
13. Lake Tanganyika (*)
         -8.9     -3.3           29.1     31.3          773.   (782)
14. Lake Malawi (Lago Niassa) (*)
        -14.5     -9.4           33.8     35.4          473.   ( ? )
=======================================================================

(*)  Hierbei ist in 'Worldbath' anstelle des Seebodens schon die
     Hoehe des Wasserspiegels angegeben, was unser Ergebnis nicht
     beeinflusst, da wir sowieso die Wasserspiegelhoehe benoetigen.
```

TOPO source code ⟶

```fortran
!=================================================================!
!                                                                 !
!                                                                 !
!                       T O P O    (gfortran)                     !
!                                                                 !
!                                                                 !
!   Programm zur Berechnung des Erdvolumens und weiterer Parameter  !
!   unter Beruecksichtigung der Land- und Wassermassen ueber Meeres- !
!   spiegelniveau bei Verwendung der topographischen Daten von    !
!   "Worldbath" und unter Beruecksichtigung der Ellipsoid- bzw.   !
!   Sphaeroidgestalt der Erde.                                    !
!                                                                 !
!                          Hans Jelitto                           !
!                      Hamburg, 3. August 2025                    !
!                                                                 !
!                                                                 !
!   Zum Programm gehoert die Datei der topographischen Hoehendaten  !
!   '[X]data.tsv'. Sie wurde von der Website (Worldbath):         !
!                                                                 !
!      http://iridl.ldeo.columbia.edu/SOURCES/.WORLDBATH/.bath/   !
!                                                                 !
!   unter 'Data Files' und 'Text with tab-separated-values' [X+]  !
!   heruntergeladen.                                              !
!                                                                 !
!   Als simplen Ansatz koennte man annehmen, dass die durchschnitt- !
!   liche Hoehe zur Berechnung des Volumens der Landmassen genuegt. !
!   Leider ist dieser Wert fuer die Berechnung des Erdvolumens nicht !
!   brauchbar, da nur der Mittelwert der Daten berechnet wird. Dies !
!   ist fuer kleine Gebiete, in denen sich der Abstand der geogra- !
!   phischen Gitterpunkte kaum aendert, zulaessig. Ueber groessere  !
!   Gebiete und speziell inklusive der Pole liefert diese Methode  !
!   jedoch keine korrekten Ergebnisse, da die unterschiedlichen    !
!   Gitterpunktabstaende nicht beruecksichtigt werden. Zum Vergleich !
!   wird dennoch dieser vereinfachte Wert durch das Programm mit   !
!   berechnet und als arithmetisches Mittel in Klammern angegeben. !
!   Entsprechende Zahlen fuer die Landmassen ueber dem Meeresspiegel !
!   sind unter "(arithm. mean)" aufgefuehrt. Sie sind jedoch auch  !
!   meist nicht korrekt wegen dieses zu einfachen Rechenansatzes.  !
!                                                                 !
!   Darueber hinaus kann die normale Mittelwertbestimmung sowieso  !
!   nicht verwendet werden, weil zur Berechnung des Erdvolumens in !
!   unserem Fall auf den Meeresflaechen nicht der Meeresboden, son- !
!   dern der Meeresspiegel als Grundlage dient.                   !
!                                                                 !
!   Die Zahlen unter "vol.-based" entsprechen der mittleren Hoehe  !
!   gemaess dem tatsaechlichen Volumen. Die Berechnung wurde abge- !
!   leitet aus Gleichung (78) mit gemitteltem Erdradius und Vernach- !
!   laessigung des Terms hoechster Ordnung (h^3). Darueber hinaus  !
!   existieren zwei alternative Berechnungsmethoden (in Kommentare !
!   umgewandelt - siehe unten "!c"). Die Zahlen unter "ell.-based" !
!   (ellipsoid-based) beruhen auf einem einfacheren Ansatz "Flaeche !
!   mal Hoehe", das heisst ohne Volumeneffekte hoeherer Ordnung.   !
!                                                                 !
!   In diesem Programm werden die groessten Binnenseen mit Ober-   !
!   flaechen, welche sich ueber- oder unterhalb des Meeresspiegels !
!   befinden, ebenfalls erfasst. Bei einigen Seen in Worldbath ist !
!   der Boden und bei anderen Seen die Wasserspiegelhoehe angegeben. !
!   Insgesamt betraegt die Volumenkorrektur, die sich durch solche !
!   Seen mit Angabe Seeboden ergibt, ueber 2600 Kubikkilometer, was !
!   prozentual gesehen jedoch immer noch gering ist.              !
!                                                                 !
```



```fortran
!                      --------------------                       !
!                                                                 !
!      Zum Programm 'Topo' gehoeren nachfolgende 6 Dateien:       !
!                                                                 !
!      Datei                   Kurzbeschreibung                   !
!   --------------------------------------------------------------   !
!     topo          Ausfuehrbare Programmdatei                    !
!     topo.f95      FORTRAN-Quellcode, vorliegender Text          !
!     [X]data.tsv   Topograph. Daten (5 minute-grid, Worldbath)   !
!     zlakes.txt    Korrekturdaten, hoch und tief liegende Seen   !
!     readme.pdf    Kurzinformation zum Programm                  !
!     out.txt       Ergebnis-Datei (wird mit jedem Programmlauf   !
!                   ueberschrieben)                               !
!   --------------------------------------------------------------   !
!                                                                 !
!=================================================================!

      module constants; real(8) :: pi,a,c,dl,dnx
        integer(4), parameter :: NX5 = 4320, NY5 = 2161
      end module
      module lakes; real(8) :: se(5,50),zero
        integer(4) :: il(5,50),nlake
      end module

      program topo
!-----Hauptprogramm----------------------------------------------------

      use constants; use lakes
      implicit double precision (A-H,O-Z); integer(4) ivap(NX5)
!     Bei folgenden Deklarationen (0:2) bedeuten die Indizes drei
!     verschiedene Hoehen: "0" bedeutet Meeresboden, "1" Meeresspiegel
!     und "2" Meeresspiegel mit hoch und tief liegenden Binnenseen.
      real(8) ev(0:2),hmitt(0:2),hmitv(0:2),hmitw(0:2)
      real(8) sumh(0:2),sumo(0:2),sumw(0:2),Vg(0:2),Vges(0:2)
      character(38) :: title1,title2,line
      character(23) :: dummy; character(11) :: tfile

!-----Parameter eingeben
      title1 = '   EARTH`S VOLUME INCLUDING LANDMASS  '
      title2 = ' (Worldbath, 5 arc-minute resolution) '
      line   = '======================================'
      open(unit=1,file='out.txt')
      write(6,'(//4(21x,A38/))') line,title1,title2,line
      write(1,'(/2(21x,A38/))') title1,title2
      write(*,'(17x,''latitude .......... (min. -90.0) from :  '')' &
     &      ,advance='no')
      read(*,*) gbmin
      if (gbmin<-90.d0 .or.gbmin>90.d0) go to 100
      write(*,'(17x,''latitude .......... (max.  90.0)   to :  '')' &
     &      ,advance='no')
      read(*,*) gbmax
      if(gbmax<-90.d0 .or.gbmax>90.d0 .or.gbmax<gbmin) go to 100
      write(*,'(17x,''ext. longitude east (min.   0.0) from :  '')' &
     &      ,advance='no')
      read(*,*) glmin
      if (glmin<0.d0 .or.glmin>360.d0) go to 100
      write(*,'(17x,''ext. longitude east (max. 360.0)   to :  '')' &
     &      ,advance='no')
      read(*,*) glmax
      if (glmax<0.d0 .or.glmax>360.d0 .or.glmax<glmin) go to 100
```

```
      !-----5x5-Bogenminuten-Gitter (Worldbath)
            tfile = '[X]data.tsv'
            NX = NX5; NY = NY5

      !-----Programmstart
            dnx   = dfloat(NX)
            dgrad = dnx/360.d0
            ibmin = idnint((gbmin + 90.d0)*dgrad + 1.d0)
            ibmax = idnint((gbmax + 90.d0)*dgrad + 1.d0)
            ilmin = idnint(glmin*dgrad + 1.d0)
            ilmax = idnint(glmax*dgrad)
            if (ilmax<ilmin) go to 100
            db    = dfloat(ibmax-ibmin+1)
            dl    = dfloat(ilmax-ilmin+1)
            write(*,'(/''   Computation started.  '')',advance='no')
            do i=1,idint((gbmax+90.d0)/5.d0)
              write(*,'(''>'')',advance='no')
            enddo
            write(*,'(/''   Output file: out.txt  '')',advance='no')

      !-----Einlesen der Daten hoch bzw. tief liegender Seen
            zero = 0.d0
            open(unit=10,file='zlakes.txt')
            do i=1,14
              read(10,*)
            enddo
            read(10,'(A23,I4)') dummy,nlake
            do i=1,5
              read(10,*)
            enddo
            do i=1,nlake
              read(10,*)
              read(10,*) (se(j,i),j=1,5)
            enddo
            do i=1,nlake
              do j=1,2
                il(j,i) = idnint((se(j,i)+ 90.d0)*dgrad + 1.d0)
              enddo
              if (se(3,i)>=zero) then
                il(3,i) = idnint(se(3,i)*dgrad + 1.d0)
              else
                il(3,i) = idnint((se(3,i)+360.d0)*dgrad + 1.d0)
              endif
              if (se(4,i)>=zero) then
                il(4,i) = idnint(se(4,i)*dgrad)
              else
                il(4,i) = idnint((se(4,i)+360.d0)*dgrad)
              endif
      !c      write(*,*)'i, il(1..5) = ',i,(il(j,i),j=1,4),se(5,i)
            enddo
            close(10)

      !-----Weitere Konstanten
      !                                        Aequatorradius  Polradius
      !     Referenz-Ellipsoid Erde:                 a [m]        c [m]
      !     ==============================================================
      !     K.R. Lang, Astophys. Data, Planets ...   6378140      6356775
      !     World Geod. System's Ref. Ellipsoid      6378137      6356752.3
      !     IERS Conventions (1989)                  6378136      6356751.3
      !     IERS Conventions (2003)                  6378136.6    6356751.9
      !     ==============================================================
```



```
            a     = 6378136.6d0
            c     = 6356751.9d0
            pi    = 3.14159265358979324d0
            V0    = (4.d0*pi/3.d0)*a**2*c * 1.d-9
      ! . . Ellipsoid-Oberflaeche, Hauptachsen: a = b > c (oblat)
            e     = dsqrt(1.d0 - (c/a)**2)
            ath   = 0.5d0 * dlog((1.d0 + e)/(1.d0 - e))
            Feli  = 2.d0 * pi * (a*a + c*c * ath/e)  ! [m**2]
      ! . . Flaeche an den Polen (fuer -90 und 90 Grad geogr. Breite)
            dwi   = pi/(180.d0*dgrad)
            call koord(-0.5d0*pi+0.5d0*dwi,x0,y0)
            Fpol  = pi * x0**2
      ! . . Kugel-Oberflaeche bei gleichem Volumen wie Ellipsoid
            rm    = (a**2*c)**(1.d0/3.d0)
            Fkug  = 4.d0*pi*rm**2  ! [m**2]
      ! . . Weitere Initialisierungen
            imo   = 5 * idnint(dgrad)
            inum  = 0
            Fges  = zero
            do i = 0,2
              sumw(i) = zero
              Vg(i)   = zero
              Vges(i) = zero
            enddo

      !-----Berechnung - Volumen der Landmassen und Erdoberflaeche
            open(unit=5,file=tfile,status='unknown', &
           &     access='sequential',RECL=NX*NY)
            do k=1,ibmax
              if (mod(k,imo).eq.0) write(*,'(''>'')',advance='no')
              read(5,*) (ivap(i),i=1,ilmax)
              if (k>=ibmin) then
                do i=0,2; sumh(i) = zero; sumo(i) = zero; enddo
                breite = -0.5d0*pi + dfloat(k-1)*dwi
                call koord(breite,x,y)
                r = dsqrt(x**2 + y**2)
      ! . . . . Innere Hauptschleife (Beruecksichtigung des Meeresspiegels)
                do l=ilmin,ilmax
                  ev(0) = dfloat(ivap(l))
                  ev(1) = ev(0)
                  if (ev(1)<zero) ev(1) = zero
                  call levels(ev(0),k,l,ev(2))
                  do i=0,2
                    ek = ev(i)
                    sumh(i) = sumh(i) + ek
      ! . . . . . . . mit Volumenanteilen hoeherer Ordnung
                    sumo(i) = sumo(i) + ek*(1.d0 + ek/r + ((ek/r)**2)/3.d0)
                  enddo
                  inum = inum + 1
                enddo
      ! . . . . Flaechenberechnung (Meeresspiegel)
                call stripe(breite,dwi,100,Fi,Fh)
                if (k.eq.ibmin) Fi = Fi - Fh
                if (k.eq.ibmax) Fi = Fh
                if (k.eq.1.or.k.eq.NY) Fi = Fpol * dl/dnx
                Fges = Fges + Fi
                do i=0,2
                  sumw(i) = sumw(i) + sumh(i)/dl
      ! . . . . . . Vg: Volumen zur Berechnung der mittleren Hoehe
                  Vg(i)   = Vg(i)   + Fi*sumh(i)/dl
                  Vges(i) = Vges(i) + Fi*sumo(i)/dl
```

```
            enddo
          endif
        enddo
        close(5)

!-----Ergebnisse der mittleren Hoehe
        do i=0,2
! . . . Mittlere Hoehe nach Flaechenanteilen auf Meeresspiegelniveau
!         (Naeherung gemaess Volumen = Flaeche * Hoehe)
          hmitt(i) = Vg(i)/Fges

! - - - 1. Mittlere Hoehe gemaess Volumen: Berechnung abgeleitet aus
!         Gleichung (78) mit mittlerem Erdradius und Vernachlaessigung
!         des Terms hoechster Ordnung (Hoehe^3)
          hmitv(i) = -rm/2.d0 + dsqrt(Vges(i)*rm/Fges + rm**2/4.d0)
!         write(*,'(/3x,a7,f10.3)')'1.  h =',hmitv(i)

! . . . 2. Alternative Berechnung mit Ellipsoidform und auch Vernach-
!         laessigung des Terms hoechster Ordnung (Hoehe^3)
!c        cube = 0.75d0*Vges(i)/pi
!c        q = a*(a/2.d0 + c)/(2.d0*a + c)
!c        hmitv(i) = (-q + dsqrt(cube/(2.d0*a + c) + q**2)) * Feli/Fges
!         write(*,'(3x,a7,f10.3)')'2.  h =',hmitv(i)

! . . . 3. Alternative Berechnung mit Kugelvolumen und Korrektur ge-
!         maess Ellipsoid-Oberflaeche (bei sehr kleiner Flaeche ungenau
!         bzw. falsch, wie z.B. bei einem oder wenigen Punkten nahe Pol)
!c        cube = rm**3 + 0.75d0*Vges(i)/pi
!c        hmitv(i) = (-rm + cube**(1.d0/3.d0)) * Fkug/Fges
!         write(*,'(3x,a7,f10.3)')'3.  h =',hmitv(i)

! . . . Arithmetisches Mittel
          hmitw(i) = sumw(i)/db
          Vges(i) = Vges(i)*1.d-9
        enddo
! . . Ergebnisse (Flaeche und Volumen)
        Fges  = Fges*1.d-6          ! Gesamtflaeche (aufintegriert)
        Verdl = Vges(2) - Vges(1) ! Volumen der hoeher gelegenen Seen
        Verdg = V0 + Vges(1)      ! Erdvolumen (Ellipsoid) + Landmassen
        Verds = V0 + Vges(2)      ! Erdvolumen + Landmassen + h. g. Seen

!-----Ergebnis-Ausgabe
        write(*,'(/)')
        do iu=1,6,5
         write(iu,'(1x,78a1)') ('=',i=1,78)
         write(iu,'(7x,A26,f13.1,A29)')'Earth`s equator. radius  :', &
      &  a,' m   (IERS conventions, 2003)'
         write(iu,'(7x,A26,f13.1,A29)')'Earth`s polar radius     :', &
      &  c,' m   (IERS conventions, 2003)'
         write(iu,'(7x,A26,f13.1,A2)')'Earth`s mean radius      :', &
      &  rm,' m'
         write(iu,'(7x,A13,A16,f8.2,A4,f8.2,3X,A16)')'Geograph. lat', &
      &  'itude [deg] :   ',gbmin,'  to',gbmax,'used grid points'
         write(iu,'(7x,A13,A16,f8.2,A4,f8.2,5X,I14/)')'Extended long', &
      &  'itude [deg] :   ',glmin,'  to',glmax,inum
         write(iu,'(A33,A44)') '       Kind of average           ', &
      &  '  ell.-based    vol.-based   (arithm. mean)'
         write(iu,'(7x,70a1)') ('-',i=1,70)
         write(iu,'(A33,F11.3,A2,F12.3,A6,F10.3,A3)') &
      &  ' (A)   Average height (sea bed) :',hmitt(0), &
      &  ' m',hmitv(0),' m   (',hmitw(0),' m)'
```



```
         write(iu,'(A33,F11.3,A2,F12.3,A6,F10.3,A3)') &
      &  ' (B)   Average h.   (sea level) :',hmitt(1), &
      &  ' m',hmitv(1),' m   (',hmitw(1),' m)'
         write(iu,'(A33,F11.3,A2,F12.3,A6,F10.3,A3)') &
      &  ' (B*)  Average h. (upper lakes) :',hmitt(2), &
      &  ' m',hmitv(2),' m   (',hmitw(2),' m)'
         write(iu,'(7x,70a1/)') ('-',i=1,70)
         write(iu,'(A24,A27,1p,1e19.9,A5)')'       Covered area ....', &
      &  '. (sea level, integrated) :',Fges,' km^2'
         write(iu,'(A24,A27,1p,1e19.9,A5)')'       Ellipsoid surface', &
      &  ' ........... (analytical) :',Feli*1.d-6,' km^2'
         write(iu,'(A23,A28,1p,1e19.9,A5/)')'       Surface of spher', &
      &  'e ......... (equal volume) :',Fkug*1.d-6,' km^2'
         write(iu,'(A24,A27,1p,1e19.9,A5)')'       Volume correction', &
      &  ' ............. (as per A) :',Vges(0),' km^3'
         write(iu,'(A24,A27,1p,1e19.9,A5)')' (C)   Volume of landmas', &
      &  's ............ (as per B) :',Vges(1),' km^3'
         write(iu,'(A24,A27,1p,1e19.9,A5)')' (C*)  Volume of landmas', &
      &  's + lakes ... (as per B*) :',Vges(2),' km^3'
         write(iu,'(A22,A29,1p,1e14.4,A10/)')'       Volume of upper', &
      &  ' lakes ........... (C* - C) :',Verdl,'      km^3'
         write(iu,'(A24,A27,1p,1e19.9,A5)')' (D)   Earth`s volume ..', &
      &  '.. (ellipsoid, sea level) :',V0,    ' km^3'
         write(iu,'(A23, A28,1p,1e19.9,A5)')' (E)   Earth`s volume +', &
      &  ' landmass ........ (D + C) :',Verdg,' km^3'
         write(iu,'(A24,A27,1p,1e19.9,A5)')' (F)   Earth`s vol. + la', &
      &  'ndm. + lakes ... (D + C*) :',Verds,' km^3'
         write(iu,'(1x,78a1/)') ('=',i=1,78)
        enddo
        go to 200
  100 continue
        do iu=1,6,5
          write(iu,'(/A36,A20)')'  ----->   Insert a correct number:  ',&
      &   'lat-max >= lat-min,'
          write(iu,'(37x,a25//)')'lon-max >= lon-min + 0.05'
        enddo
  200 close(1)
        stop
        end program topo

        subroutine levels(ev0,k,l,ev2)
!-----Erfassung von hoch oder tief liegenden Binnenseen. Falls
!       zutreffend wird die Hoehenangabe 'ev2' korrigiert. (Anmerkung:
!       Die Gebiete duerfen nicht ueber den Nullmeridian reichen, was
!       auch nicht vorkommt. Gegebenenfalls waere eine Loesung, das
!       Gebiet entlang des Nullmeridians zu teilen.)
        use lakes
        implicit double precision (a-h,o-z)
        ev2 = ev0
        do i=1,nlake
          elev = se(5,i)
          if (il(1,i)<=k.and.k<=il(2,i).and. &
      &       il(3,i)<=l.and.l<=il(4,i)) then
            if (ev0<elev) ev2 = elev; go to 10
          else
            if (i>=nlake.and.ev0<zero) ev2 = zero
          endif
        enddo
   10 return
        end
```

```fortran
      subroutine koord(br,x,y)
!-----Input: 'br' geograph. Breite; Output: 'x' Abstand des Brei-
!     tenkreises zur Erdachse, 'y' Abstand des Breitenkreises zur
!     Aequatorebene (mit Vorz.); Berechnung fuer abgeplattete
!     Sphaeroidgestalt.
      use constants
      implicit double precision (a-h,o-z)
      x = a / dsqrt(1.d0 + (c*dtan(br)/a)**2)
      y = c * dsqrt(1.d0 - (x/a)**2)
      if (br<0.d0) y = -y
      return
      end

      subroutine stripe(b,dwi,n,F,Fh)
!-----berechnet die Flaeche F des Streifens (Breite 'dwi') zwischen
!     zwei Breitenkreisen mit den geographischen Breiten b-dwi/2 und
!     b+dwi/2 auf der Ellipsoid-Oberflaeche. Der Winkel dwi ist der
!     geographische Breiten- bzw. Laengenunterschied zweier benach-
!     barter Gitterpunkte im Bogenmass. Je groesser der Parameter 'n'
!     ist, desto genauer ist die numerische Integration der Oberflae-
!     che. (Der Parameter n teilt den Streifen der Flaeche F in 2*n
!     parallele duenne Streifen, wobei n = 100 hier voellig ausrei-
!     chend ist.)
      use constants
      implicit double precision (a-h,o-z)
      ti = 1.d0/dfloat(2*n)
      F  = 0.d0
      call koord(b-0.5d0*dwi,x1,y1)
      do i=1,2*n
        call koord(b+(dfloat(i)*ti-0.5d0)*dwi,x2,y2)
        dbr = dsqrt((x2-x1)**2 + (y2-y1)**2)
        F = F + (x1+x2)*dbr
        if (i.eq.n) Fh = F * pi*dl/dnx
        x1 = x2
        y1 = y2
      enddo
      F = F * pi*dl/dnx
      return
      end
!     Number of lines: 406
```

Use of programs P5, TOPO, and description

Further copyrights

Internet addresses

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List of Internet addresses

No.	URL
1	https://pyramiden-jelitto.de/downloads.html
2	https://lte.observatoiredeparis.psl.eu/ (Merger of SyRTE and IMCCE on January 1, 2025, new name LTE)
3	https://ftp.imcce.fr/pub/ephem/planets/vsop87/
4	https://www.kit.edu/english/index.php
5	https://eclipse.gsfc.nasa.gov/eclipse.html
6	https://eclipse.gsfc.nasa.gov/SEhelp/deltatpoly2004.html
7	https://archive.org/details/SkyGlobe_1020
8	https://stellarium.org/
9	https://eclipse.gsfc.nasa.gov/transit/catalog/MercuryCatalog.html
10	https://eclipse.gsfc.nasa.gov/transit/catalog/VenusCatalog.html
11	https://ssd.jpl.nasa.gov/planets/eph_export.html
12	https://eclipse.gsfc.nasa.gov/SEhelp/uncertainty2004.html
13	https://iridl.ldeo.columbia.edu/SOURCES/.WORLDBATH/.bath/

14 https://www.ncei.noaa.gov/products/etopo-global-relief-model

15 https://ssd.jpl.nasa.gov/diagrams/mb_hist.html

16 https://www.eso.org/public/images/eso1440a/

17 https://www.fourmilab.ch/cgi-bin/Solar

18 https://ssd.jpl.nasa.gov/tools/orbit_viewer.html

19 https://pyramiden-jelitto.de/pup-bautechnik.html

20 https://pyramiden-jelitto.de/pub-aufsaetze-downloads.html

21 https://shopatsky.com/products/astronomical-algorithms-2nd-edition

22 https://shopatsky.com/collections/willmann-bell

23 https://shopatsky.com/products/transits

24 https://gcc.gnu.org/onlinedocs/gcc-11.2.0/gfortran.pdf

25 https://www.iap.kit.edu/english/index.php

26 https://katalog.bibliothek.kit.edu/cgi-bin/koha/opac-detail.pl?biblionumber=778639

27 https://katalog.bibliothek.kit.edu/cgi-bin/koha/opac-detail.pl?biblionumber=778640

28 https://www.inah.gob.mx

29 https://ui.adsabs.harvard.edu/abs/1982A%26A...114..278B/abstract

30 https://ui.adsabs.harvard.edu/abs/1988A%26A...202..309B/abstract

31 https://pyramiden-jelitto.de/buch-pyramiden-und-planeten.html

32 https://archive.org/details/pyramidstempleso0000unse/

33 https://www.gizapyramids.org/pdf_library/petrie_gizeh.pdf

34 https://zamalekbc.com/

35 https://openlibrary.org/books/OL5972546M/L'_architettura_delle_piramidi_menfite

36 https://archive.org/details/lexikondergyptol00helc

37 https://heasarc.gsfc.nasa.gov/W3Browse/star-catalog/ppm.html

38 https://simbad.u-strasbg.fr/simbad/

39 https://www.youtube.com/watch?v=FNjwrctdZ1Y

40 https://archive.org/details/lifeworkgreatpy1smyt/page/iii/mode/1up

41 https://archive.org/details/lifeworkgreatpy2smyt/page/iii/mode/1up

42 https://archive.org/details/lifeworkgreatpy3smyt/page/iii/mode/1up

43 https://archive.org/details/bub_gb_7FM3AQAAMAAJ/page/n6/mode/1up

44 https://archive.org/details/GreatPyramidPassagesVol11910Edition

45 https://archive.org/details/GreatPyramidPassagesVol21913Edition

46 https://archive.org/details/operationscarrie01howa/page/n8/mode/1up

47 https://archive.org/details/operationscarrie02howa/page/n10/mode/1up

48 https://archive.org/details/isbn_9785873847167/page/n4/mode/1up

49 https://digi.ub.uni-heidelberg.de/diglit/perring1839bd1?sid=2bbaa7e72798c61bd258bd5b42f1c350

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60 https://www.agi.com/getmedia/c85a440a-cf71-4e08-ad78-fa73736cee6c/Precession-nutation-Theories-and-their-Implementation.pdf?ext=.pdf)

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66 https://meetingorganizer.copernicus.org/EPSC-DPS2011/EPSC-DPS2011-137-8.pdf

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72 https://www.youtube.com/watch?v=GVvVMOV5oI0

73 https://www.hangar18b.com/die-planetenkorrelationen-von-teotijuacan-hans-jelitto-aas-odm-2023/

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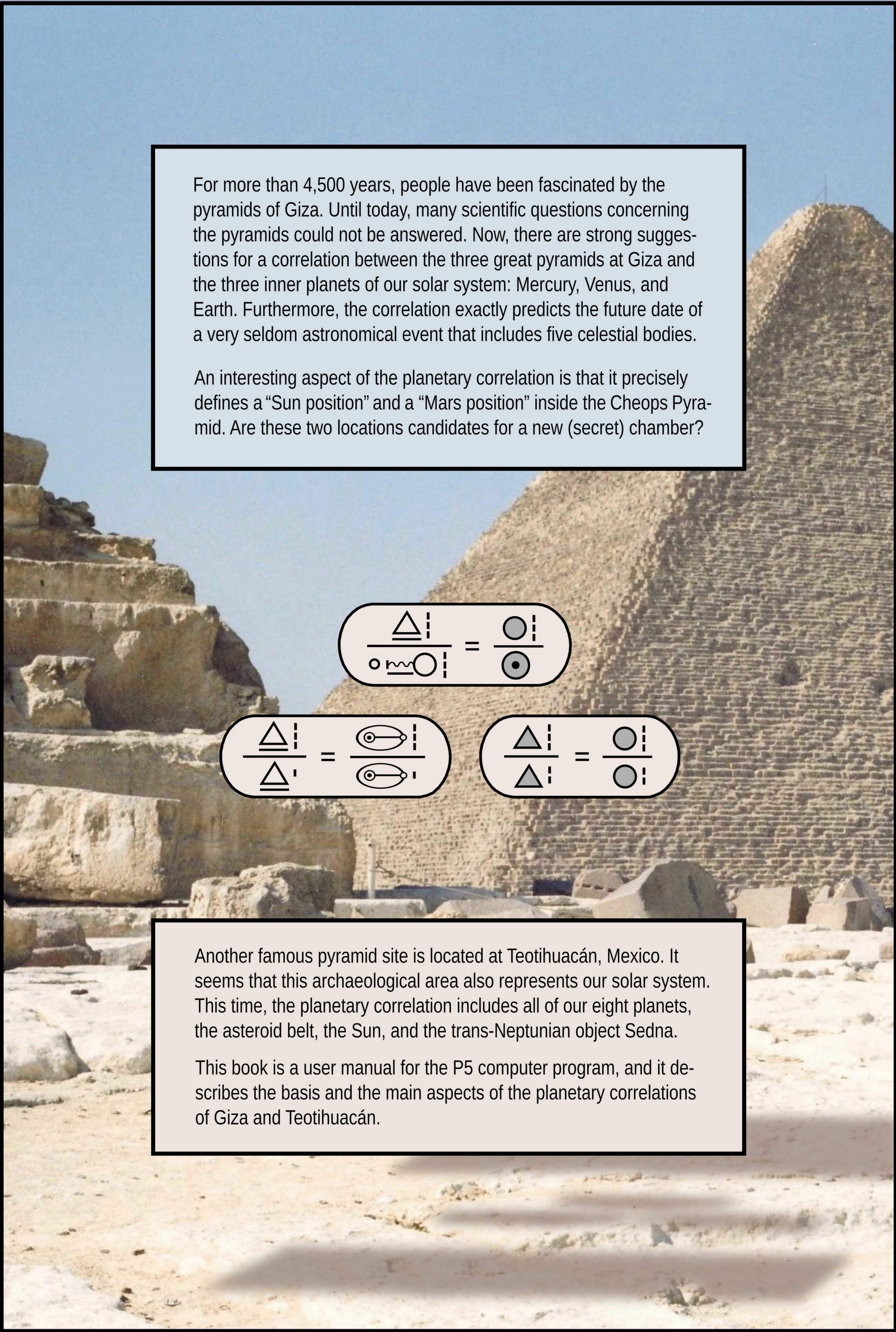
For more than 4,500 years, people have been fascinated by the pyramids of Giza. Until today, many scientific questions concerning the pyramids could not be answered. Now, there are strong suggestions for a correlation between the three great pyramids at Giza and the three inner planets of our solar system: Mercury, Venus, and Earth. Furthermore, the correlation exactly predicts the future date of a very seldom astronomical event that includes five celestial bodies.

An interesting aspect of the planetary correlation is that it precisely defines a “Sun position” and a “Mars position” inside the Cheops Pyramid. Are these two locations candidates for a new (secret) chamber?



Another famous pyramid site is located at Teotihuacán, Mexico. It seems that this archaeological area also represents our solar system. This time, the planetary correlation includes all of our eight planets, the asteroid belt, the Sun, and the trans-Neptunian object Sedna.

This book is a user manual for the P5 computer program, and it describes the basis and the main aspects of the planetary correlations of Giza and Teotihuacán.