SUPPLEMENT TO:

Coincidence Cross Sections within the Quasi Free Break-up Model for Elastic Projectile Break-up

Hans Jelitto

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Abstract

This article contains additional information to the Report KfK-4960 (Kernforschungszentrum Karlsruhe, today: KIT), published by V. Corcalciuc and the author in 1991. Therein, the Quasi Free Break-up Model, introduced originally by Robert Serber, was used to calculate triple differential cross sections for particle-particle coincidences in nuclear break-up reactions. In that report, the approach of Serber's model had not been changed. Instead, the area of application had been extended by pure analytical calculations. To obtain triple differential cross sections, one calculation step was a coordinate transformation from the system on the circumference of the target nucleus to the laboratory system by evaluating the determinant of a five-dimensional Jacobi matrix. For the connection of the corresponding sets of variables, five equations were provided. However, these equations did not explicitly connect the variables that were used in the matrix. Therefore, it could be difficult to follow the calculations. In the present article, the missing set of five equations is provided, which should improve the understanding and facilitate the validation of the Jacobian.

The so-called Serber model [1], developed by the nuclear physicist Robert Serber in 1947, describes the quasi free break-up of nuclei, when they pass or hit the target nucleus after being accelerated in a particle accelerator. For example, a deuteron-ion can break up into proton and neutron or a ^{6}Li-ion can disintegrate into alpha particle and deuteron. The latter reaction was investigated in detail by the author and colleagues [2]. However, Serber calculated the (single) differential cross sections $d\sigma/dE$ and $d\sigma/d\Omega$, with σ being the cross section, and E and Ω the energy and emission direction of the observed fragment. About 37 years later, H. Utsunomiya derived double differential cross sections $d^2\sigma/(d\Omega\, dE)$ on the same basis [3]. In 1991, together with V. Corcalciuc, we derived triple differential cross sections $d^3\sigma/(d\Omega_1 d\Omega_2 dE_1)$ on the basis of Serber's model [4], describing coincidence experiments in which both fragments are detected simultaneously. Here and in the following, the indices 1 and 2 correspond to the two fragments. Actually, this triple differential cross section depends on five variables, which are

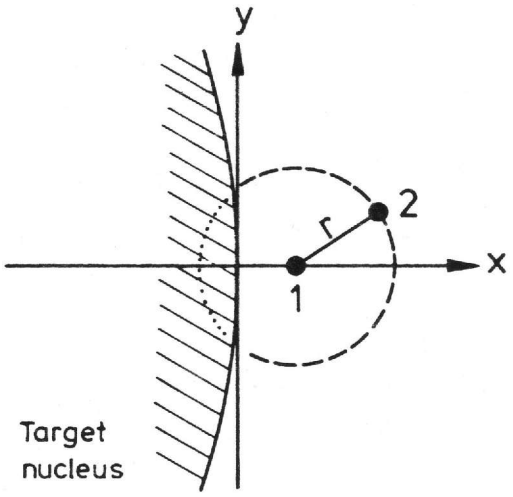

Figure 1: *Local coordinate system at the target surface and the two projectile clusters 1 and 2, projected into the x-y plane with a given separation r. The viewpoint of this figure is in the direction of the incoming projectiles (taken from [4]).*

θ_1, ϕ_1, θ_2, ϕ_2, and E_1. The angles θ and ϕ specify the emission direction of a fragment, indicating that the whole reaction takes place not necessarily *in plane* but also *out of plane*. E_1 is the kinetic energy of fragment 1.

The model is also called the "Spectator Model," which indicates that the projectile has no interaction with the target, except the momentum transfer. In other words, the target nucleus remains in the ground state. The break-up of the projectile is triggered by constriction of the projectile space by the target nucleus (Fig. 1). The relative momentum between the two fragments, e.g., alpha particle and deuteron, is determined by the internal momentum distribution between the alpha and deuteron cluster in the projectile. Fig. 2 illustrates the geometric situation of the momenta of the fragments after break-up.

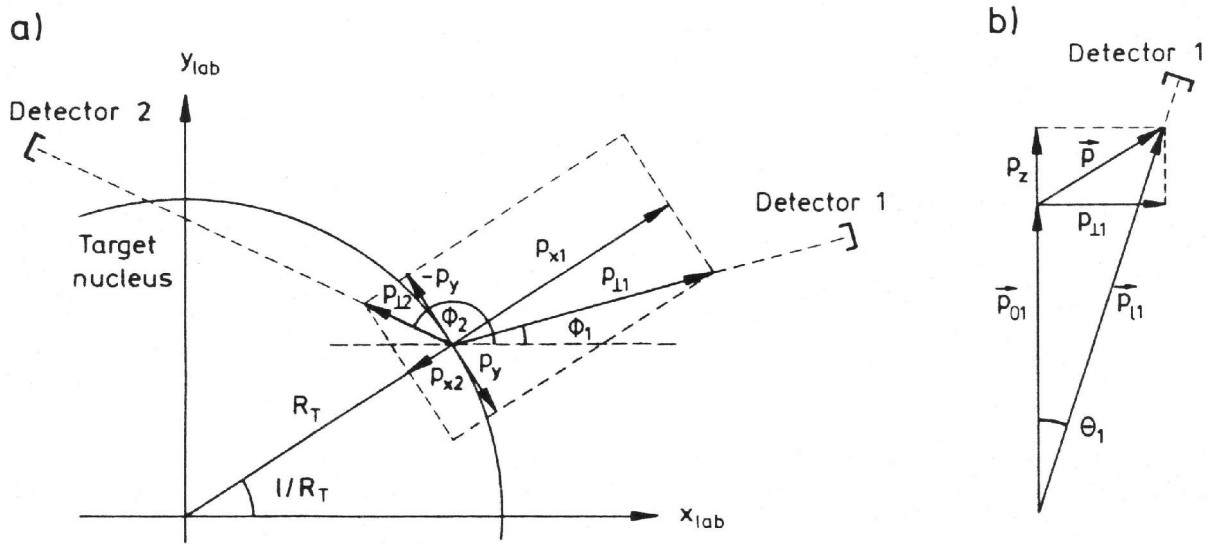

Figure 2: *Momenta of the break-up fragments* ***a)*** *in the p_x-p_y plane and* ***b)*** *in the p_z-$p_{\perp 1}$ plane [4].*

The quantities become clear in Fig. 2. Here, R_T is the radius of the target nucleus, $p_{\perp 1}$ the momentum component of fragment 1 perpendicular to the incoming beam, and $\boldsymbol{p}$ the relative momentum of the clusters. Furthermore, $\boldsymbol{p}_{l1}$ is the momentum of fragment 1 in the laboratory system, $\boldsymbol{p}_{01}$ the momentum of cluster 1 according to the projectile velocity, and l is the arc length along the surface of the target nucleus, defining the break-up location.

After calculating the appropriate quantum mechanical wave function for the break-up reaction [4], which is not repeated here, a coordinate transformation was necessary from the momentum components at the target circumference (Fig. 2a) to the quantities, measured in the laboratory system. This means a transformation of the five "coordinates" p_{x1}, p_{x2}, p_z, p_y, and l to the five coordinates θ_1, ϕ_1, θ_2, ϕ_2, and E_1. Therefore, we have to calculate the following Jacobian $\boldsymbol{J}_{op}$ for the "opaque" target nucleus [4]:

$$\boldsymbol{J_{op}} = \frac{\partial(p_{x1}, p_{x2}, p_z, p_y, l)}{\partial(\theta_1, \theta_2, E_1, \phi_1, \phi_2)} = \begin{vmatrix} \frac{\partial p_{x1}}{\partial \theta_1} & \frac{\partial p_{x2}}{\partial \theta_1} & \frac{\partial p_z}{\partial \theta_1} & \frac{\partial p_y}{\partial \theta_1} & \frac{\partial l}{\partial \theta_1} \\ \frac{\partial p_{x1}}{\partial \theta_2} & \frac{\partial p_{x2}}{\partial \theta_2} & \frac{\partial p_z}{\partial \theta_2} & \frac{\partial p_y}{\partial \theta_2} & \frac{\partial l}{\partial \theta_2} \\ \frac{\partial p_{x1}}{\partial E_1} & \frac{\partial p_{x2}}{\partial E_1} & \frac{\partial p_z}{\partial E_1} & \frac{\partial p_y}{\partial E_1} & \frac{\partial l}{\partial E_1} \\ \frac{\partial p_{x1}}{\partial \phi_1} & \frac{\partial p_{x2}}{\partial \phi_1} & \frac{\partial p_z}{\partial \phi_1} & \frac{\partial p_y}{\partial \phi_1} & \frac{\partial l}{\partial \phi_1} \\ \frac{\partial p_{x1}}{\partial \phi_2} & \frac{\partial p_{x2}}{\partial \phi_2} & \frac{\partial p_z}{\partial \phi_2} & \frac{\partial p_y}{\partial \phi_2} & \frac{\partial l}{\partial \phi_2} \end{vmatrix} \tag{1}$$

The five equations provided in Ref. [4], which connect the target and laboratory system, are:

$$p_{x1} = p_{\perp 1} \cos\left(\frac{l}{R_T} - \phi_1\right) \tag{2}$$

$$p_{x2} = p_{\perp 2} \cos\left(\frac{l}{R_T} - \phi_2\right) \tag{3}$$

$$p_{y1} = p_{\perp 1} \sin\left(\frac{l}{R_T} - \phi_1\right) \tag{4}$$

$$p_{y2} = -p_{y1} \tag{5}$$

$$p_{z1} = p_{l1} \cos\theta_1 - p_{01} \tag{6}$$

However, they cannot be applied directly to calculate the Jacobian. Because we have five laboratory coordinates, we need five independent variables that describe completely the kinematic situation at the target nucleus during break-up of the projectile. The five variables, already mentioned, and the corresponding five equations, which were not explicitly given in [4] but were used to calculate the Jacobian $\boldsymbol{J_{op}}$, are as follows:

$$p_{x1} = \pm\sqrt{p_{\perp 1}^2 - p_y^{\,2}} \tag{7}$$

$$p_{x2} = \pm\sqrt{p_{\perp 2}^2 - p_y^{\,2}} \tag{8}$$

$$p_z = p_{l1} \cos\theta_1 - p_{01} \tag{9}$$

$$p_y = \frac{\pm\, p_{\perp 1} p_{\perp 2} \sin(\phi_1 - \phi_2)}{\sqrt{p_{\perp 1}^2 + p_{\perp 2}^2 + 2 p_{\perp 1} p_{\perp 2} \cos(\phi_1 - \phi_2)}} \tag{10}$$

$$l = R_T\left(\phi_1 + \arcsin\frac{p_y}{p_{\perp 1}}\right) + \begin{cases} 0 & for \quad p_{x1} \geqslant 0 \\ \pi R_T & for \quad p_{x1} < 0 \end{cases} \tag{11}$$

Because $p_{y2} = -p_{y1}$, we write p_y; and, p_{z1} is renamed as p_z. With the equations

$$p_{\perp 1} = p_{l1} \sin\theta_1 \quad , \qquad p_{l1} = \sqrt{2 m_1 E_1} \quad , \tag{12a, b}$$

$$p_{\perp 2} = p_{l2} \sin\theta_2 \quad , \qquad p_{l2} = \sqrt{2 m_2 (E_p - E_1 - Q)} \quad , \tag{13a, b}$$

$$\text{and} \qquad p_{01} = (m_1/m_p) \sqrt{2 m_p (E_p - Q)} \tag{14}$$

it is possible to calculate all of the derivatives in Eq. (1). E_p is the projectile energy, Q the Q-value of the break-up reaction, and m_1, m_2, and m_p are the masses of fragments 1, 2, and the projectile. Equations (7)–(9) and (11) can be easily deduced from Fig. 2. To get p_y in Eq. (10), Eq. (2) has to be solved for l/R_T. Then, after replacing l/R_T in Eq. (3) and considering Eq. (5), Eq. (3) can be solved for p_y.[1]

In Eqs. (7)–(11) the momentum component p_y as well as $p_{\perp 1}$ and $p_{\perp 2}$ are intentionally not always resolved, because this simplifies the evaluation of the Jacobian. The double solutions ("±") correspond to the "near side" and "far side" contributions, respectively (see Figs. 4 and 6 in Ref. [4]). In Ref. [4], only the Jacobian for fragment 1 is determined. The solution for fragment 2 can be obtained by interchanging accordingly the parameters of both fragments. Therefore, Eqs. (7)–(11) should be sufficient to follow the calculations in Appendix A of Ref. [4]. This article does not yield any new results, but we hope that it helps anyone who wants to reproduce or check the calculations in [4].

For those who are interested in calculating the Jacobian $\boldsymbol{J_{op}}$ by themselves (it's a good exercise) before looking up the calculation method in [4], the final result should be:

$$\boldsymbol{J_{op}} = \frac{R_T \, p_{l1} \, p_{l2}^2 \, m_1 \sin\theta_1 \sin\theta_2 \cos\theta_2}{\sqrt{p_{\perp 1}^2 + p_{\perp 2}^2 + 2 \, p_{\perp 1} \, p_{\perp 2} \cos(\phi_1 - \phi_2)}} \tag{15}$$

Eq. (1), in which the given Jacobian corresponds to the energy spectrum of fragment 1, has to be evaluated by using Eqs. (7)–(13b). Although this can be done in a straightforward manner by determining the partial derivatives and finally the determinant, the calculation is still not easy.

[1] R. Serber: The Production of High Energy Neutrons by Stripping. **Phys. Rev. 72** (1947) 1008

[2] H. Jelitto, J. Buschmann, V. Corcalciuc, H. J. Gils, N. Heide, J. Kiener, H. Rebel, C. Samanta, S. Zagromski: Inclusive Measurements of the Break-up of 156 MeV ^{6}Li-Ions at Extreme Forward Angles. **Z. Phys. A332** (1989) 317-330 (full text)

[3] H. Utsunomiya: Breakup of 11 MeV/A-^{7}Li by ^{158}Tb. **Phys. Rev. C30** (1984) 1748

[4] V. Corcalciuc, H. Jelitto: Coincidence Cross Sections within the Quasi Free Break-up Model for Elastic Projectile Break-up. Report **KfK-4960**, Kernforschungszentrum Karlsruhe (1991), today: KIT (full text)

1 In this context, on p. 20 after Eq. (A.2) in Ref. [4], a sentence needs to be corrected. Instead of "From equations (3.2), (3.4), and (3.5) ...", it should be "From equations (3.2), (3.3), and (3.5) ...".